An analysis of self tuning Fuzzy PID-IMC for coupled water two tank system

Pankaj A. Valand, Amit Patel, Hetal Solanki

Abstract—**Nowadays the simplest and effective solutions to most of the control engineering applications are provided by PID controllers. However PID controllers are poorly tuned in practice with most of the tuning is done manually which is difficult and time consuming. This article comes up with new approaches of Fuzzy-IMC to design of PID controller for coupled water tank system at tank-1. The coupled water two tank system has limitations and it is difficult to control optimally using only PID controllers as the parameters of the system are changing constantly. The liquid level of two coupled water tank system is taken as an object. MATLAB software is used to design Fuzzy-IMC control at tank-1 then analyze the control effect. As a result, it is found that Fuzzy-IMC gives faster rise time, no overshoot, good steady quality with shorter adjusting time and smaller steady state error.**

Keywords-fuzzy self tuning PID-IMC, Mathematical model of coupled tank systemEvaluationary Algorithm

I. Introduction

Liquid level control is a typical representation of process control and is widely used in iron and steel, chemicals, petroleum and other industries. The control quality directly affects the quality of products and safety of the equipment's. However, the liquid level control system of water tank system is a large-lag, time-varying and non-linear complex system and is very difficult to control. Now, the liquid level control has been an active area in the process control over last decades and various different approaches has been devised.

PID controller is a generic control loop feedback mechanism widely used in industrial control systems. It calculates an error value as the difference between measured process variable and a desired set point [2]. The PID controller calculation involves three separate parameters P, I and D. The goal of PID controller tuning is to determine parameters that meet closed loop system performance specifications, and the robust performance of the control loop over a wide range of operating conditions should also be ensured. Practically, it is often difficult to simultaneously achieve all of these desirable qualities [9][11].In 'Self-tuning PID control structures', Multiple-model self-tuning PID controllers give a neat way of handling nonlinear or time varying systems [7]. Mary, P.M. et al. had given an improvement over the existing conventional fuzzy logic approach, based on a self-tuning fuzzy logic controller (FLC), for the design of a temperature control process, capable of providing optimal performance over the entire operating range of the process [18]. The proposed control system had the advantages of self-tuning FLC schemes. To evaluate the performance of the proposed control system methods, the results from the simulation of the process were presented [10]. An industrial interest in fuzzy logic control as evidenced by the many publications on the subject in the control literature has created an awareness of its interesting importance by the academic community[11-13].

II. MATHEMATICAL MODEL OF COUPLED TANK SYSTEM

In this the pump feeds into tank-1 and that tank-2 is not considered at all [1]. Therefore, the input to the process is the voltage to the pump and its output is the water level in tank-1.

A. EQUATION OF MOTION:

The out flow rate from tank-1, F_{01} can be expressed by:

$$F_{01} = A_{01}V_{01} \tag{1}$$

Where, cross-section area of tank-1 can be calculated by:

$$A_{01} = \frac{1}{4}\pi D_{01}^2 \tag{2}$$

The outflow velocity from tank-1 V_{01} can be expressed by, Bernoulli's equation for small orifices:

$$V_{01} = \sqrt{2}.\sqrt{g.L_{10}} \tag{3}$$

$$F_{01} = A_{01}.\sqrt{2}.\sqrt{g.L_{10}} \tag{4}$$

The first order differential equation in L_1 is given by:

$$At_1\left(\frac{d}{dt}\right)L_1 = F_{i1} - F_{01} \tag{5}$$

Where,

$$F_{i1} = K_pV_p. \tag{6}$$

Substituting in equation (5) F_{i1} and F_{01} with their expressions, and rearranging results in the following equation of motion for the tank-1 system:

$$\frac{d}{dt}L_1 = \frac{K_pV_p - A_{01}.\sqrt{2}.\sqrt{g.L_{10}}}{A_{t1}} \tag{7}$$

Due to the square root function applied to L_1, the first order differential equation expressed by equation (7) is non-linear.

B. NOMINAL PUMP VOLTAGE:

The nominal pump voltage V_{p0} for the pump-tank-1 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (V_{10}, L_{p0}) is characterized by the water in tank-1 being at a constant position level L_{10} due to the constant inflow rate generated by V_{p0}.

At equilibrium, all time derivative terms equate zero and equation (7) becomes:

$$K_p.V_{p0} - A_{01}.\sqrt{2}.\sqrt{g.L_{10}} = 0 \tag{8}$$

Where,
K_p= pump flow constant

Solving equation (8) for V_{p0}, gives the pump voltage at equilibrium. So,

$$V_{p0} = \frac{A_{01}.\sqrt{2}.\sqrt{g.L_{10}}}{K_p} \quad (9)$$

Using the system's specifications and the design requirements the result of the equation (9) is given by,

$$V_{p0} = 9.26\ [v] \quad (10)$$

C. *EQUATION OF MOTION LINEARIZATION AND TRANSFER FUNCTION:*

In order to design and implement a linear level controller for the tank-1 system, the Laplace open-loop transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the Equation of motion should be linearized around a quiescent point of operation.

In the case of the water level in tank-1, the operating range corresponds to small departure heights L_{11}, and small departure voltages V_{p1},from the desired equilibrium point(L_{10}, V_{p0}).

Therefore, L_1 andV_p can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11} \text{and} V_p = V_{p0} + V_{p1} \quad (11)$$

For a function, f, of two variables,L_1 andV_p, a first order approximation for small variations at a point $(L_1, V_p) = (L_{10}, V_{p0})$ is given by the following Taylor's series approximation:

$$f(L_1, V_p) = f(L_{10}, V_{p0}) + \left[\frac{d}{dL_1} f(L_{10}, V_{p0})\right](L_1 - L_{10}) + \left[\frac{d}{dV_p} f(L_{10}, V_{p0})\right](V_p - V_{p0}) \quad (12)$$

Now, equation (7) can be linearized as,

$$\frac{d}{dt} L1 = \frac{K_p V_{p0} - A_{01}.\sqrt{2}.\sqrt{g.L_{10}}}{A_{t1}} - \frac{1}{2}\frac{A_{01}.\sqrt{2}.g.L_{11}}{\sqrt{g.L_{10}}.A_{t1}} + \frac{K_p V_{p1}}{A_{t1}} \quad (13)$$

Substitute V_{p0} in equation (10) with its expression given in equation (9) results to the following linearized EOM for the tank-1 water level system:

$$\frac{d}{dt} L_{11} = -\frac{1}{2}\frac{A_{01}.\sqrt{2}.g.L_{11}}{\sqrt{g.L_{10}}.\ A_{t1}} + \frac{K_p V_{p1}}{A_{t1}} \quad (14)$$

Now, applying the Laplace transform to equation (11) and rearranging yields the desired open-loop transfer function for the coupled-Tank's tank-1 system, such that:

$$G_1(s) = \frac{L_{11}(s)}{V_{p1}(s)} \quad (15)$$

Therefore, by expressing the open-loop transfer functions DC gainK_{dc_1} and time constantτ_1, as functions of L_{10} and system parameters:

$$G_1(s) = \frac{K_{dc\_1}}{\tau_1 s + 1} \quad (16)$$

Where,

$$K_{dc_1} = \frac{K_p.\sqrt{2}.\sqrt{g.L_{10}}}{A_{01}.g} \quad (17)$$

$$\tau_1 = \frac{A_{t1}.\sqrt{2}.\sqrt{g.L_{10}}}{A_{01}.g} \quad (18)$$

Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, $G_1(s)$ is of type zero.

According to the system's parameters and the desired design requirements,

$$K_{dc\_1} = 3.2\ [\frac{v}{cm}] \quad (19)$$

$$\tau_1 = 15.2\ [s] \quad (20)$$

As a remark, it is obvious that linearized models, such as the coupled-Tank, tank 1's voltage-to-level transfer function are only approximate models. Therefore, they should be treated as such and used with approximate caution that is to say within the valid operating range and/or conditions.

The controller object is taken to the transfer function,

$$G(s) = \frac{K_{dc\_1}}{\tau_1 s + 1} \quad (21)$$

Hence,

$$G(s) = \frac{3.2}{15.2\ s+1} \quad (22)$$

III. EVALUTIONARY ALGORITHM

fuzzy self tuning pid-imc:

At firstly, the principle of fuzzy self-tuning PID is find out the fuzzy relationship between three parameters of PID (K_p, K_i and K_d) and error(e) and error changes(ec). Fuzzy inference engines modify three parameters to be content with the demands of the control system online through constantly checking e(e=r-y) and ec (ec=de/dt). Thus, the plant will have better dynamic and steady performance. The structure of fuzzy self-tuning PID is shown in fig. 1. [2]

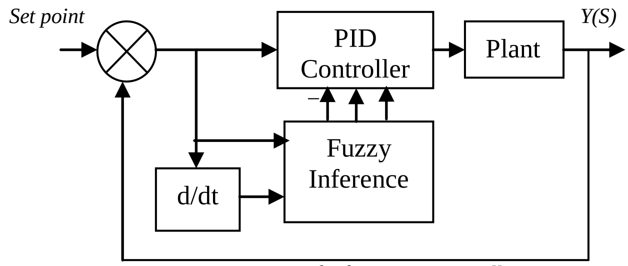

Fig. 1. Basic structure of a fuzzy-PID controller

The unity feedback controller can be realized by a PID controller with filter, and then the internal model control can be found approximately through parameter-tuning of PID controller.

a. Controller design procedure:

The fuzzy logic based self-tuning PID IMC design consists of the following steps.

1) Identification of input and output variables.

2) Construction of control rules.

3) Establishing the approach for describing system state in terms of fuzzy sets, i.e. establishing fuzzification method and fuzzy membership functions.

4) Selection of the compositional rule of inference.

5) De-fuzzification method, i.e., transformation of the fuzzy control statement into specific control actions.

The above steps are explained with reference to fuzzy logic based PID-IMC in the following sections.

b. Fuzzy Logic based self-tuning PID-IMC:

The structure of fuzzy self-tuning is already discussed in figure 1.

1) Selection of input and output variables

Define input and control variables, that is, determine which states of the process should be observed and which control actions are to be considered. Fuzzy self-tuning PID controller is adopted two input variables and three output variables. Taking e and ec as input variables and kp, ki, kd as output variables. The dynamic performance of the system could be evaluated by examining the response curve of these variables. The values of kp, ki and kd is taken as the output from the fuzzy logic controller and then further these values are utilized to next module of control system.

2) Selection of Membership Function

The number of linguistic variables describing the fuzzy subsets of a variable varies according to the application, which is usually an odd number. A reasonable number is seven. However, increasing the number of fuzzy subsets results in a corresponding increase in the number of rules. Each linguistic variable has its fuzzy membership function. The membership function maps the crisp values into fuzzy variables. The triangular membership functions are used to define the degree of membership which plays an important role in designing a fuzzy controller. Each of the input and output fuzzy variables is assigned seven linguistic fuzzy subsets varying from negative big (NB) to positive big (PB).The membership functions for all inputs and outputs are shown in fig. 2.

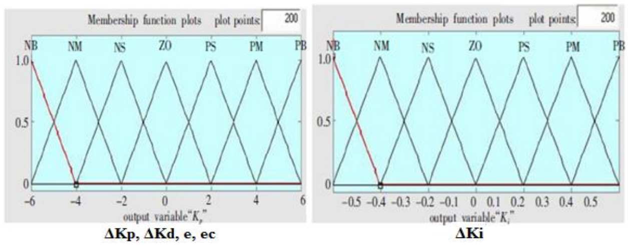

Fig. 2. Membership functions for fuzzy PID-IMC controller

3) Making Fuzzy rule base:

According to the knowledge of designing system, a set of rules which define the relation between the input and output of fuzzy controller can be found. The self-tuning rules are different according to different $e, ec, k_p, k_i and k_d$ which are defined using the linguistic variables. The two inputs, error and rate of change in error, result in 49 rules. The typical rules are:

Rule 1: If (e is NB) and (ec is NB) then (k_p is PB) (k_i is NB) (k_d is PS)

Rule 2: If (e is NB) and (ec is NM) then (k_p is PB) (k_i is NB) (k_d is NS)

Rule 3: If (e is NB) and (ec is NS) then (k_p is PM) (k_i is NM) (k_d is NB)

Rule 3: If (e is NB) and (ec is Z) then (k_p is PM) (k_i is NM) (k_d is NB)

And so on….

All the 49 rules governing the mechanism for each output are explained in Table-I.

TABLE I. RULES FOR FUZZY PID-IMC CONTROLLER

ΔK_p, ΔK_i, ΔK_d							
EC	**E**						
	NB	***NM***	***NS***	***Z***	***PS***	***PM***	***PB***
NB	PB/NB/PS	PB/NB/NS	PM/NM/NB	PM/NM/NB	PS/NS/NB	Z/Z/NM	Z/Z/PS
NM	PB/NB/PS	PB/NB/PS	PM/NM/NB	PS/NS/NM	PS/NS/NM	Z/Z/NS	NS/PS/Z
NS	PM/NM/Z	PM/NM/NS	PM/NM/NM	PS/NS/NM	Z/Z/NS	NS/PS/NS	NS/PS/Z
Z	PM/NM/Z	PM/NM/NS	PS/NS/NS	Z/Z/NS	NS/PS/NS	NM/PM/NS	NM/PM/Z
PS	PS/NS/Z	PS/NS/Z	Z/Z/Z	NS/PS/Z	NS/PS/Z	NM/PM/Z	NM/PB/Z
PM	PS/Z/PB	Z/Z/NS	NS/PS/PS	NM/PS/PS	NM/PM/PS	NM/PB/PS	NB/PB/PB
PB	Z/Z/PB	Z/Z/PM	NM/PS/PM	NM/PM/PM	NB/PB/PS	NB/PB/PS	NB/PB/PB

Using min-max inference, the activation of the i^{th} rule consequent is a scalar value which equals the minimum of the two antecedent conjuncts values. The knowledge required to generate the fuzzy rules can be derived from an offline simulation. Some knowledge can be based on the understanding of the behavior of the dynamic system under control. However, it has been noticed in practice that, for monotonic systems, a symmetrical rule table is very appropriate, although sometimes it may need slight adjustment based on the behavior of the specific system. If the system dynamics are not known or are highly nonlinear, trial-and-error procedures and experience play an important role in defining the rules.

4) Defuzzification

The input for the defuzzification process is a fuzzy set and the output is a single crisp number. As much as fuzziness helps the rule evaluation during the intermediate steps, the final desired output for each variable is generally a single number. However, the aggregate of a fuzzy set encompasses a range of output values, and so must be defuzzified in order to resolve a single output value from the set. The most popular defuzzification method is the centroid calculation, which returns the center of area under the curve and therefore is considered for defuzzification.

For the given system **MAMDANI** type of rule-base model is used.In the given fuzzy inference system, this work is done using centroid defuzzification principle. In this min implication together with the max aggregation operator is used.

IV. CASE STUDY

Here, I take a coupled water-tank system as a test system. The control algorithm is used to control the liquid level of the tank-1. For this, MATLAB software is used. The purpose of the coupled tank experiment is to design a control system that regulates the water level in a multiple coupled tank system. The controller can then track the liquid level to a desired trajectory. The coupled tank parameters are given below in Table-II:

TABLE II. PARAMETERS OF COUPLED WATER TANK SYSTEM

NAME	SYMBOL	VALUE	UNITS
		PUMP	
Flow constant	K_m	4.6	$(cm^3/sec)/volt$
Maximum Voltage	V_{max}	22	volts
Maximum Flow	F_{max}	100	cm^3/sec
Out1 Orifice Diameter	O_1	.635	cm
Out2 Orifice Diameter	O_2	.4763	cm
		TANK-1	
Height	L_{max}	30	cm
Inside Diameter		4.445	cm
Cross Section Area	A_1	15.5179	cm^2
Sensor Sensitivity		5	cm/volt
		TANK-2	
Height	L_{max}	30	cm
Inside Diameter		4.445	cm
Cross Section Area	A_2	15.5179	cm^2
Sensor Sensitivity		5	cm/volt
		OUTFLOW ORIFICES Diameters	
Small	D_1	0.3175	cm
Medium (Typical)	D_1& D_2	0.47625	cm
Large	D_2	0.555625	cm

V. RESULTS DISCUSSION

The FLC is applied to the plant. The simulation results are obtained using a 49 rule FLC. Rules shown in Rule Editor provide the control strategy. Here these rules are implemented to the above control system using IMC-PID. As expected, FLC provide good performance in terms of oscillations and overshoot in the absence of a prediction mechanism. The FLC algorithm adapts quickly to longer time delays and provides a stable response. To strictly limit the overshoot, using Fuzzy Control can achieve great control effect. Especially it can give more attention to various parameters, such as the time of response, the error of steadying and overshoot. It indicates that the fuzzy logic controller significantly reduced overshoot and steady state error.

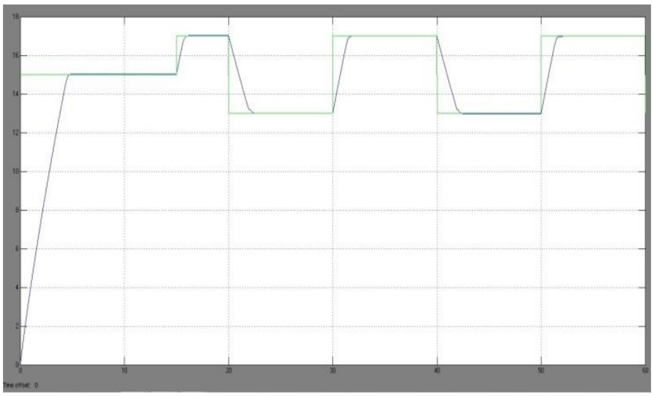

Fig. 3. Simulation of Fuzzy-PID-IMC controller

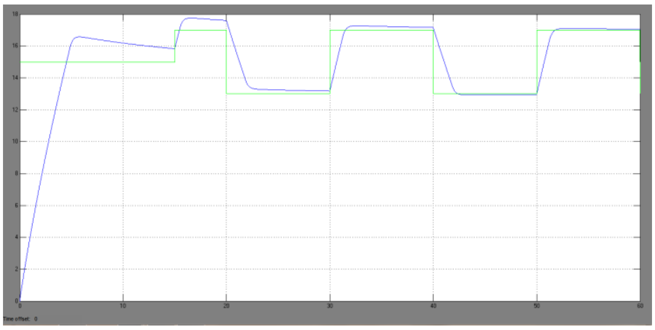

Fig. 4. Simulation of conventional PID controller

Results of Fuzzy-PID-IMC & Conventional PID areshown below:

i) PID PARAMETER:

	FUZZY PID-IMC	PID Controller
k_p	19	17
k_i	0.19	1.21
k_d	0	0.142

ii) RESULT ANALYSIS:

	FUZZY PID-IMC	PID Controller
Rise time	3.6 sec	4.6 sec
Peak overshoot	No overshoot	more
Settling Time	less	more

VI. Conclusion

In this work, design and tuning method for PID controller using fuzzy-IMC is proposed. A coupled two water tank system was taken as the control object (test-bench). Simulation was carried out using MATLAB to get the output response.

Simulation results shows that the fuzzy self-tuning PID-IMC gives smaller overshoot and faster rise time. The amount of overshoot for the output response was successfully decreased using the Fuzzy PID-IMC.

For the future perspective we can make design of an efficient fuzzy logic based self-tuning PID-IMC which involves the optimization of parameters of fuzzy sets and proper choice of rule base. There may be several techniques based on neural network and genetic algorithms to learn and optimize a fuzzy logic based controller parameters. Genetic-Fuzzy and Neuro-Fuzzy approaches may be able to learn rule base and identify membership function parameters accurately.

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