

# Parameter Based Transient Response Analysis for Pendulum System Using LabVIEW

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**Abstract**— This paper introduce useful concept for transient response analysis for pendulum system using graphical programming language - LabVIEW. Transient response analysis is the most general method for computing forced dynamic response. The purpose of transient response analysis is to compute the behavior of a structure subjected to time varying excitation. Once the virtual state space model of pendulum system is obtained it is very easy to analyze and identify the parameters of transient response of pendulum system.

**Index terms** – Pendulum State-Space Model and Transfer Function, Transient Response and Stability Analysis, LabVIEW.

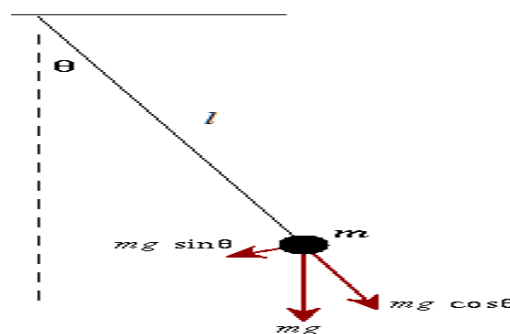
## I. INTRODUCTION

There are many methods already introduced for pendulum system analysis but the pendulum system analysis with the use of LabVIEW is better method to find the Transient response parameters of the pendulum system [1, 5, 7, 8]. Once the state space model of pendulum system has been derived then it can be converted into the equivalent transfer function using LabVIEW. Transfer function is the ratio of the Laplace output to the Laplace input of the system assuming all the initial conditions set to Zero [7, 8]. With the use of the Control System Toolbox in LabVIEW all the transient response parameters can be determined; those parameters help to identify the transient response of the pendulum system. Transient response is very useful to find the behavior of the system. With the use of the LabVIEW control system tool box transient system parameters such as peak overshoot, Delay Time, Settling Time, Rise Time and Frequency parameters Such as phase margin ,gain margin, phase cross over frequency ,gain cross over frequency can be determined[7, 8].LabVIEW represents a resourceful tool for the development and analysis of control system. The graphical user programming language allows programming without special experience because it uses terminology, icons and theories familiar to scientists and engineers [5].

A simple pendulum may be described ideally as a point mass suspended by a massless string from some point about which it is allowed to swing back and forth in a plane. A simple pendulum can be approximated by a small metal sphere which has a small radius and a large mass when compared relatively to the length and mass of the light string from which it is suspended [7, 8, 9]. If a pendulum is set in motion so that it swings back and forth, its motion will be periodic. The time that it takes to make one complete oscillation is defined as the period  $T$ . Another useful quantity used to describe periodic motion is the frequency of oscillation. The frequency  $f$  of the oscillations is the number of oscillations that occur per unit time and is the inverse of the period,  $f = 1/T$ . Similarly, the period is the inverse of the frequency,  $T = 1/f$ . The maximum distance that the mass is displaced from its equilibrium position is denoted as the amplitude of the oscillation. Once the state space model of the pendulum has been derived with the use of the LabVIEW all the parameters of pendulum system can be determined [5]. State space model of pendulum is the input parameter of this system then all the parameters can be determined. [5, 7, 8]

## II. PENDULUM SYSTEM

A simple pendulum[9] may be described ideally as a point mass suspended by a massless string from some point about which it is allowed to swing back and forth in a plane as shown in Fig.1 [8, 9]



**Fig.1 Pendulum System**

Parameters of Virtual Pendulum System [8, 9]:

- Length of rod (l) = 0.495 Meter
- Mass of system (m) = 0.43 Kg
- $g = 9.81 \text{ Meter / Sec}^2$
- Spring Constant (k) = 0.00035
- Torque (u) = 1 N/M<sup>2</sup>

**State Space Analysis:**

A state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors. [3, 4]

The linearized state-space equation around operating point can be written as shown below [3, 4]:

$$\left. \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \right\} \quad (1)$$

Where,

- A = state transition matrix
- B = Input matrix
- C = Output matrix
- D = Feed forward matrix

**State Model of the Pendulum System [7, 8, 9]**

- X<sub>1</sub>      Position of ball (  $\theta$  )
- X<sub>2</sub>      Velocity of system (  $\dot{\theta}$  )
- Output(y):    Position of Pendulum
- Input (u):      Unit step of Torque

As per System dynamics under impulsive torque, the state Equations can be written as,

$$\left. \begin{aligned} \ddot{\theta}(t) &= -(g/l)\sin\theta(t) - (k/m)\dot{\theta}(t) + u \\ ml\ddot{\theta}(t) &= -mg\sin\theta(t) - kl\dot{\theta}(t) + u \end{aligned} \right\} \quad (2)$$

Let us define the state variable as,

$$x_1 = \theta, x_2 = \dot{\theta}, \dot{x}_1 = \dot{\theta}, \dot{x}_2 = \ddot{\theta}$$

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(g/l)\sin x_1 - (k/m)x_2 \\ y &= x_1 \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -g/l & -k/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= x_1 \end{aligned} \right\} \quad (4)$$

Considering the pendulum system parameters state-space model as below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -19.81 & -0.00081 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(5)

From mentioned state-space model system matrices A, B, C and D can be written as mention below:

$$A = \begin{bmatrix} 0 & 1 \\ -19.81 & -0.00081 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

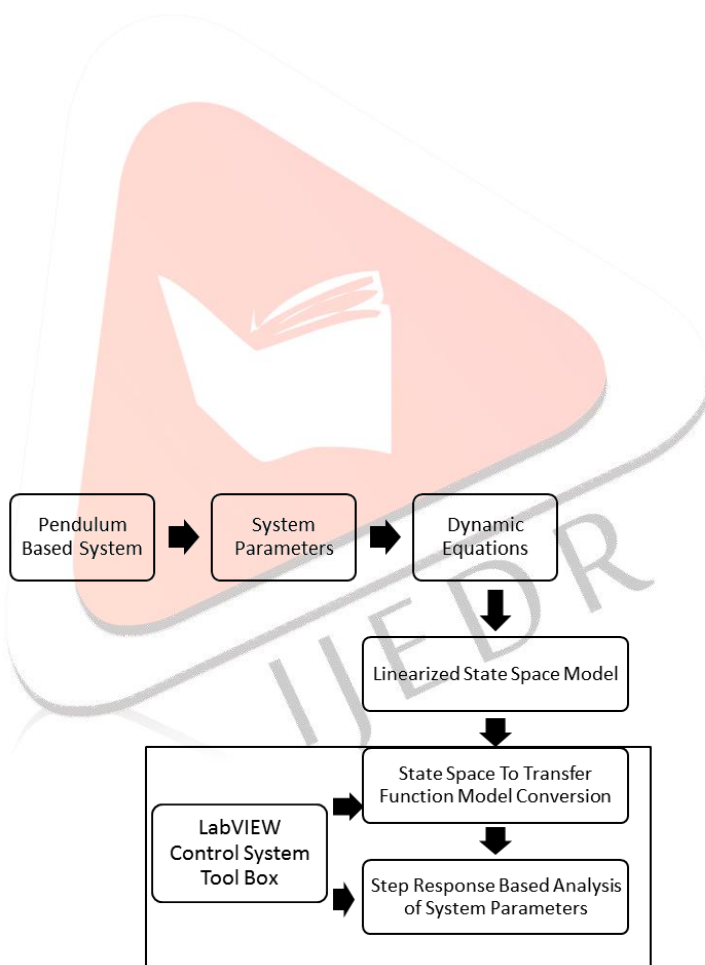
$$C = [10]$$

$$D = 0$$

Where,

- A = System Matrix
- B = Input Matrix
- C = Output Matrix
- D = Feed forward Matrix

**Flow Chart of Entire Procedure**



**Fig.2 Functional Block diagram of Entire Procedure**

In this Figure2 Represents entire block diagram of the Procedure. First System parameters and dynamic equations of the system have been derived, with the use of it Linearized State Model of the System has been derived, and that State Model can be converted into the equivalent transfer Function Model using Control System Tool Box LabVIEW [5].

Following System Parameters are being determined using LabVIEW Control System Tool Box [4, 5]:

- 1) **Delay Time:** it is time required for the response to reach 50% of its final value in the first attempt.
- 2) **Rise Time:** the time required for the response to reach 10% to 90% of the final value for over damped system.
- 3) **Settling Time:** it is the time required for the response curve to reach and stay within a specified percentage of the final value.
- 4) **Peak Time:** it is the time required for the response to reach the first peak.
- 5) **Phase Margin:** it is measure of relative stability. The phase margin indicates how much the system angle can be increased to cause system to become unstable from a stable condition.
- 6) **Gain Margin:** the gain margin indicates how much gain can be increased to cause system instability.

These are the basic parameters of the system but other than lots more parameters are being determined with the use of this system.

### III. RESULTS

Once the State Space model of the Pendulum System has been derived that can be converted into the equivalent Transfer Function Model using LabVIEW Control System tool box [5]. From that all the parameters are being determined.

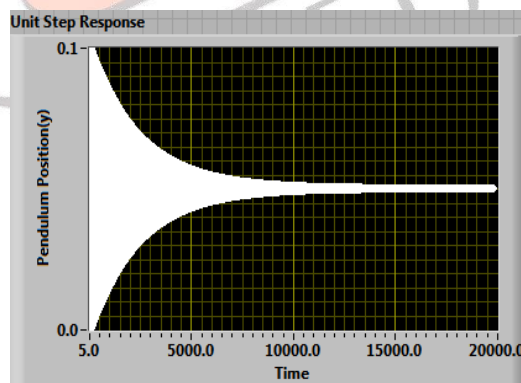
Following figures provides overall information about the System setup on LabVIEW software:

**Equation**

$$\frac{1}{s^2 + 0.00081s + 19.81}$$

*Fig.3 LabVIEW screen shot of Block diagram of pendulum system*

Fig.3 is the equivalent block diagram model of the system. This can be obtained by state space to transfer function conversion using LabVIEW control System Tool box.



*Fig.4 Unit Step Response of Pendulum Position*

Fig.4 shows Step response of the Pendulum System. Transient response of the system can be analyse and determined with the use of this graph.

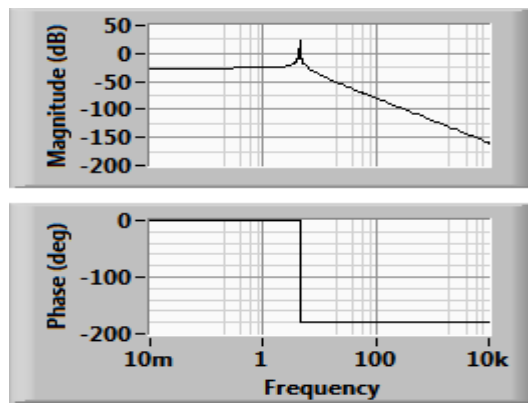


Fig.5 Frequency Response of Pendulum System

Fig.5 shows the magnitude and phase plot of the pendulum system that can be used to find the frequency response of the system.

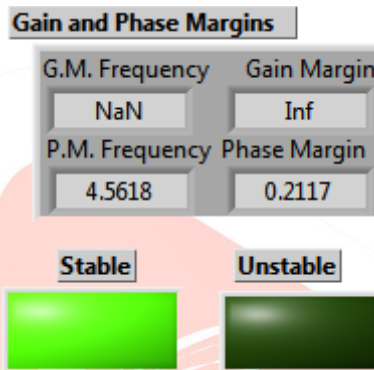


Fig.6 frequency parameters of the system

From the fig. 6 it is obtained that the system is stable because phase margin and gain margin is positive. The value of phase margin is 0.2117 and gain margin is infinitive so both are positive so the system is stable, and the Led in the LabVIEW Screen indicating that the system is Stable [5, 7, 8]

Transient Response Parameters	Frequency Response Parameters
Natural frequency( $\omega_n$ )=4.45	Phase Cross over frequency=4.56
Damping ratio =0.00009	Phase margin=0.2117
Delay Time( $T_d$ )=1.001s	Gain margin=infinitive
Rise Time( $T_r$ )=2.788s	Stability=Stable System
Peak time( $T_p$ )=0.705s	
Settling Time( $T_s$ )=195564s	

Table-1 observation Table

#### IV. CONCLUSION

Transient Response Analysis is applicable for a wide range of Control Systems and can be used to analyze the behavior of control system with respect to time by finding the Time response specification parameters as well as Frequency Response parameters like Delay time, Rise Time, Peak Time, Settling Time, G.M, P.M... etc. LabVIEW is very useful tool to develop basic GUI of the system and the Control System Toolbox is the classic tool of LabVIEW to analyze the detail time behavior and stability of the control system. Once the state space model of the pendulum system is achieved, the system analysis can be very easily done by Control System Toolbox of LabVIEW and pendulum based system stability can be checked.

#### V. REFERENCES

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