Literature Review on Modelling and Finite Element Analysis of Delamination of Composite

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***Abstract*— Laminated composite material have found extensive application in the mechanical, aerospace, marine, automotive industry due to their high fatigue life and high strength to weight ratio. Predication of the failure of composite laminate structure, mathematical modelling, finite element analysis of composite laminate and load that can laminate take have become an important topic of research and has drawn close attention in recent year. Accurate prediction of failure of composite laminate structure has become more challenging to design due to non-isotropic and non linear material property. This paper presents literature review on modelling and finite element analysis of composite laminate plate. The literature review is devoted to the different finite element method based on the various laminated plate theories.**

***Index Terms*— Composite laminated plate, Buckling and post buckling, Delamination, Finite Element Analysis .**

I. Introduction *(Heading 1)*

Composite laminated materials have been increasingly used in a various industrial area due to their long fatigue life, high stiffness, high strength to weight ratio, resistance to electrochemical corrosion and many more other material property. A proper understanding of their structural behaviour is required such as the deflection, buckling load, de-lamination and modal characteristics, the through thickness distribution of stress and strain [1]. Finite element method is especially versatile and efficient for the analysis of complex structural behaviour of the composite laminated structures. Using finite element method a significant amount of research has been devoted to the analysis of de-lamination, vibration, dynamics, buckling, post-buckling failure and damage analysis [15].

It is necessary to clarify some of the terminology that was included in this paper. Composite material are material made from two or more constituent material with significant different physical or chemical properties that when combined produce a material with characteristics different from the individual components. The individual components remain separate and distinct within finished structure. There are two main categories of constituent materials, matrix and reinforcement. The matrix material surrounds and supports the reinforcement materials by maintaining their relative position. The reinforcement imparts their special mechanical and physical properties to enhance the matrix properties [4].

Mechanical properties of a composite material depend on:

- Properties of constituent materials.
- Orientation of each layer.
- Volume fractions of each constituent.
- Thickness of each layer.
- Nature of bonding between adjacent layers[1].

Composite laminate s are assemblies of layer of fibrous composite materials which can be joined to provide required engineering properties including in-plane stiffness, bending stiffness, strength and coefficient of thermal expansion.. If ply with different material is used then resulting composite laminate is called hybrid composite laminate [16]. The mechanical properties of laminate are determined by its configuration, including fiber volume fraction and in-plane fiber distribution of each single ply and layup arrangement such as the number of plies and ply thickness and fiber orientation angle [8].

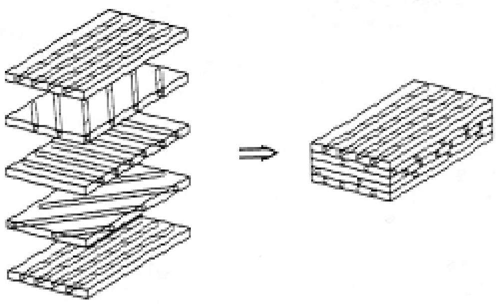


Fig. 1: composite laminate.

Under repeated or impact load significant shear stresses develop in between two adjacent ply due to the tendency of each layer to deform independently. These stresses are maximum at edge of a laminate and may cause de-lamination at such locations [2]. In laminated composite due to repeated cyclic stresses, impact load can cause layers to separate forming a mica like structure of separate layer with significant loss of mechanical toughness such failure mechanism of laminate is called de-lamination.

Causes of de-lamination [3]:

- Impact damage.
- Matrix micro cracking.
- Overload or fatigue.
- Inadequate bonding between layers.
- Environmental condition.

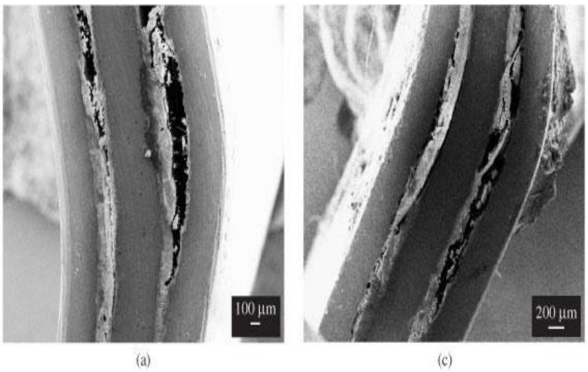


Fig.2: De-lamination in composite (plywood)

Such damage becomes an obstacle to the more extensive usage of composite material. Therefore the monitoring of internal or hidden damage in composite material is critical in engineering practices [17]. The effective damage monitoring for this kind of material or structure depends largely on the accurate prediction or estimation of mechanical or dynamic behaviours of both intact and damaged composite panels [18]. It is difficult to obtain accurate exact solutions for multi-layered panels having arbitrary lamination sequence and boundary conditions. This difficulty increases considerably when such structures are de-laminated. Thus computational approaches like finite element method play an important role in detecting damage for laminated composites. There are numerical and experimental investigations of delaminated multi-layered composites [14].

Ousset and poudolff [21] analyzed the de-laminated multi-layer composite plate based on Mindlin Reissner plat model. Sankar bv.[19] modelled a delaminated beam as two sub laminates by offsetting beam finite element. Gadelrab [23] discussed the modal variation of delaminated beam under different boundary conditions. Zak etal [20] and Ousset and Roudolff [26] developed models of finite element for beams and plates with boundary de-lamination. K.Alnefaie [23] analyzed finite element modelling of composite plates with internal de-lamination.

II. MATHEMATICAL MODELLING OF COMPOSITE LAMINATE:

Composite material is anisotropic material having the modulus E1 in the fiber direction will typically be larger than those in the transverse direction (E2 and E3) and properties in the plane transverse to the fiber direction to be isotropic to a good approximation (E2=E3) such a material is called transversely isotropic. The elastic constitutive law for such material becomes [3]

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma\gamma 12 \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -v_{21}/E_2 & 0 \\ -v_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \gamma_{12} \end{Bmatrix} \tag{1}$$

The parameter v_{12} is the principal Poisson's ratio of the strain induced in the direction-2 by a strain applied in the direction-1 and v_{21} gives the strain induced in the direction-1 by a strain applied in the direction -2. Since the direction-2 usually has much less stiffness than the direction-1 hence [5]

$$v_{12} > v_{21} \tag{2}$$

There are five constant in the above equation (E1, E2,v_{12}, v_{21} and G12) and only four of them are independent since the S matrix is symmetric therefore we have

$$\frac{v_{21}}{E_2} = \frac{v_{12}}{E_1} \tag{3}$$

The simple form of above eqn. with zeroes in the terms representing coupling between normal and shearing components is obtained only when the axes are aligned along the principal material direction but it is important to be able to transform the axes to and from the laboratory x-y frame to a natural material frame in which the axes might be labelled 1-2 corresponding to the fiber and transverse direction as shown in figure-3.

The transformation law for Cartesian Cauchy stress can be written as [11] :

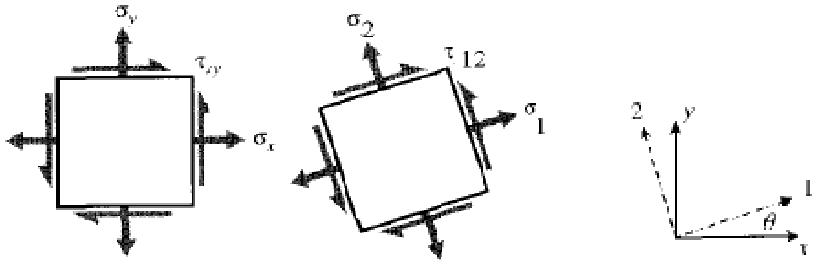


Fig.3: Rotation of axes.

$$\sigma_1 = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2r_{xy}\sin\theta\cos\theta \quad (4)$$
$$\sigma_2 = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2r_{xy}\sin\theta\cos\theta \quad (5)$$
$$r_{12} = (\sigma_y - \sigma_z)\sin\theta\cos\theta + r_{xy}(\cos^2\theta - \sin^2\theta) \quad (6)$$

Where Ɵ is the angle from the x axis to the 1 (fiber) axis. These relations can be written in matrix form as

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ r_{12}\end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc\\ s^2 & c^2 & -2sc\\ -sc & sc & c^2-s^2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ r_{xy}\end{Bmatrix} \quad (7)$$

Where $c = \cos\theta$ and $s = \sin\theta$. This can be abbreviated as

$$\sigma' = A\sigma \quad (8)$$

Where A is the transformation matrix and particular form of A given in eqn. is valid in two dimensions and for Cartesian coordinates. Using mathematical arguments it can be shown that the components of infinitesimal strain transform by almost the same relations.[9]

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_{12}\end{Bmatrix} = A\begin{Bmatrix}\varepsilon_z\\ \varepsilon_y\\ \frac{1}{2}\gamma_{xy}\end{Bmatrix} \quad (9)$$

The factor of $\frac{1}{2}$ on shear components arises from the classical definition of shear strain which is twice the tensorial shear strain. This factor introduces some awkwardness in to the transformation relations, which can be reduced by introducing the Reuter's matrix as [7]

$$[R] = \begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix} \quad or \quad [R]^{-1} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&\frac{1}{2}\end{bmatrix} \quad (10)$$

We can now write:

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} = R\begin{Bmatrix}Z\begin{matrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_{12}\end{matrix}\end{Bmatrix} = RA\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \frac{1}{2}\gamma_{xy}\end{Bmatrix} = RAR^{-1}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \text{Or } \varepsilon' = RAR^{-1}\varepsilon \text{ [10]} \quad (11)$$

The transformation law for compliance can now be developed from the transformation laws for strain and stress. The final grouping of transformation matrices relating the x-y strains to the x-y stresses is then the transformed compliance matrix in the x-y direction.

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = R\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \frac{1}{2}\gamma_{xy}\end{Bmatrix} = RA^{-1}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_{12}\end{Bmatrix} = RA^{-1}R^{-1}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} = RA^{-1}R^{-1}S\begin{Bmatrix}\sigma_1\\ \sigma_2\\ r_{12}\end{Bmatrix} = RA^{-1}R^{-1}S\begin{Bmatrix}\sigma_1\\ \sigma_2\\ r_{12}\end{Bmatrix}$$

$$= RA^{-1}R^{-1}SA\begin{Bmatrix}\sigma_x\\ \sigma_y\\ r_{xy}\end{Bmatrix} = \bar{S}\begin{Bmatrix}\sigma_z\\ \sigma_y\\ r_{xy}\end{Bmatrix} \quad (12)$$

Where $\bar{S}$ is the transformed compliance matrix relative to x-y axes. The inverse of S is D that is stiffness matrix relative to x-y axes.

$$\bar{S} = RA^{-1}R^{-1}SA, \bar{D} = \bar{S}^{-1} \quad (13)$$

A.1 Formulation for laminated composite plate:

Fig.4 shows a typical composite laminate with thin ply with thickness 0.2mm. the orientation of each ply is arbitrary and the layup sequence is tailored to achieve the properties desired of the laminate. In this section we outline how such laminates are designed and analyzed.

"Classical laminate theory" is an extension of the theory for bending of homogeneous plates but with an allowance for in-plane tractions in addition to bending moments and for the varying stiffness of each ply in the analysis. In general cases, the determination of the tractions and moments at a given location will require a solution of the general equation for equilibrium and displacement compatibility of plates. This theory is treated in a number of standard texts and will not be discussed here [7]. We begin by assuming a knowledge of the traction N and moment M applied to a plate at a position x, y as shown in fig. 5

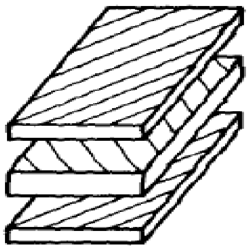


Fig.4: Typical composite laminate.

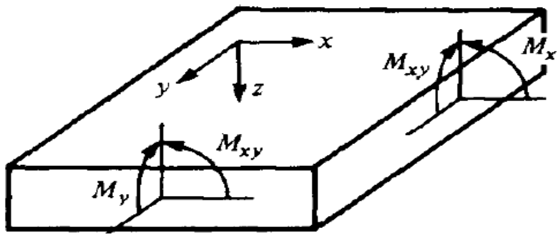


Fig.5: Applied moments in plate bending.

$$N = \begin{Bmatrix} N_z \\ N_y \\ N_{xy} \end{Bmatrix} \quad M = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \qquad (14)$$

Co-ordinate x and y are the direction in the plane of the plate and z is customarily taken as positive downward. The reflection in the z direction is termed w and also taken as positive downward.

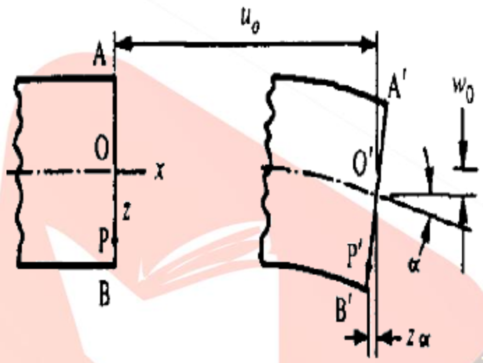


Fig.6: Displacement of a point in a plate.

Analogously with the Euler assumption for beam, the Kirchhoff assumption for plate ending takes initially straight vertical line to remain straight but rotate around the mid-plane (z=0). As shown in fig.6 the horizontal displacement u and v in the x and y direction due to rotation can be taken to a reasonable approximation from the rotation angle and distance from mid-plane and this rotational displacement is added to the mid-plane displacement (u_0 , v_0).

$$u = u_0 - (z\, w_0\, x)$$
$$v = v_0 - (z\, w_0\, y) \; [13] \qquad (15)$$

The strains are just the gradients of the displacement using matrix notation these can be written,

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} - z w_{0,xx} \\ v_{0,y} - z w_{0,yy} \\ (u_{0,y} + v_{0,x}) - 2z w_{0,xy} \end{Bmatrix} = \varepsilon^0 + zk \qquad (16)$$

Where, ε^0 is the mid-plane strain and K is the vector of second derivatives of the displacement called the curvature.

$$k = \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix} \qquad (17)$$

The component k_{xy} is a twisting curvature stating how the x- direction mid-plane slop changes with y. The stress relative to each x-y axes are now determined from the strain and this must take consideration that each ply will in general have a different stiffness depending on its own properties and also its orientation with respect to the x-y axes. The properties of each ply must be transformed to common x-y axes chosen arbitrarily for the entire laminate. The stresses at any vertical position are then,

$$\sigma = \bar{D}\epsilon = \bar{D}\epsilon^0 + \bar{D}\, k \; [6] \qquad (18)$$

Where $\bar{D}$ is the transformed stiffness of the ply at the position at which the stresses are being computed.
Each of these ply stresses must add to balance the traction per unit width N,

$$N = \int_{\frac{-h}{2}}^{\frac{+h}{2}} \sigma\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \sigma_k\, dz \qquad (19)$$

Where σ_k is the stress in the K^{th} ply and Z_k is the distance from the laminate mid-plane to the bottom of the K^{th} ply.

$$N = \sum_{k=1}^{N} \left(\int_{z_k}^{z_{k+1}} \bar{D}\ \epsilon^0\, dz + \int_{z_k}^{z_{k+1}} \bar{D}\ kz\, dz \right) \qquad (20)$$

The curvature K and mid-plane strain ε^0 are constant throughout Z and transformed stiffness $\bar{D}$ does not change within a given ply.

$$N = \sum_{k=1}^{N}\left(\bar{D}\,\epsilon^0 \int_{z_k}^{z_{k+1}} dz + \bar{D}k \int_{z_k}^{z_{k+1}} z\,dz\right) \quad (21)$$

After evaluating this expression can be written in the compact form:

$$N = A\epsilon^0 + Bk \quad (22)$$

Where A is an “extensional stiffness” matrix defined as,

$$A = \sum_{k=1}^{N} \bar{D}\,(z_{k+1} - z_k) \quad (23)$$

and B is a ”coupling matrix” defined as

$$B = \frac{1}{2}\sum_{k=1}^{N} \bar{D}\,(z_{k+1}^2 - z_k^2) \quad (24)$$

The A matrix gives the influence of an extensional mid-plane ε^0 on the in-plane traction N and the B matrix gives the contribution of curvature K to the traction. It may not be obvious why bending the plate will require an in-plane traction or conversely why pulling the plate in its plane will cause it to bend but visualize the plate containing plies all of the same stiffness except for some very low modulus plies somewhere above its mid plane. When the plate is pulled the more compliant plies above the mid-plane will tend to stretch more than the stiffer plies below the mid-plane. The top half of the laminate stretches more than the bottom half so it taken on a concave downward curvature [7, 13].

Similarly, the moment resultant per unit width must be balanced by the moment contributed by the internal stresses.

$$M = \int_{-h/2}^{+h/2} \sigma z dz = B\epsilon^0 + Dk \quad (25)$$

Where D is a “bending stiffness matrix” defined as

$$D = \frac{1}{3}\sum_{k=1}^{N} \bar{D}(z_{k+1}^3 - z_k^3) \quad (26)$$

The complete set of relation between applied force and moment and the resulting mid-plane strain and curvature can be summarized as,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{Bmatrix} \epsilon^0 \\ k \end{Bmatrix} \text{[13]} \quad (27)$$

The presence of non - zero element in the coupling matrix B indicates that the application of an in-plane traction will lead to warping of plate and applied bending moment will also generate an extensional strain. These effect are usually undesirable and they can be avoided by making the laminate symmetric about the mid-plane.[12]. If desired the individual ply stresses can be used in a suitable failure criterion to assess the likelihood of the ply failing. The Tsai-Hill criterion is popularly used for this purpose.

$$\left(\frac{\sigma_1}{\hat{\sigma}_1}\right)^2 - \frac{\sigma_1\sigma_2}{\hat{\sigma}_1^2} + \left(\frac{\sigma_2}{\hat{\sigma}_2}\right)^2 + \left(\frac{\tau_{12}}{\hat{\tau}_{12}}\right)^2 = 1 \quad (28)$$

This criterion predicts failure (de-lamination in composite laminate) whenever the left hand side of the above equation equal or exceeds unity. The above relation provide a straightforward path to determining stress and displacement in laminated composites subjected to in plane traction.[12]

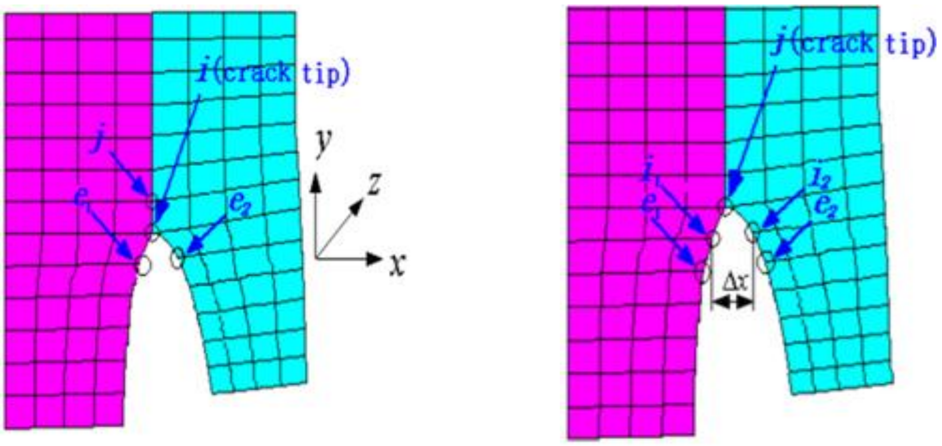
III. FINITE ELEMENT ANALYSIS OF COMPOSITE LAMINATE USING VIRTUAL CRACK CLOSURE TECHNIQUE

Buckling and post-buckling are special failure mode of composite laminates with multiple interlaminar de-laminations under compressive load. This virtual crack closure technique is detail explained in Finite element analysis of post-buckling and de-lamination of composite laminates using virtual crack closure technique by P.F. Liu a, S.J. Hou a, J.K. Chu a, X.Y. Hub, C.L. Zhou a, Y.L. Liu a, J.Y. Zheng a A. Zhao a, L. Yana Composite Structures 93 (2011) 1549–1560.

In general this type of failure mode can be divided in to two categories local buckling and global buckling [24]. The sub-laminates under compressive load may locally buckling and impose the additional bending stress on the neighbouring sub-laminates which may lead to the failure of remaining sub-laminates. The global buckling for composite laminates with through the width de-lamination appears before the immediate unstable de-lamination process with increasing compressive strain. Chai et al [25] established the one dimensional and two dimensional de-lamination buckling models by evaluating the crack tip energy release rate (ERR) which was defined as the drive force for crack propagation. Now a day the ERR as atypical fracture parameter is widely used to predict the de-lamination crack propagation.

The ERR can be efficiently solved by the virtual crack closure technique (VCCT) which was proposed by rybicki and kannien [26] based on the lrwin’s crack tip energy analysis [27]. The sole assumption of VCCT is the energy required for the crack propagation length Δa is equal to that for closing two separate crack surface with crack lengthΔa. Krueger [28] gave a full scale overview on the VCCT in terms of the solid shell element approach the calculation formula for the 2D and 3D problems the modified VCCT with geometrically non-linear FEA and the de-lamination growth behaviour for dissimilar materials. Gaudenzi et al [29] explored the non-linear behaviour of de-laminated composite panels under compressive load using an incremental continuation method and modified VCCT.

As the FEA is associated with the VCCT fig.7 show crack propagation from the crack tip node i to j with the increment crack lengthΔa. Node i is separated in to two nodes i_1 and i_2 after crack propagation. For node i the relative displacement in three directions (x, y, z) are Δu_{ix}, Δu_{iy}, Δu_{iz} after propagation and node force before propagation are F_{ix}, F_{iy}, F_{iz}. The total ERR due to crack propagation is expressed as



(7.a) (7.b)

Fig.7:.Schematic representation of crack propagation between composite layer (a) before propagation and (b) after propagation.

$$G = G_1+G_2+G_3 = \lim_{\Delta a \to 0} \frac{1}{2S} \int_0^{\Delta a} F_{ix}(a)\Delta U_{ix}(a)da + \int_0^{\Delta a} F_{iy}(a)\Delta U_{iy}(a)da + \int_0^{\Delta a} F_{iz}(a)\Delta U_{iz}(a)da \quad (29)$$

Where G_1, G_2, G_3 are ERR for the mode-1, mode-2, mode-3 and S is the new area generated due to a crack propagation length Δa.

Assume a two-step method is used based on the node force before crack propagation and node relative displacement after crack propagation the ERR G due to crack propagation on the crack closure surface is calculated as [28]

$$G = \frac{1}{2S}\left[F_{ix}\Delta U_{ix} + F_{iy}\Delta U_{iy} + F_{iz}\Delta U_{iz}\right] \quad (30)$$

Often, the two-step method can be approximately substituted by the one step method if the mesh size on the crack closure surface is sufficiently small where the relative displacement after crack propagation can be substituted by the relative displacement between nearest node pair before crack propagation [30]. For example the relative displacement in three direction for the node pair i_1 and i_2 can be approximately substituted by those for the node pair e_1 and e_2. In this analysis the one step method is used for the calculation of ERR.

In the FEA the node bonding technique is used to simulate crack propagation which divides the node pair at the same position in to node by releasing the coupling freedom degree if the following crack propagation criterion is satisfied.

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$$G_{equ}/ G_{equc} \geq 1 \quad (31)$$

Where G_{equ} and G_{equc} are the equivalent and critical ERR respectively.

Currently three typical crack propagation criteria are used

(1) B-K law:

$$G_{equc} = G_{IC} + (G_{2c} - G_{1c})\left(\frac{G_2+G_3}{G_1+G_2+G_3}\right)^{\eta} \quad (32)$$

(2) Power law :

$$\frac{G_{equ}}{G_{equc}} = \left(\frac{G_1}{G_1C}\right)^{am} + \left(\frac{G_2}{G_2C}\right)^{an} + \left(\frac{G_3}{G_3C}\right)^{a0} \quad (33)$$

(3) Reeder law:

$$G_{equc} = G_{1c} + (G_{2c} - G_{1c})\left(\frac{G_2+G_3}{G_1+G_2+G_3}\right)^{\eta} + (G_{3c} - G_{2c})\left(\frac{G_3}{G_2+G_3}\right)\left(\frac{G_2+G_3}{G_1+G_2+G_3}\right)^{\eta} \quad (34)$$

In the buckling analysis the multi-point constraint is used to tie the node at the same position on de-lamination surface. The buckling analysis is performed using the subspace iterative method from which the Eigen value of buckling modes provides an initial imperfection for the post-buckling analysis. The post-buckling analysis employs the master slave node technique in which the slave nodes on the slave surface are to be de-bonded from those on the master surface. The de-lamination of composite laminate is simulated by releasing the tied node pair at the crack tip front in to two separate nodes if a specified fracture criterion f reaches 1.0 within a given tolerance F_{tol}. In general the B-K failure criterion for crack propagation is used.

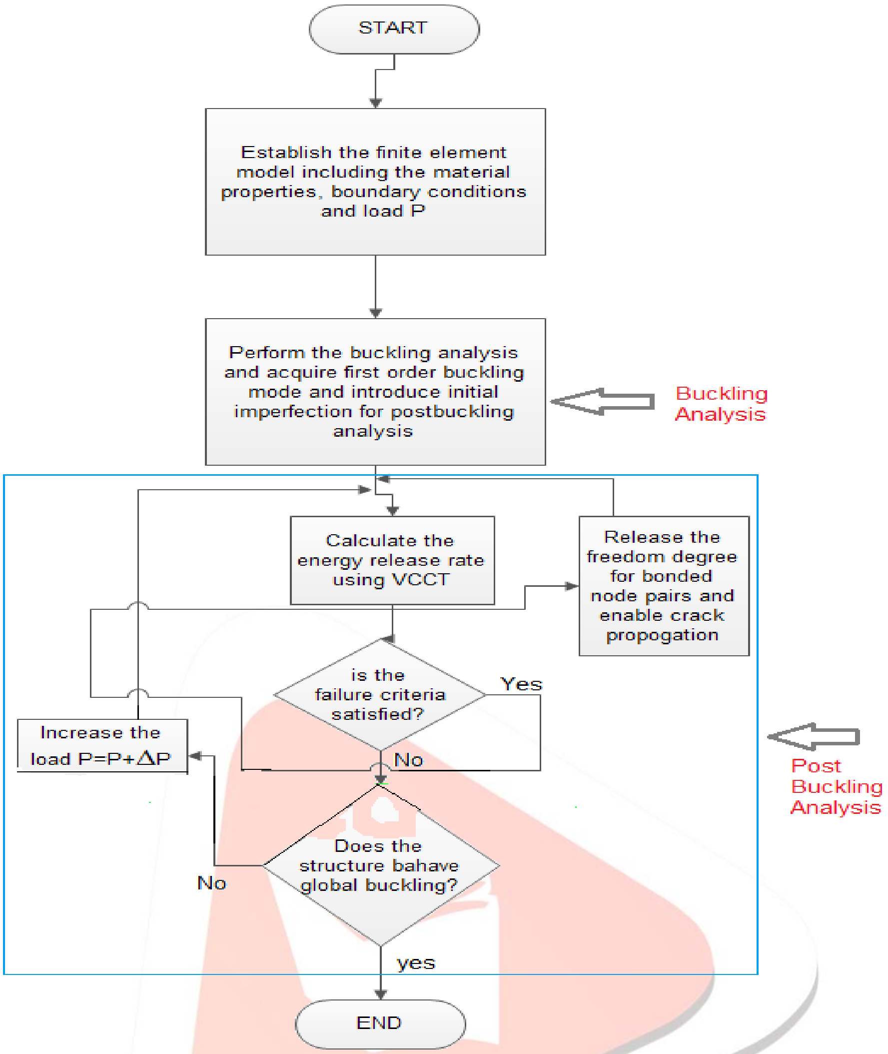

Chart no.1: FEA Flowchart using VCCT

IV GEOMETRIC NONLINEAR FINITE ELEMENT ANALYSIS OF LAMINATED COMPOSITE PLATES

For accurate prediction for the static structural responses of composite laminated plates, geometric nonlinearity should be included in the finite element analysis. Some literatures on the geometric nonlinear finite element analysis of laminated composite plates existed. A procedure for the reliability analysis of laminated composite plate structures with large rotations but moderate deformation under random static loads was presented via a co-rotational total Lagrangian finite element formulation which was based on the von Karman assumption and first-order shear deformation theory [32].

An eight-node C0 membrane-plate quadrilateral finite element-based on the Reissner–Mindlin plate theory was presented to analyse moderately large deflection, static and dynamic problems of moderately thick laminates including buckling analysis and membrane-plate coupling effects [33]. The hierarchical finite element method to carry out the geometrically nonlinear analysis of laminated composite rectangular plates. Based on the first-order shear deformation theory and Timoshenko's laminated composite beam functions, the current authors developed a unified formulation of a simple displacement-based 3-node, 18- degree-of-freedom flat triangular plate/shell element [34] and two simple, accurate, shear-flexible displacement based 4-node quadrilateral elements and for linear and geometrically nonlinear analysis of thin to moderately thick laminated composite plates. The deflection and rotation functions of the element boundary were obtained from Timoshenko's laminated composite beam functions. Based on a higher-order shear deformation theory involving four dependent unknowns and satisfying the vanishing of transverse shear stresses at the top and bottom surfaces of the plate, geometrically nonlinear flexural response characteristics of shear deformable un-symmetrically laminated rectangular plates were investigated using a four-node rectangular C1 continuous finite element having 14 degrees of freedom per node [35].

A high-order plate model which exactly ensured both the continuity conditions for displacements and transverse shear stresses at the interfaces between layers of a laminated structure, and the boundary conditions at the upper and lower surfaces of the plates was used to study the geometrically nonlinear behaviour of multi-layered plates [36], and based on this refined plate model, a six-nodeC1 conforming displacement-based triangular finite element was developed, with the Argyris interpolation used for transverse displacement, the Ganev interpolation used for membrane displacements and transverse shear rotations, and the transverse shear strain distributions represented by cosine functions. A three-dimensional element with two-dimensional kinematic constraints was developed for the geometric nonlinear analysis of laminated composite plates [37] using a total Lagrangian description and the principle of virtual displacements. The large deformation analysis of circular composite laminated plates [38] was studied using a 48- DOF four-node quadrilateral laminated composite shell finite element.

V. BUCKLING AND POST BUCKLING ANALYSIS OF LAMINATED COMPOSITE PLATES

An assumed hybrid-stress finite element model together with a composite multilayer element were developed to study the buckling of generally laminated composite An assumed hybrid-stress finite element model together with a composite multilayer element were developed to study the buckling of generally laminated composite plates with arbitrary thickness and edge conditions under an in plane stress system [39]. The equilibrium conditions within each layer, the interlaminar traction reciprocity conditions, and the stress-free boundary conditions on the top and bottom surfaces of the laminate, were satisfied by the assumed stress field and thus the composite shear correction factors were not required. A shear deformable finite element was developed for the buckling analysis of laminated composite plates based on Mindlin's theory in which shear correction factors were derived from the exact expressions for orthotropic materials [40].

The effects of material properties, plate aspect ratio, length-to-thickness ratio, number of layers and lamination angle on the buckling loads of symmetrically and anti-symmetrically laminated composite plates were investigated. An 8-node isoparametric plate finite element with 5-DOF per node was developed based on the first-order shear deformation theory associated with von Karman's nonlinear strain–displacement relationships to investigate the buckling and post-buckling of moderately thick laminated plates subjected to uni or bi-axial compression [41]. The effects of boundary conditions, aspect ratio, side to thickness ratio and lay-up sequence on the buckling and post-buckling behaviour were studied in detail. The linear buckling analysis of multi-laminated composite plate-shell structures was analysed using a discrete finite element model based on an eight-node isoparametric element with 10 degrees of freedom per node and the higher-order theory [42].

The geometric stiffness matrix was developed taking into consideration the effects of the higher-order terms on the initial in-plane and transverse shear stresses. The element was then used to study the buckling and free vibrations of multi-laminated structures of arbitrary geometry and lay-up [43].

A generalized layer-wise stochastic finite element formulation was developed for the buckling analysis of both homogeneous and laminated plates with random material properties [44]. The pre-buckled stresses were considered in the derivation of geometric stiffness matrix and the effect of variation in these stresses on the mean and coefficient of variation of buckling strength was studied. The post buckling behaviour of laminated composite plates under the combination of in-plane shear, compression and lateral loading was investigated using an element- based Lagrangian formulation based on the assumed natural strain method for composite structures [44]. Natural coordinate-based strains, stresses and constitutive equations were used in the element and the element based Lagrangian formulation was computational efficient and had the ability to avoid both membrane and shear locking.

Effects of material nonlinearity on buckling and post-buckling behaviour of composite laminated plates:

In the literature, most stability studies of composite laminated plates have been limited to the geometrically nonlinear analysis and the research on the effect of nonlinear effective constitutive material properties on composite structural buckling and post buckling responses has been very limited. The nonlinearity of in-plane shear is significant for composite materials [45]. With the nonlinear composite constitutive properties, a few attempts have been made to study buckling of thin composite laminate panels and post-buckling of thick-section composite laminate plates. The influence of in- plane shear nonlinearity on buckling and post-buckling responses of composite plates under uniaxial compression and biaxial compression and of shells under end compression and hygrostatic compression. They also investigated the nonlinear buckling of simply-supported composite plates under uniaxial compression, and of composite laminate skew plates under uniaxial compressive loads. The effect of material nonlinearity on buckling and post-buckling of fibre composite laminate plates and shells subjected to general mechanical loading, together with the interaction between the material and geometric nonlinearity was investigated and it was concluded that the composite material nonlinearity had significant effects on the geometrically nonlinearity, structural buckling load, post-buckling structural stiffness, and structural failure mode shape of composite laminate plates and shells.

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