

# A Review on Failure Modes of Composite Pressure Vessel

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**Abstract**— Modern composites, using continuous fibres in a resin matrix, are important candidate materials in the engineering of energy-efficient structures. In many applications, fibre/matrix materials are lighter, stronger and more cost effective when compared with traditional materials like metals. Filament-wound tubular structures, more specifically pressure vessels, offer significant weight saving over conventional all metallic ones for containment of high pressure gases and liquids. The main advantage of COPV's over similar sized monolithic metallic pressure vessels is a much better strength-to-density ratio due to significant mass reductions. Currently, a large amount of research works has been concentrated on the stress and failure analysis of the cylindrical part of the composite-metallic vessels. The study of the stress and strain distribution in the structure is of prime interest for designing the vessel. The complicated failure mechanisms and degradation mechanisms are distinct characteristic of composites although they exhibit high stiffness- and strength-density ratios. In this paper various failure modes of composite pressure vessels are studied such as failure due to hygrothermal stresses, influence of flaws, effect of thermal loads etc.

**Index Terms**—Composite overwrapped pressure vessel, Composite laminate theory , Failure criteria.

## I. INTRODUCTION

All Composite overwrapped pressure vessels (COPV's) are made of a thin metallic liner wrapped with a high-strength low-density composite. The metallic liner provides shape, toughness, tightness and interface with the gas feeding systems while the overwrapped composite ensures mechanical strength to withstand high pressures and protects the vessel against scratches, indents and other forms of impact damage. The utilization of COPV's in the aerospace industry goes back to 1970's when the advances in composite materials allowed replacing heavier, thick walled, pressure vessels made of high-strength metals by thin-walled metallic liners of aluminium, stainless steel, titanium or inconel wrapped with epoxy or polyamide polymer resins reinforced with high-strength carbon, kevlar or glass fibres. Other examples of pressure vessels are fuel tanks, rocket motor cases, diving cylinders, recompression chambers, distillation towers, pressure reactors, autoclaves, vessels in mining operations, oil refineries and petrochemical plants, nuclear reactor vessels, submarine and space ship habitats, Pneumatic reservoir , hydraulic reservoir under pressure, rail vehicle airbrake , reservoirs, road vehicle airbrake, reservoirs and storage vessels for liquefied gases such as ammonia, chlorine, propane, butane, and LPG.

The basic design requirements for the storage vessel are safety, reliability and economy. However, the composite pressure vessel may work under the high-pressure and high-temperature environment. This not only presents a strong challenge to the physical and mechanical performance, but also to the reliable and economical design about how to achieve a perfect combination of safety performance and low cost. Conventional movable metal pressure vessel can no longer be competent for the rigorous need of high strength - and stiffness - weight ratios. In this case, the composite filament wound technology is introduced to improve the storage efficiency.

Modern composites, using continuous fibres in a resin matrix, are important candidate materials in the engineering of energy-efficient structures. In many applications, fibre/matrix materials are lighter, stronger and more cost effective when compared with traditional materials like metals. Many high-strength composite products are fabricated using the filament winding process. In this process, bands of resin-impregnated fibres are wound over a cylindrical mandrel using a computer controlled fibre placement machine. Fibre application to the metallic mandrel is usually accomplished by a transverse feed head over a rotating surface, capable of achieving any desired winding orientation. Filament-wound tubular structures, more specifically pressure vessels, offer significant weight saving over conventional all metallic ones for containment of high pressure gases and liquids.

The main advantage of COPV's over similar sized monolithic metallic pressure vessels is a much better strength-to-density ratio due to significant mass reductions .Currently, a large amount of research works has been concentrated on the stress and failure analysis of the cylindrical part of the composite-metallic vessel [1,2,3,4,5]. The study of the stress and strain distribution in the structure is of prime interest for designing the vessel. Two theoretical approaches are used to investigate the behavior of the composite vessels; the classical theory laminates [5] and the so-called theory of elasticity [6, 7]. The first theory assumes that the composite laminates are in a state of planes stress and provides no stress in the direction of thickness. The second shows that the stress developed through radial thickness have a large influence on the choice of stacking sequences [6].For reliability assurance an accurate prediction of failure process of laminated composite structures and the maximum loads that the structure can withstand before failure occurs has thus become an important topic of research. In this paper various failure modes of composite pressure vessels are studied such as failure due to hygrothermal stresses, influence of flaws, effect of thermal loads etc.

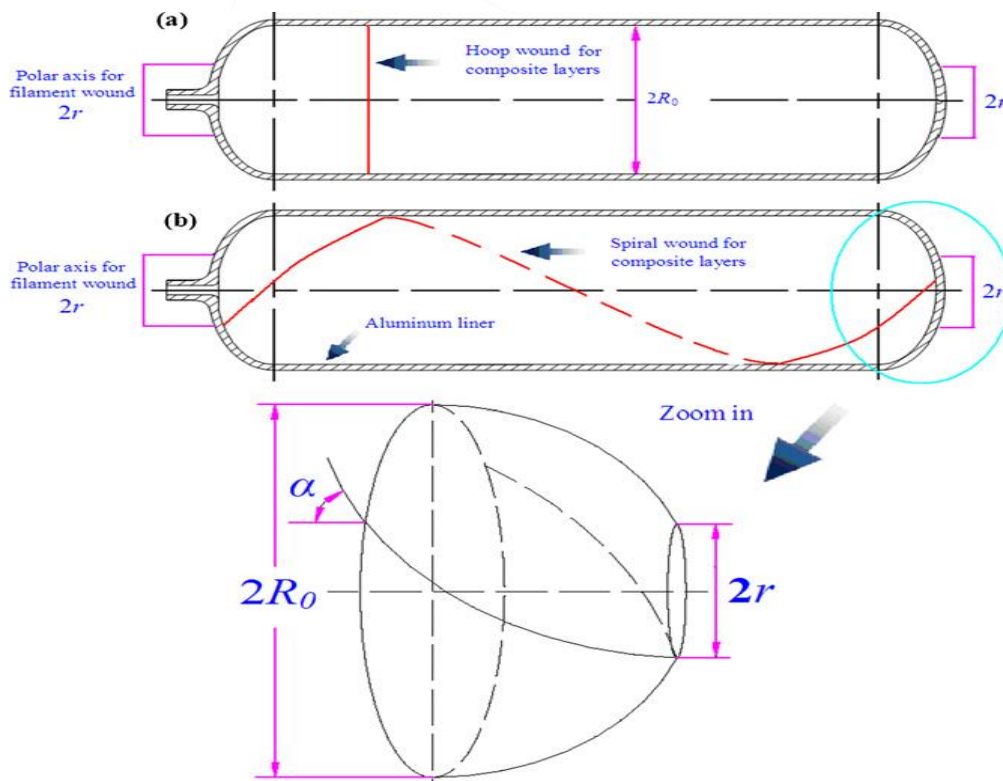
**II. GEOMETRY STRUCTURE OF COMPOSITE HYDROGEN STORAGE VESSEL:-**

Fig. 1 shows a typical composite vessel used in the hydrogen fuel car. Fig. 2 shows the basic structure and wound pattern of composite vessel. The aluminum layer prevents the gas leakage and provides a mould for filament wound, and the composite layers are responsible for resisting the internal pressure. For the cylinder part, spiral wound and hoop wound patterns are used. Yet, only spiral wound pattern appears at the head part. From the manufacturing principle, the wound trace for composite layers is a combination of the rotation of aluminum liner and the axial movement of wound machine. The spiral wound angle  $\alpha_0$  at the cylinder is determined by [8].

$$\alpha = \arcsin\left(\frac{r}{R_0}\right) \dots \dots \dots (1)$$



**Fig. 1.** Lightweight composite high-pressure hydrogen storage vessel used in the hydrogen fuel-cell vehicle.



**Fig. 2.** Basic structure and wound pattern of composite vessel: (a) hoop wound and (b) spiral wound.

Where  $r$  is the radius of the polar axis and  $R_0$  denotes the inner radius of the cylinder. The spiral wound at the head obeys the geodesic path algorithm, marking the shortest distance between any two points on the head. The spiral wound angle  $\alpha$  at the head changes from  $90^\circ$  at the polar axis to  $\alpha_0$  at the cylinder [8]

$$\alpha = \arcsin\left(\frac{r}{R_0}\right) \dots \dots \dots (2)$$

The thickness  $H$  of the composite layer at any radius  $R$  of the head is calculated by [8]

$$H = h \sqrt{\frac{(R_0^2 - r^2)}{R^2 - r^2}} \dots \dots \dots (3)$$

Where  $h$  is the wound thickness at the cylinder. The polar radius  $r$  affects the wound angle  $\alpha$ , and the thickness  $H$  at the head.

**III. DESIGN THEORIES:-**

In general, the composite vessel is considered as the composite laminated structure. Now, there are two main design theories: grid theory and composite laminate theory.

**1. Grid theory:-**

The basic assumptions according to the grid theory are: (1) only the longitudinal carbon fiber bears the pressure and (2) effects of wound patterns are neglected. The grid theory can be used to calculate the thickness of each composite layer and the longitudinal in situ fiber strength. The strength value is inversely solved by the burst pressure of composite vessel. Chen [9] gave a set of design method of the composite vessel based on the grid theory, and the total composite wound thickness  $h_f$  is expressed as

$$h_{f\alpha} = \frac{R_0 P_b}{2 \sigma_{fb} \cos^2 \alpha_0}$$

$$h_{f\theta} = \frac{R_0 P_b}{2 \sigma_{fb}} (2 - \tan^2 \alpha_0)$$

$$h_f = h_{f\alpha} + h_{f\theta} = \frac{3R_0 P_b}{\sigma_{fb}} \dots\dots\dots(4)$$

Where  $h_{f\alpha}$  and  $h_{f\theta}$  are longitudinal and hoop wound thickness,  $P_b$  is the burst pressure of composite vessel, and  $\sigma_{fb}$  is the in situ strength of carbon fiber.

It should be pointed out the grid theory is merely an ideal design method which largely depends on the processing parameters and the sizes of the experimental composite vessel. This show seeking a more efficient design theory is necessary.

**2. Composite laminate theory**

The basic assumptions of the classical composite laminate theory (CCLT) are: (1) perfect bonding appears at the interface between each layer, (2) the mechanical properties of composite laminates are substituted by those of a middle plane, and (3) the normal stress is neglected at the section parallel to the middle plane. Lif- shitz et al. [10] proposed a method using the CCLT to calculate the stress and strain in non-symmetric wound pressure vessel with thick metal liners. However, the last two assumptions above do not strictly hold for 3D composite vessel. In this case, Parnas et al. [11] derived the elastic stress and displacement solutions for 3D cylindrical composite laminates based on the CCLT and generalized plane strain assumption. As the liner is assumed isotropic and the composite layers are considered transversely isotropic,Chapelle et al. [12], further derived the analytical solutions of the stress, strain and displacement for 3D cylindrical composite laminates.

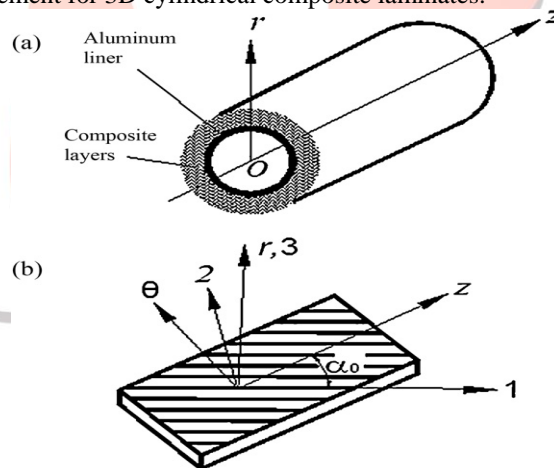


Fig. 3(a) shows the cylindrical part of composite vessel and Fig. 3(b) describes a representative volume element taken from Fig. 3(a), showing the principal direction (1, 2, and 3) of a composite layer under the cylindrical coordinate ( $r$ ,  $\theta$ , and  $z$ ).

Fig.3(a) Composite hydrogen storage vessel (transverse section) and (b) schematic illustration of the relationship between on axis coordinate (1,2,3) and the off axis coordinate.

If the axial strains at all layers are assumed constant and the shear strains are independent of  $z$ , the strain - displacement relationship is given by [12]

$$\epsilon_r^{(k)} = \frac{du_r^{(k)}}{dr} \quad , \quad \epsilon_\theta^{(k)} = \frac{u_r^{(k)}}{r} \quad , \quad \epsilon_z^{(k)} = \frac{du_z^{(k)}}{dz} = \epsilon_\theta \quad , \quad \gamma_\theta^{(k)} = \frac{du_\theta^{(k)}}{dz} = \gamma_\theta r \quad , \dots\dots\dots(5)$$

Where  $\epsilon_z$  is the axial strain,  $\gamma_{\theta z}$  is the shear strain and  $\gamma_\theta$  is twist per unit length.

The off-axis stress-strain relationship under the hygro-thermomechanics multiphysics field is expressed as

$$\begin{bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \sigma_{z\theta} \end{bmatrix}^{(k)} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{36} \\ C_{61} & C_{62} & C_{63} & C_{66} \end{bmatrix}^{[K]} \begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{z\theta} \end{bmatrix}^{(k)} - \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_6 \end{bmatrix} \times^{(k)} \Delta T - \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_6 \end{bmatrix}^{(k)} \times \Delta C \dots\dots\dots(6)$$

Where the symbols  $C_{ij}$  (i, j) = 1,2,3,6 are off-axis elastic constants of composites.

The symbols  $\sigma_z, \sigma_\theta, \sigma_r,$  and  $\tau_{z\theta}$  are the off axis axial, hoop, radial and shear stresses, respectively.  $\varepsilon_z, \varepsilon_\theta, \varepsilon_r,$  and  $\gamma_{z\theta}$  are corresponding strains respectively.  $\Delta T$  and  $\Delta C$  are temperature and moisture concentrations, respectively.  $\alpha_i$  (i = 1, 2, 3, 6) and  $\beta_i$  (i = 1, 2, 3, 6) are the corresponding Thermal and moisture expansion coefficients.

For the  $k^{th}$  layer, the equilibrium equation under the cylindrical coordinate is given by [12]

$$\frac{1}{r} = \frac{\partial(r\sigma_r^{(k)})}{\partial r} - \frac{\delta\theta^{(k)}}{r} = 0 \dots\dots\dots (7)$$

After introducing the displacement continuity conditions, the radial stress continuity conditions as well as the axial equilibrium and zero torsion conditions between two neighboring layers, the analytical solutions for the strain, stress and displacement can be derived by Eqs. (5)– (7).

**IV. ENVIRONMENTAL EFFECTS ON COMPOSITE MATERIALS :-**

The influence of environmental factors, such as elevated temperature, humidity and corrosive fluids must be taken into consideration since they affect mechanical and physical properties of composite materials resulting in a change of the mechanical performance. The effect of the elevated temperature can be seen in the composite material properties with a decrease in the modulus and strength because of thermal softening. For example, the longitudinal strength and modulus of a unidirectional composite specimen remain almost constant but off-axis properties of the same specimen are significantly reduced as the temperature approaches the glass transition temperature of the polymer. When exposed to humid air or water environment, many polymeric matrix composites absorb moisture by instantaneous surface absorption followed by diffusion through the matrix. Analysis of moisture absorption shows that for epoxy and polyester matrix composites, the moisture concentration increases initially with time and approaches an equilibrium (saturation) level after several days of exposure to humid environments [13].

The analysis of composites due to elevated temperature and moisture absorption is called as hygrothermal problem. It can be solved mainly in three steps: First, the temperature distribution and the moisture content inside the material are calculated. Then from known temperature and moisture distribution, the hygrothermal deformations and stresses are calculated. Finally, the changes in performance due to both affects are determined.

**1. Hygrothermal stresses:-**

The hygrothermal and mechanical strains can be superposed in strain level to obtain total strains as,

$$\varepsilon_{ij}^{tot} = \varepsilon_{ij}^{mech} + \varepsilon_{ij}^{hygr}$$

Or

$$\varepsilon_{ij}^{tot} = \varepsilon_{ij}^{mech} + \alpha_{ij} \Delta T + \beta_{ij} c$$

Where  $\alpha_{ij}$  and  $\beta_{ij}$  are thermal and moisture expansion coefficients, respectively. Total stresses however, can be obtained using anisotropic stress–strain relations. Total stresses due to hygrothermal and mechanical loads can be written as

$$\sigma_{ij}^{tot} = [\alpha]^{-1} \varepsilon_{ij}^{tot}$$

**2. Failure criteria for composites:-**

Hinton et al. [14] explored existing failure theories for composites by launching a world-wide failure exercise, and tested some leading failure theories by comparing experimental data.

Nahas [15], Tsai [16] and Sleight [17] performed extensive literature surveys on the failure theories of composites and classified the failure criteria into two types: the non-interactive failure criteria and interactive failure criteria. The non-interactive failure criterion is defined as one without interactions between the stress or strain components. These criteria compare the individual stress or strain components with the corresponding material allowable strengths, and indicate the type of failure modes. The maximum stress, Hashin, Christensen, Rotem, McCartney, Yamada- Sun, Chang-Chang, Puck and Huang’s criteria belong to this type.

In contrast, the interactive failure criteria consider interactions between stress components. These theories use a polynomial based on the material strengths to describe a failure surface. The Tsai-Wu, Tsai-Hill and Hoffman failure criteria belong to this type. Currently, there is also an evolving trend to develop the fracture energy-based failure criteria to predict the failure of composites using such as the cohesive fracture energy-based criterion [18,19] and finite element failure-based criterion [20,21]. However, accurate prediction of fracture properties of composites requires advanced experimental technique.

**3. Progressive failure analysis:-**

The flow chart for the progressive failure analysis of the composite vessel using finite element method is illustrated in Fig. 7, as summarized by five points [17]: (1) for each load step, the finite element analysis is performed and the on-axis stresses/strains at each element are obtained, (2) the stresses/strains at each element are compared with the material allowable values and used to

determine whether some elements fail according to the failure criteria. If no failure is detected, the applied load is increased and the analysis continues, (3) if some elements fail, the damage variables are updated, and the element stiffness constants are degraded according to the damage model. After that, the equilibrium of the composite vessel is re-established using the modified stiffness. This adjustment accounts for the nonlinear solution due to the stiffness change, (4) the calculations are performed continuously under the same load until there is no failed element anymore, (5) the iterative process repeats until the catastrophic failure showing the burst of the composite vessel. Chang [22] and Ju et al. [23] performed the initial and progressive failure analysis of the composite vessel based on the CCLT. However, the analysis is only limited to the analytical solution. Further, Perreux and Thiebaud [24], Liu and Zheng [25] performed the progressive failure analysis of the composite vessel by proposing damage models and developed the corresponding finite element technique. The numerical convergence problem is solved by introducing the arc-length algorithm or viscous stabilization method [25, 26]. The fracture localization problem is addressed by adding a characteristic parameter into the damage model to eliminate the mesh effect [27]. However, to develop the finite element technique for the failure analysis under the hygrothermo- mechanics multiphysics field may still require a long way.

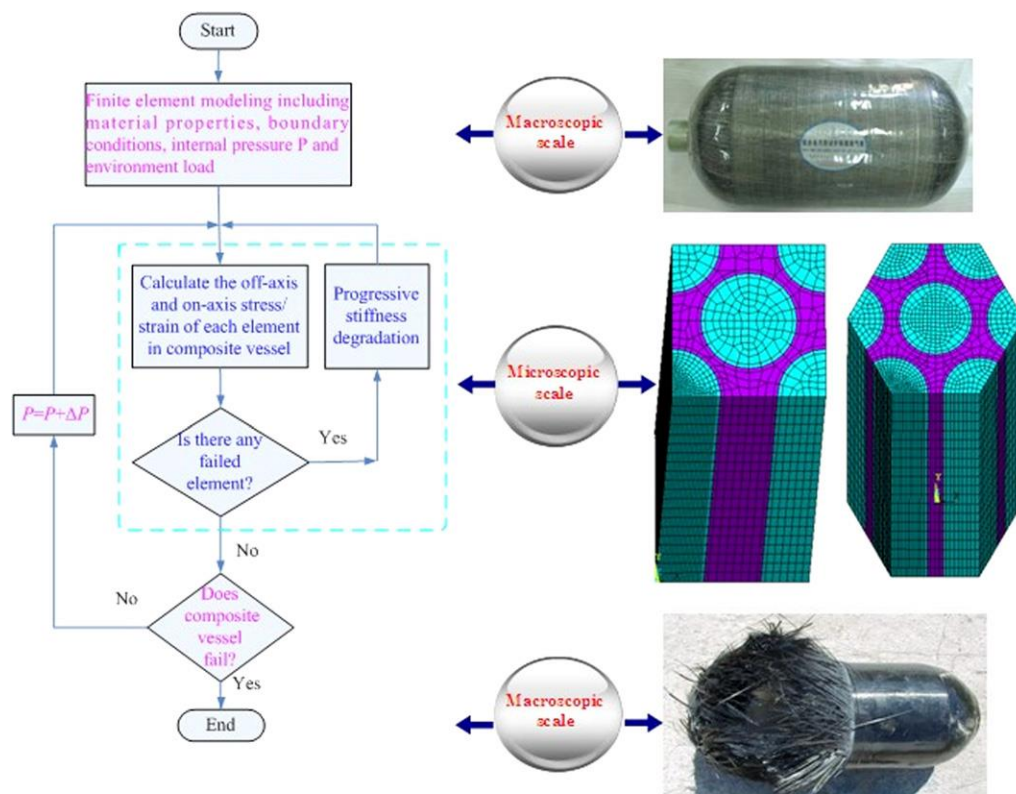


Fig. 4. Flow chart of progressive failure analysis of composite vessel

## V. CONCLUSIONS

1) Hygrothermal effects are analyzed in this study in two levels. The effect of temperature and moisture to the performance of the materials is determined by using the micromechanics of the composite materials. By taking a linear variation of temperature and constant value for moisture content throughout the body, hygrothermal stresses and strains are determined.

If the material has a tendency of expanding due to a positive temperature difference, the increasing operating temperature reduces the mechanical performance of the system.

2) Optimum winding angle-

In literature [3], the optimum winding angle for filament wound composite pressure vessels is given as 54.74° by netting analysis. Using the current procedure for the internal pressure loading, the optimum winding angle is obtained as ranging between 52.1 °C and 54.2°C depending on geometry and failure criteria used.

3) It should be pointed out that, the negative temperature for constant moisture content also cause an increase in the mechanical properties of the composite material. It can be concluded that if the operation temperature is less than the curing temperature, burst pressure is increased.

Although, the performance of the composite material is negatively influenced by the presence of moisture, it creates less residual strains compared with the thermal ones and does not change the burst pressure, significantly.

Conclusions were drawn by analyzing the influence of composite flaws in high pressure vessels for compressed natural gas vehicles on their fatigue life.

(1) For the pressure vessel models selected for the study, the fatigue life of flawless high pressure vessels for compressed natural gas vehicles is more or less 25,000 cycles, and the fatigue life of flawed vessels (flaw depth: 1.5–4.0 mm, flaw length: 50–200 mm) is between 8032 and 24,308 cycles. (2) Generally, as the flaw depth increased the fatigue life was reduced; however, such changes were not clear with flaw depth of 2.0 mm, and the results of analyses also showed that the stress of the flawed regions on the outer surface of liners increased to a maximum value with a flaw depth more than 3.0 mm. This confirms that the flaw depth of 3.0 mm was the critical value upon which they decisively affected the fatigue life of high pressure vessels.

(3) The effect of flaw length on the fatigue life of high pressure vessels was obvious. The fatigue life of COPV was reduced when flaw depth was more than 3.0 mm and the flaw length was more than 100 mm. However, they did not show specific pattern or tendency below the flaw depth of 2.0 mm and the flaw length 50 mm. Due to the small number of tested specimens; additional tests need to be performed to confirm the effect of flaw lengths of COPV quantitatively.

**Thermal load** has a significant influence on the stress distribution of a multilayered composite pressure vessel because it is made of materials with different coefficients of thermal expansion and constructed by shrink fitting.

When the vessel is loaded from 0 to 50 bars, all the load is initially taken by the liner until the gap is closed. Whatever the stacking sequence is, an identical behavior characterized the early beginning response of the structure. The increase in pressure allows the contact between the liner and the composite to take place as well as the load to be transferred. A loss of rigidity is also observed after 150 MPa hoop stress. This loss of rigidity is justified by the damage which occurs in the composite, but also by the non linear plastification of the liner.

The TSAI-WU criterion is used to evaluate the fracture behavior of composite layers. Failure is assumed when the threshold of this criterion is exceeded.

The model makes it possible to conclude that the failure of the first composite ply involves the failure of the others for only one additional increment of the loading pressure.

The hoop strain decreases while the winding angle increases. A loss of rigidity is observed on the various curves around 200 MPa. This loss is mainly due to the plastification of the liner what influences the behavior of the composite part.

#### **Further prospect of composite hydrogen storage vessel:-**

The complicated failure mechanisms and degradation mechanisms are distinct characteristic of composites although they exhibit high stiffness- and strength-density ratios. Further, the hygro-thermo-mechanics environment is also a large problem in the design of the composite vessel. More fundamental research on the failure properties of composites under the multiphysics field remains to be performed.

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