

Composite Fibre-Resin Lamina and Comparison between Finite Element Analysis and Analytical Solution for Study of Ligament

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Abstract— The objective of this work is to compare ligament system with the composite fibre-resin lamina which is not exactly but matches nearer to the ligament structure. The skeletal ligaments are short bands of tough fibrous connective tissue which are same as composite material. This paper presents 3D finite element analysis performed for a composite Fibre-Resin lamina and compare this result with the orthotropic stress -strain relationship formulation for validation of the result. Nonlinear static analysis performed for composite material to evaluate stress-strain relationship for a composite material.

Index Terms— composite lamina, FEA, Ligament

I. INTRODUCTION

Ligaments are composed of closely packed collagen fibre bundles oriented in a parallel fashion to provide for stability of joints in the musculoskeletal system. Ligaments can be compared with the composite material because of the similar structural system. Composites are made from two or more constituent materials with significantly different chemical and material properties that when combined, produce a material which characteristics different from the individual components. The current paper attempt to combine the two approaches and the result for stress-strain relationship is compared.

Analytical solution are developed based on proposed solution by (P. Boresi and Richard J.Schmidt).The fibre-resin lamina stress-strain relationship case from Advanced Mechanics of Materials. FEM analysis was carried out by simulating the composite model using ANSYS 14.5.the stress strain relation obtained by applying shear force on one face of model along the axis of fibre.

II. ANALYTICAL SOLUTION

Advance Mechanics of Material(P. Boresi and Richard J.Schmidt) proposed the solution for unidirectional model of a lamina of a section of an airplane wing composed of fibres and resin. The volume fraction is consider for the determination of stress strain relations of the lamina.

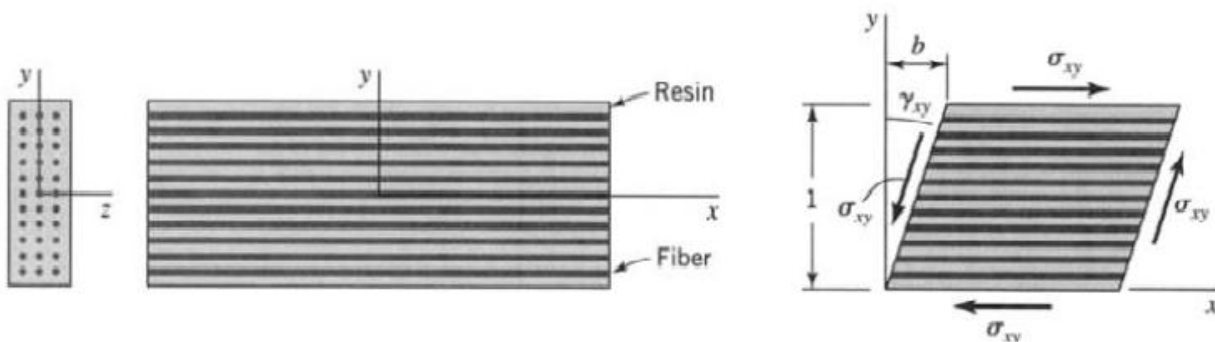


Fig.1 Fiber-Resin lamina, fiber: resin volume fraction = f, resin volume fraction=1-f

The modulus of elasticity and Poisson's ratio of Fibre and Resin be denoted as E_F, μ_F and E_R since the lamina is thin, the effective state of stress in lamina is approximately one of plane stress in the x-y plane of the laminate. stress strain relations for the fibres and resins are

$$\epsilon_{xxF} = \frac{1}{E_F} (\sigma_{xxF} - \mu_F \sigma_{yyF})$$

$$\epsilon_{yyF} = \frac{1}{E_F} (\sigma_{yyF} - \mu_F \sigma_{xxF})$$

$$\begin{aligned}\epsilon_{xxR} &= \frac{1}{E_R} (\sigma_{xxR} - \mu_R \sigma_{yyR}) \\ \epsilon_{xxR} &= \frac{1}{E_F} (\sigma_{yyR} - \mu_F \sigma_{xxR})\end{aligned}\quad (1)$$

Where $(\sigma_{xxF}, \sigma_{yyF}), (\sigma_{xxR}, \sigma_{yyR}), (\epsilon_{xxF}, \epsilon_{yyF})$ and $(\epsilon_{xxR}, \epsilon_{yyR})$ denotes stress and strain components in fiber(F) and resin(R) respectively.

Since the fibers and resin are bonded, the effective lamina strain ϵ_{xx} is same as that in the fibers and in the resin ;x direction.

$$\epsilon_{xx} = \epsilon_{xxF} = \epsilon_{xxR} \quad (2)$$

In y- direction, the effective lamina strain ϵ_{yy} is proportional to the amount of fiber per unit length in y direction,

$$\epsilon_{yy} = f \epsilon_{yyF} + (1 - f) \epsilon_{yyR} \quad (3)$$

Also, by equilibrium of the lamina in x direction, the effective lamina stress σ_{xx} is

$$\sigma_{xx} = f \sigma_{xxF} + (1 - f) \sigma_{xxR} \quad (4)$$

In the y direction, the effective lamina stress σ_{yy} is same as in the fibers and in resin

$$\sigma_{yy} = \sigma_{yyF} = \sigma_{yyR} \quad (5)$$

solving equations.(a) through (e) for ϵ_{xx} and ϵ_{yy} in terms of σ_{xx} and σ_{yy} ,

$$\begin{aligned}\epsilon_{xx} &= \frac{1}{E} (\sigma_{xx} - \mu \sigma_{yy}) \\ \epsilon_{yy} &= \frac{1}{E} (\beta \sigma_{yy} - \mu \sigma_{xx})\end{aligned}\quad (6)$$

Where,

$$E = f E_F + (1 - f) E_R$$

$$\mu = f \mu_F + (1 - f) \mu_R$$

$$\beta = f(1 - f) \left[(1 - \mu_R^2) \frac{E_F}{E_R} + (1 - \mu_F^2) \frac{E_R}{E_F} + 2\mu_F \mu_R + \frac{1-f}{f} + \frac{f}{1-f} \right] \quad (7)$$

To determine the shear stress-strain relation, apply a shear stress σ_{xy} to a rectangular element of the lamina and calculated angle change γ_{xy} of the rectangle. by figure the relative displacement b of the top of the element is

$$b = f \gamma_F + (1 - f) \gamma_R \quad (8)$$

where γ_F and γ_R are the angle changes attributed to the fiber resin.

$$\gamma_F = \frac{\sigma_{xy}}{G_F} \quad \gamma_R = \frac{\sigma_{xy}}{G_R} \quad (9)$$

and G_F and G_R are the shear modulus of elasticity for fiber and resin respectively. hence ,the change γ_{xy} in angle of element(the shear strain is)

$$\gamma_{xy} = 2\epsilon_{xy} = \frac{b}{1} = \left[\frac{f G_R + (1-f) G_F}{G_F G_R} \right] \sigma_{xy} \quad (10)$$

by equation (j) shear stress relation is

$$\sigma_{xy} = G \gamma_{xy} = 2G \epsilon_{xy} \quad (11)$$

where

$$G = \frac{G_F G_R}{f G_R + (1-f) G_F} \quad (12)$$

by equation (6),(7),(11),(12) we obtained the stress-strain relations of lamina

$$\sigma_{xx} = C_{11} \epsilon_{xx} + C_{12} \epsilon_{yy}$$

$$\sigma_{yy} = C_{12} \epsilon_{xx} + C_{22} \epsilon_{yy}$$

$$\sigma_{xy} = C_{33}\gamma_{xy} \quad (13)$$

where

$$C_{11} = \frac{\beta E}{\beta - \mu^2}, C_{22} = \frac{E}{\beta - \mu^2}, C_{12} = \frac{\mu E}{\beta - \mu^2}, C_{33} = G \quad (14)$$

III. FINITE ELEMENT ANALYSIS

Basically finite element method consider a structure is constructed from simple element which are connected at their nodes and fulfills equilibrium and compatibility conditions. Based on this definition the composite fiber-resin model of size 20mm×50mm×20mm is drawn with fiber of size 3mm diameter and 20mm in length inside the resin. The finite element analysis was carried out using ANSYS 14.5 software. There are two option which can be used: ANSYS Parametric Design Language (APDL) and Graphics User Interface (GUI).in this paper (GUI) is used.

Table 1
properties of composite materials:-

Glass fiber:-
EF=72.4GPa,GF=27.8GPa,poissons ratio=0.30
Epoxy resin:-
EF=3.50GPa,GF=1.35GPa,poissons ratio=0.30
The volume fraction of fiber is f=0.70

FEM analysis is done with 77475 nodes to describe 20mm×50mm×20mm block and meshing is kept automatic to generate the result. Static nonlinear analysis is done on the model. a shear force of value 10N is applied on the upper face of the block keeping lower face fixed to get shear effect.

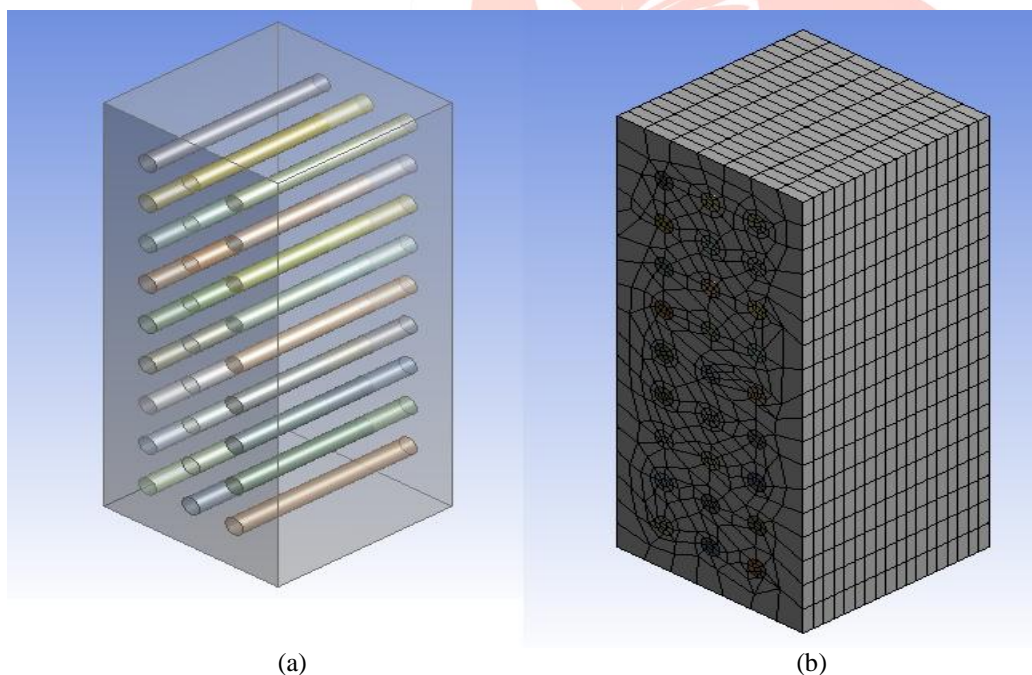


Fig.3 (a) composite Fiber-resin model. inside tubes showing fibers and outside block showing resin.(b)FEM mesh generated model.

The mesh configuration for the composite model is shown in Fig.3(b).the stress strain relation for the fiber and Resin is found by solving the problem with above boundary conditions and the following results are showing the shear elastic strain and shear stress for the model.

IV. RESULTS AND DISCUSSION

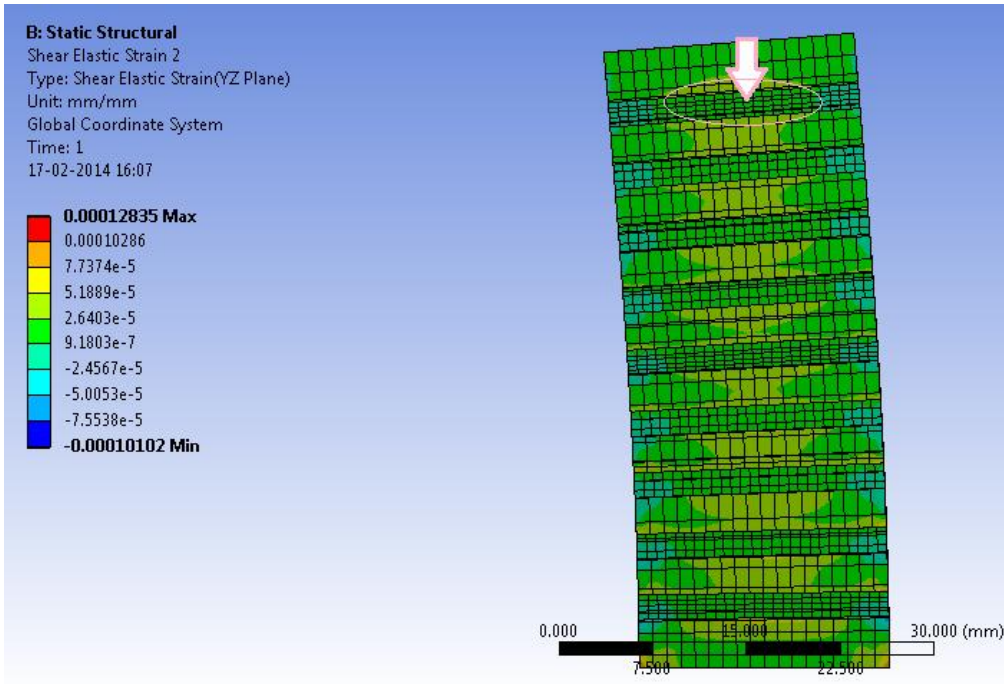


Fig.4 shear elastic strain of composite model at y-z plane.

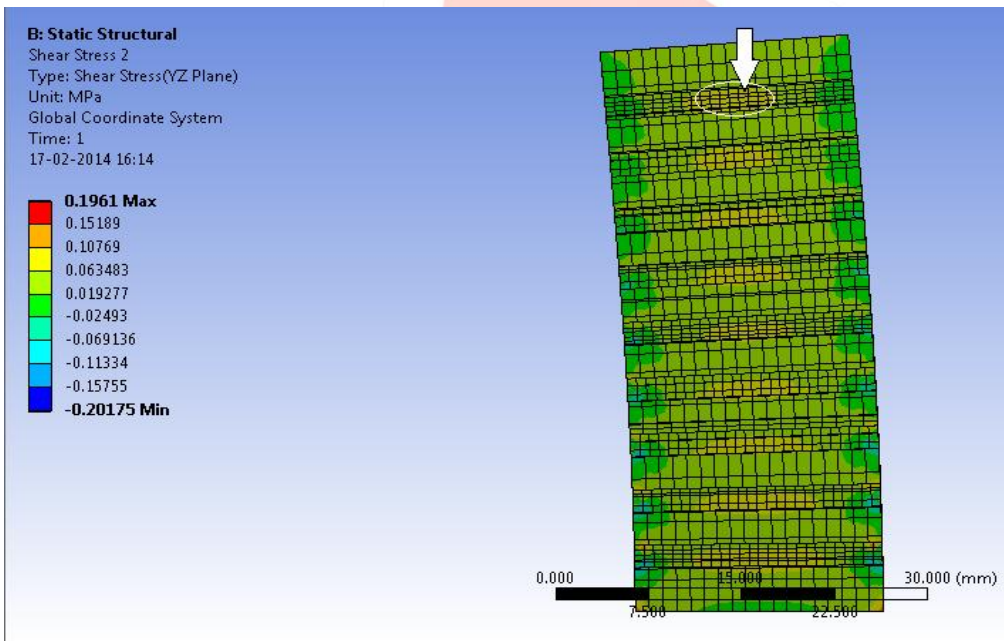


Fig.5 shear stress of composite model at y-z plane.

For the stress-strain relation the results are presented in Fig.4 and 5 for y-z plane. On this plane the shear stress and shear strain are calculated on the fiber of the model. The above model shows the circular portion at which both results are taken. The green zone of fiber showing the shear strain value and same portion is observed in yellow zone for shear stress. Evaluated value is used to validate the analytical solution, the values for analytical solution are calculated by putting the above table data in the equation (13). Results are as

$$E = [0.7 \times 72.4] + [0.3 \times 3.50] = 51.73 \text{ GPa}$$

$$\mu = (0.7 \times 0.3) + (0.3 \times 0.3) = 0.3$$

$$\beta = (0.7 \times 0.3) \left[(1 - 0.009) \frac{72.4}{3.50} + (1 - 0.009) \frac{3.50}{72.4} + (2 \times 0.3 \times 0.3) + \frac{0.3}{0.7} + \frac{0.7}{0.3} \right]$$

$$= 4.9325$$

$$G = \frac{27.8 \times 1.35}{(0.7 \times 1.35) + (0.3 \times 27.8)}$$

$$= 4.0420$$

$$C_{11} = 52.6914$$

$$C_{12} = 3.2047$$

$$C_{22} = 10.6824$$

$$C_{33} = 4.0420$$

The stress-strain relation for shear stress is given from equation(13) is given below. this equation is used to validate the FEA result by putting the value of FEA strain into the given equation.

$$\sigma_{xy} = C_{33} \gamma_{xy}$$

$$\sigma_{xy} = 4.0420e3 \times \gamma_{xy}$$

$$\sigma_{xy} = 4.0420e3 \times 2.6403e - 5$$

$$= 0.1067\text{MPa}$$

Comparison will be made with the analytical solution obtained in the equation (13). It shows that good agreement is achieved between FEA and analytical results. The difference is less than 5%.

V.CONCLUSION

From this analysis it can be conclude that a good agreement is found between the analytical solution and FEA results for shear stress of the fiber-resin lamina and the remaining results for stresses will also match with the FEA results. Finite element analysis can predict results for the composite lamina and in future it will be used to find out the Ligament analysis by referring this result. the lamina is independent from both dimension as well as force.

REFERENCES

- [1] P. Boresi, Richard J. Schmidt. "Advanced Mechanics Of Materials" 2009
- [2] Ever J. Barbero. "Finite Element Analysis of Composite Materials" 2011
- [3] Jeffrey A Wiess, John C. Gardiner, Benjamin J Ellis, Trevo J. Lujan, Nikhil S. Pathak Three dimensional finite element modeling of ligament technical aspects, medical engineering and physics 27(2005);845-861
- [4] Ligament tissue engineering: an evolutionary materials science approach. Biomaterials 26(2005)7530-7536
- [5] Savi L-Y Woo, Steven D Abramowitch, Biomechanics of knee ligaments: injury, healing, and repair; journal of mechanics 39(2006)1-20
- [6] Wrinkling of sandwich column: comparison between finite element analysis and finite element solution. composite structures 53(2001)447-482