

# A Case Study on Effect of Mean Stress on Fatigue life

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**Abstract**—Cyclic loading causes failure of component by fatigue. Due to such cyclic loading cracks formed in component which fails the material before the yield stress. Life of component is more when cyclic loading nature is pure reversible. If magnitude of loading in both directions is different, then mean stress plays vital role in fatigue life estimation. Effect of mean stress is proposed by different theories like Morrow rule, SWT relation, and Gerber-Goodman-Soderberg relations. By this theory it is concluded that as increased mean stress reduced the life of component. This paper includes above mentioned theory along with numerical justification to theory.

**Index Terms**—mean stress, fatigue-life, HCF, LCF.

## I. INTRODUCTION:-

Fatigue is cyclic loading fracture. Cyclic loading contains load variation from minimum to maximum and this cycle repeats for number of times. When minimum and maximum load is same in magnitude but opposite in direction like tension and compression or heating and cooling, then mean stress is zero. But when min and max load are not identical in magnitude then it contains some amount of residual stress which is called as mean stress. Mean stress affects the fatigue life of component. Generally positive or mean stress in tension has a bad effect on component. Its life gets reduced as mean stress gets increased. But if mean stress is in compression then it increases the life of component.

## II. FATIGUE: - NOMENCLATURE AND ITS TYPES

Fatigue is a failure phenomenon of material which occurs due to cyclic loading. Fatigue is defined as, it is a localized damage process of a component produced by cyclic loading. It is the process of progressive localized permanent structural change occurring in a material subjected to conditions that produce fluctuating stresses and strains at some point or points and that may culminate in cracks or complete fracture after a sufficient number of fluctuations. It is the result of the cumulative process consisting of crack initiation, propagation, and final fracture/failure of a component.

Fatigue process involves the following stages:

1. Crack nucleation
2. Crack growth
3. Final fracture

During cyclic loading, localized plastic deformation may occur at the highest stress site. This plastic deformation develops a crack. As the component experiences an increasing number of loading cycles, the length of the crack (damage) increases. After a certain number of cycles, the crack will cause the component to fail.

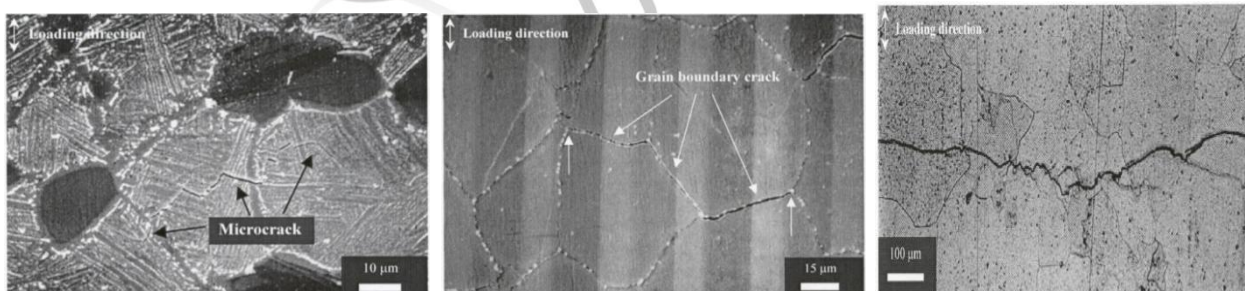


Fig2.01:- Micro-crack formation

Crack growth through grain boundary

Final Fracture

Loading conditions:-

- I. A fully reversed stress cycle, shown in 1st Fig., where the maximum and minimum stresses are equal, is commonly used in testing. In this mean stress is zero.
- II. The middle graph in Fig. shows the condition where both stresses (cyclic and applied) are tensile (greater than zero), but it is also possible to test with both stresses in compression.
- III. In addition, the maximum and minimum stresses in the cycle do not necessarily have to be equal in value. The last type of loading cycle is the random or irregular stress cycle, in which the part is subjected to random loads during service, as shown in the last graph of Fig.

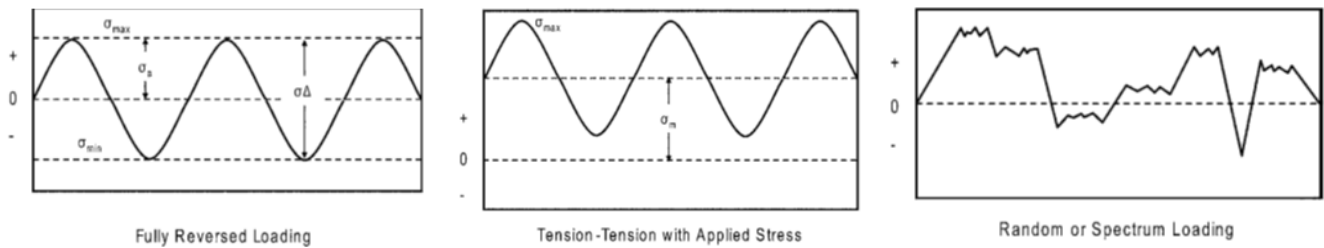


Fig.2.02:- Types of Stress cycle

Nomenclature: - Based on loading, stress can be categorized in following entities

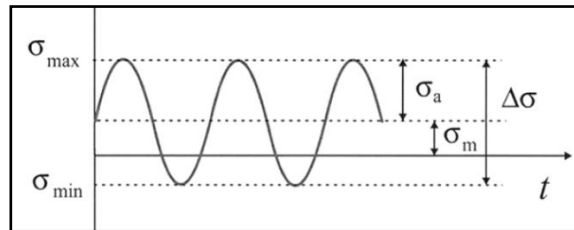


Fig.2.03:- Stress cycle

A stress range: - It is difference between maximum and minimum stress level.

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

The alternating stress: - It is half of stress range.

$$\sigma_a = \sigma_r / 2$$

Mean stress is average of both.

Stress Ratio:  $R = \frac{\sigma_{min}}{\sigma_{max}}$

**III. FATIGUE ANALYSIS METHODS:-**

- i. Stress based fatigue analysis
- ii. Strain based fatigue analysis

i. *Stress based fatigue analysis:-*

Stress life method is also referred as a S-N method. Stress based fatigue method widely used in high cycle fatigue application. In such application stress is within the elastic range of material and gives higher fatigue life. The Goodman line, Gerber line and Soderberg line relations are available in the Stress-Life module. To generate this relation line, substantial amount of testing is required. This line gives relation between alternating stress, mean stress, yield stress and ultimate stress. A graphical comparison is shown in fig. Gerber relation deals with ultimate strength but Soderberg relation deals with yield strength. So for ductile material prefer soderberg relation.

Gerber Relationship:  $-\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1.$

Goodman Relationship  $\frac{S_a}{S_e} + \left(\frac{S_m}{S_u}\right)^2 = 1$

Soderberg Relationship  $\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$

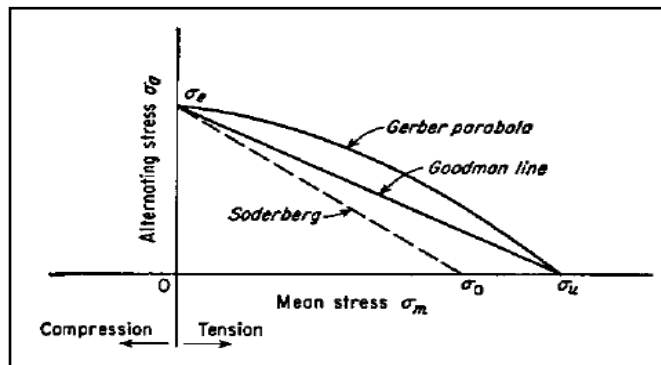


Fig.2.04: - Graphical comparison of Gerber, Goodman and Soderberg relations

ii. *Strain Based analysis:-*

Most components may appear to have nominally cyclic elastic stresses, but notches, welds, or other stress concentrations present in the component may result in local cyclic plastic deformation. Strain life method based on assumption that fatigue life is dominated by crack nucleation and small crack growth. By using this concept it is possible to determine the fatigue life at a point in a cyclically loaded component if the relationship between the localized strain in the specimen and fatigue life is known. For low cycle fatigue strain based approach is used. Because in low cycle fatigue material goes into plastic region and shows minimum life less than  $10^3$  or  $10^5$  number of cycles.

This strain-life relationship is typically represented as a curve of strain versus fatigue life and it is given by the Basquin equation which can be expressed in terms of true elastic strain amplitude as:

$$\varepsilon_e = \frac{\sigma_a}{E} = \frac{\sigma'_f}{E} (2N_f)^b$$

$\varepsilon_e$  = Elastic component of the cyclic strain amplitude

$\sigma_a$  = Cyclic stress amplitude

$\sigma'_f$  = Fatigue strength coefficient

$2N_f$  = Number of cycles to failure

$b$  = Fatigue strength exponent

Coffin and Manson's relation between plastic Strain-Life data using a power relationship:

$$\varepsilon_p = \varepsilon'_f (2N_f)^c$$

$\varepsilon_p$  = Plastic component of the cyclic strain amplitude

$\varepsilon'_f$  = fatigue ductility coefficient

$c$  = fatigue ductility exponent

The Strain-Life Curve can be formed by summing the elastic and plastic components:

$$\varepsilon_t = \varepsilon_e + \varepsilon_p$$

$$\varepsilon_t = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

#### IV. MEAN STRESS EFFECT:-

Structural members subjected to in-service cyclic loads exhibit a fatigue behavior that generally depends on the mean stress values. Mean stress is generally taken as average of minimum stress and maximum stress. Mean stress in cyclic loading plays important role. For a given fatigue load range a tensile mean normal stress has a detrimental effect on fatigue strength, whereas, in general, a compressive mean normal stress has a beneficial effect. The problem of the mean stress effect on fatigue life has been approached practically by developing empirical relationships. For metals and alloys, various criteria have been proposed to deal with the mean stress effect on fatigue life.

##### 1. For stress based fatigue:-

For stress based fatigue algorithm i.e. for high cycle fatigue we have different approaches to take account effect of mean stress like Gerber relationship, Goodman relationship, Soderberg relationship, etc. As described in Sec:-III.1, this relationship can be generated by doing number of experiment. Graph is plotted as alternating stress Vs mean stress. If point lies inside the area of lines then component is safe for that stress value. And if point is lies outside of line area then material fails.

##### 2. For strain based fatigue:-

For strain based fatigue life calculation we have Basquin and Coffine-Manson relationship. But this relationship doesn't take account for mean stress. So for strain based fatigue algorithm or for low cycle fatigue we need different types of relationships which can take account of the effect of mean stress during fatigue calculation like Morrow, SWT, Manson and Halford.

###### a) Morrow Relationship: -

Effect of mean stress is generally described by Morrow rule which states that mean stress mostly affect during early stage of loading or for higher fatigue life. In such type of life, elastic strain amplitude dominates the total life. As shown in fig., morrow

explains that as number of cycles get increased mean stress come nearly to zero. So Morrow deduced mean stress from elastic part of Basquin and Coffin-Manson equation.

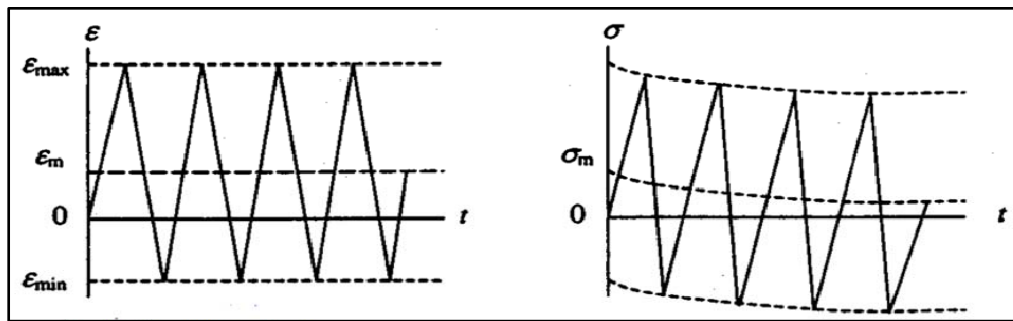


Fig.2.05: - Mean stress effect for cyclic loading

$$\epsilon_t = \frac{(\sigma'_f - \sigma_{mean})}{E} (2N_f)^b + \epsilon'_f (2N_f)^c$$

The Morrow model predicts that the mean stress has a significant effect on longer lives, where the elastic strain amplitudes dominate. The prediction trend of the Morrow mean stress correction model is consistent with observations that the mean stress has greater impact at longer lives.

**b) Manson and Halford:-**

Manson and Halford suggested that both the elastic and plastic terms of the strain-life equation should be modified to account for the mean stress effects and maintain the independence of the elastic to plastic strain ratio from the mean stress. This model over predict the result of mean stress when fatigue is at low cycle i.e. where component goes into plastic region.

$$\frac{\Delta \epsilon}{2} = \frac{(\sigma'_f - \sigma_{mean})}{E} (2N_f)^b + \epsilon'_f \left( \frac{\sigma'_f - \sigma_m}{\sigma'_f} \right)^{\frac{c}{b}} (2N_f)^c$$

**c) SWT Relationship:-**

According to Smith-Watson-Topper product of maximum tensile stress and strain amplitude controls the effect of mean stress. So they multiply  $\sigma_{max}$  on both side of Basquin-Coffine-manson equation and proposed SWT relationship. The SWT model assumes that  $\sigma_{max} \epsilon_a$  parameter at given life remains constant for different combination of strain amplitude and maximum stress.

$$\sigma_{max} \frac{\Delta \epsilon}{2} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \epsilon'_f \sigma'_f (2N_f)^{b+c}$$

All relations give the result as if tensile mean stress is acting on component then component have less life than zero mean stress condition.

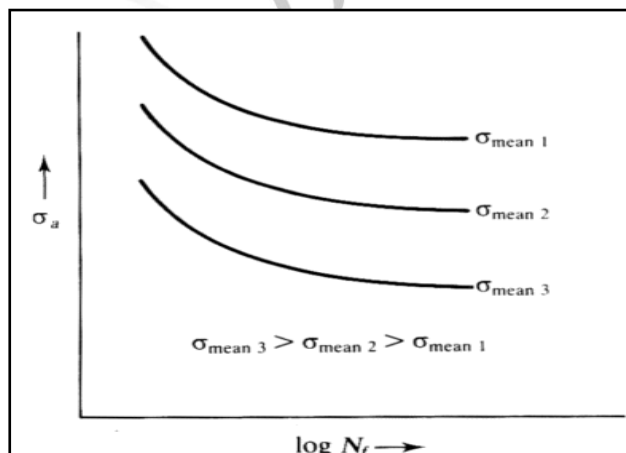


Fig.2.06:- Effect of mean stress on fatigue life

**V. NUMERICAL JUSTIFICATION:-**

Consider following example, in which alternating load is acting with no mean stress and with mean stress. Three graph shows stress cycles working. Other material properties are given in table. Calculation is done by considering elastic property and by using

Basquin equation. For mean stress correction Morrow relation is used. Values are given for different condition and comparison is done for fatigue life cycle.

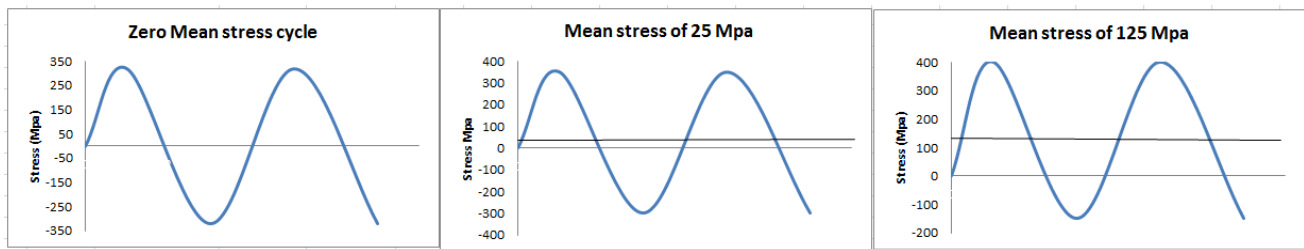


Fig.2.07:-Stress cycle for different condition.

Material properties:-

Elastic Modulus = E= 200Gpa

$\sigma'_f$  = Fatigue strength coefficient= 1100 Mpa

b = Fatigue strength exponent= -0.05

Stresses and strains for different conditions given in table. By using following formula life (2Nf) is calculated. Nf= Number of reversal of cycles. So (2Nf) term is used to define one cycle.

$$\frac{\Delta \epsilon}{2} = \frac{(\sigma'_f - \sigma_{mean})}{E} (2N_f)^b$$

Given Data			Calculated Data			
1	Strain	Stress	Alternating Strain	Mean Stress	Log10(Life)	Life
A	0.003	320	0.003	0	5.17	150800
B	-0.003	-320				
2	Strain	Stress	Alternating Strain	Mean Stress	Log10(Life)	Life
A	0.005	350	0.003	25	4.97	95217.51
B	-0.001	-300				
3	Strain	Stress	Alternating Strain	Mean Stress	Log10(Life)	Life
A	0.0055	400	0.003	125	4.13	13509.52
B	-0.0005	-150				

Calculations conclude that as mean stress increased, life of components gets reduced. Fatigue life for mean stress of 125 Mpa is less than fatigue life of zero mean stress condition.

**VI. CONCLUSION:-**

Mean stress affect the fatigue life of component. By using Morrow relation or SWT relation or Manson and Halford relation, we can estimate the effect of mean stress on fatigue life of component. From numerical example solved in paper, it is clear that increased mean stress reduced the fatigue life of component. But if mean stress is compressive then it will increase the fatigue life of component.

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