Aperiodic Fibonacci Series Multiple Quantum Wells Lead to Topological Quantum Computation

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Abstract
Quantum computation has recently emerged as one of the most exciting approaches to constructing a fault-tolerant quantum computer. Quantum information is stored in states with multiple quasiparticles, which have a topological degeneracy. Topological quantum computation is concerned with two-dimensional many body systems that support excitations.
The lack of periodicity in the Fibonacci series MQW results in strong emission. Anyons are elementary building block of quantum computations. When anyons tunneling in a double-layer system can transition to an exotic non-Abelian state and produce Fibonacci anyons, which are powerful enough for universal topological quantum computation (TQC).Here we theoretically studied the resonant and exotic behavior of Fibonacci Superlattice by using unified analytical transfer matrix methods and hence Fibonacci anyons. This Fibonacci anyons can build a quantum computer which is very emerging and exciting field in today's Nanophotonics and quantum computation.

Keywords: Quantum Computing, Quasicrystals, Multiple Quantum wells (MQWs), Transfer Matrix Method, Fibonacci Anyone. Nanophotonics.

1. Introduction

A quasicrystals is a structure that is in ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but lacks translational symmetry [7]. Crystals can possess only two, three, four, and six-fold rotational symmetries .The Bragg diffraction pattern of quasicrystals shows sharp peaks with other symmetry orders as for example five-fold symmetry. Quasicrystals had been investigated and observed earlier but, until the 1980s, they were disregarded in favour of the prevailing views about the atomic structure of matter. The natural quasicrystals were found after a dedicated search in 2009. The Fibonacci-based constructions are an obvious example of quasicrystals that do not possess a 'forbidden symmetry' The mathematical properties of the Fibonacci chain multiple quantum wells are well researched and readily applied in such studies. The elements of a Fibonacci crystal structure are arranged in one or more spatial dimensions according to the sequence given by the Fibonacci word. The Fourier transform of such arrangements consists of discrete values and X-rays diffraction pattern would produce Bragg peaks[1-7].Periodic crystals are formed by a periodic repetition of a single building block the so-called unit cell exhibiting a long range translational and orientational symmetry [19]. Only 2-, 3-, 4-, and 6-fold non-trivial rotational symmetries are allowed in the periodic crystals and their diffraction patterns give sharp Bragg in long range order. In contrast to periodic crystals, quasicrystals exhibit a long range order in spite of their lack of translational symmetry and often possess n-fold (n = 5 and > 6) rotational symmetries. Most of the quasicrystalline structures can be described by using quasiperiodic [9,12].The Fibonacci quasicrystal is a model used to study systems with aperiodic structure. Fibonacci' quasicrystal is a specific type of quasicrystal. Fibonacci 'chains' or 'lattices' are used with regard to the dimensionality of the model,used mostly as a theoretical construct, Most of its applications pertain to various areas of solid state physics. The diraction pattern of quasicrystals shows a dense set of Bragg peaks with their positions related by the irrational number where $\tau = 2\cos(\pi/5) = (1 + \sqrt{5})/2 = 1{:}618$, the so-called golden mean, which is related to the geometry of pentagonal and decagonal symmetries.

Here we will present the most celebrated non-Abelian anyons which is connected Fibonacci series MQWS. In this model, there are two different types of anyons 0 and 1 that have the following fusion rules: 0X0=0, 0X1=1 and 1x1=0+1.It is interesting to study all the possible outcomes when we fuse many anyons of type 1. For this we fuse the first two anyons and then their outcome is fused with the third 1 anyon and the single outcome is fused with the next one and so on. As the 1 anyons have two possible fusion outcome states, it is natural to ask what are the number of ways, $d_1(n)$, one can fuse n+1 anyons of type 1 to yield a 1. During the first fusing step the possible outcomes are 0 or 1, giving $d_1(2)=1$. When we fuse the outcome with the next anyon then 0×1=1 and 1×1=0+1, resulting in two possible 1s coming from two different processes and a 0, i.e. $d_1(3)=2$. Taking the possible outcome and fusing it with the next anyon gives a space of 1s that is three-dimensional, $d_1(4)=3$. The series of the space dimension $d_1(n)$ when n+1 anyons of type 1 are fused is actually the Fibonacci series. This dimension, $d_1(n)$, is also called the dimension of the fusion space and is given approximately by the following formula:$d_1(n)\alpha\ \theta^n$ where $\theta= (1+\sqrt{5})/2$,where is θ the golden mean. The latter has been used extensively by artists, such as Leonardo da Vinci, in geometrical representations of nature (plants, animals or humans) to describe the ratios that are aesthetically appealing. The above calculation is helpful to illustrate the counting of Hilbert space dimension; however, there is a more systematic way to compute the state space dimension using the concept of quantum dimension [14-16]. This anionic system is a good example for realization of quantum computation. We are interested in encoding information in the fusion space of anyons and then processing it appropriately. For the Fibonacci anionic system the particle types are denoted as **1** and τ, and the fusion rules are given by:

1 ¤ τ=τ , τ ¤ 1 and τ ¤ τ=1 ¤ τ Where the ¤ denotes the two possible fusion channels. There have been several proposals of quantum computation that are conceptually different, but equivalent to the circuit model. By equivalent we mean that any model can simulate another with utmost a polynomial overhead in the number of operations that determine the complexity of implementation. Topological quantum computation is yet another 'exotic' way of encoding and processing information. The fusion space in which information is encoded is the space of possible different outcomes from the fusion of anyons. For the case of the Fibonacci anyons, the encoding of a qubit can be visualized by employing four 1 anyons.one can braid the anyons before fusing them. For example, one can fuse the anyons a, b and c by two distinctive ways. One can first fuse a with b and then with c or first fuse b with c and then with a. these two processes are related by a six-index object F. This complex-valued function is the 6 –j symbol for quantum

$$F_{cdj}^{abi} = \begin{Bmatrix} a & b & j \\ c & d & i \end{Bmatrix}_q . \qquad (1)$$

Spin networks which are analogous to angular momentum recoupling. If the triples {*abi*}, {*cdi*}, {*adj*} and {*cbi*} are allowed products under fusion. With the proper normalization, for each set of labels (*abcd*) involved in a two-vertex interaction, where the matrix $F\ (abcd)^{i}_{j}$ is unitary. The Fibonacci anyon model allows us to universal computation on logical qubits using 4*n* physical anyons[17-18].

In classical system, a bit would have to be in one state or the other. However quantum mechanics a qubit to be in a superposition of both states at the same time, which is a fundamental in quantum computing In computing, a qubit or quantum bits is a unit of quantum information which is the quantum analogue of the classical bit. A qubit is a two-state quantum-mechanical system, such as the polarization of a single photon: here the two states are vertical polarization and horizontal polarization. A qubit has a few similarities to a classical bit, but is overall very different. There are two possible outcomes for the measurement of a qubit-usually 0 and 1, like a bit. The difference is that whereas the state of a bit is either 0 or 1, the state of a qubit can also be a superposition of both. It is possible to fully encode one bit in one qubit. However, a qubit can hold even more information, e.g. up to two bits using superdense coding. A pure qubit state is a linear superposition of the basis states. This means that the qubit can be represented as a linear combination of |0> and |1> as given bellow

$$|\Psi\rangle=\alpha|0\rangle+\beta|1\rangle \qquad (2)$$

Where α and β are probability amplitudes and can in general both be numbers. When we measure this qubit in the standard basis, the probability of outcome|0> is $|\alpha|^2$ β and the probability of outcome |1> is$|\beta|^2$. The absolute squares ofthe amplitudes equate to probabilities. It follows that α and β must be connected by the equation

$$|\alpha|^2 + |\beta|^2=1 \qquad (3)$$

This above equation ensures that we must measure either one state or the other as the total probability of all possible outcomes must be 1.Entanglement is a nonlocal property that allows a set of qubits to express higher correlation than is possible in classical systems. An important distinguishing feature between a qubit and a classical bit is the multiple qubits, that can exhibit quantum entanglement. Let us consider two entangled qubits:

$$1/\sqrt{2}\ (|00\rangle + |11\rangle).$$

Here we called an equal superposition in this state, there are equal probabilities of measuring either |00> or |11> as given by:$|1/\sqrt{2}|^2=1/2$.Suppose two entangled qubits are separated, given one each X and Y. X makes a measurement of his qubit, obtaining with equal probabilities either|0> or |1> .As qubits' entanglement X measures as |0> Y must measure the same, because |00> is the only state where X qubit is |0> . Entanglement also allows multiple states to be acted on simultaneously, unlike classical bits that can only have one value at a time. Entanglement is a necessary ingredient of any quantum computation that cannot be done efficiently on a classical computer. Many of the successes of quantum computation and communication that make use of entanglement, suggesting that entanglement is a resource that is unique to quantum computation.

Fibonacci sequence is the most well-known example of 1D aperiodic structures, along with Thue-Morse structures and the Cantor structure [13,16].So we choose to study Fibonacci series MQW structure. Another interesting property of Fibonacci structures is their direct connection with the 2D and 3D quasicrystals, the Penrose lattices [17]. The Fibonacci numbers can be obtained from the recurrence relation $F_n = F_{n-1} + F_{n-2}$ which is also true for $n < 1$. Here $F_0 = 0$, $F_1=1$, $F_2=1$ and so on. The nth element in the Fibonacci series as an analytic function of n and s obtained from the relation

$$F_n = \frac{1}{\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \cdot \left(\frac{1-\sqrt{5}}{2}\right)^n . \qquad (4)$$

The one –dimensional Fibonacci lattice, being one of the most studied quasicrystals, is determined by the substitution rule: L→LS, S→L where L(large) and S(small) are two elements.

The parameters of this structure are given by $\tau = (\sqrt{5}+1)/2 \approx 1.618$, which is known as golden mean. $\Delta = S_p - L_p$ and $d = S_p + (L_p - S_p)/\tau$, where d is the mean period of the lattice structure. When Lp = Sp the structure becomes periodic [24] and when Lp/Sp= τ (golden mean) , it becomes the Fibonacci chain multiple quantum wells structures [9],[19].There is currently intense interest in the realization of exotic quantum phases of matter that host quasiparticles with non-Abelian anyons partially due to the possibility of topological quantum computation (TQC)[15].Fibonacci anyons are powerful enough for universal TQC. So we are interested to study tunneling properties of Fibonacci quasicrystals, in aperiodic and asymmetric multiple quantum wells systems. We study resonant photonic states in photonic multiple quantum well (MQW) heterostructures in Fibonacci series consisting of two different photonic crystals. Using the transfer matrix method [6,11], we have obtained an expression for the transmission coefficient and hence tunneling probabilities of the MQWs Fibonacci chain heterostructures. From this, we have performed numerical simulations of the transmission spectra for GaAs/AlGaAs MQW heterostructures in Fibonacci sequence. We found the resonant tunneling and exotic nature of Fibonacci chain multiple quantum wells nanostructures that produce non Abelian anyons and suggest that Fibonacci quasiparticle' could form basis of future quantum computers [18].

2. Theory

Here we consider the most generalized aperiodic MQW structures where the well and barrier widths are all unequal as shown in Fig.1. The well and barrier materials are also different with different barrier heights and effective electron masses. The barrier widths are narrow enough so that the adjacent wells are coupled through the intervening barrier. We have considered the general quantum structure as shown in Fig. 1 where the widths of the three barriers are 2d, 2b and 2f (d≠ b≠ f.) and two wells having widths 2a and 2c (a ≠ c).

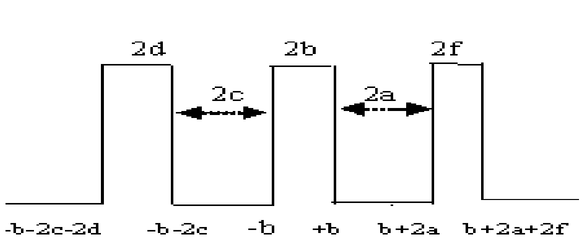


Fig.1: Array of aperiodic and Asymmetric MQWS

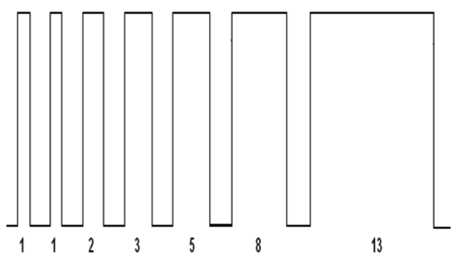


Fig.2: Array of Fibonacci series MQWS.

We solved the Schrödinger equation for the Quantum well and barrier region obtained by putting the appropriate values of the potential energy V, which may differ from region to region. The solutions take the following form as given by Equations (2) to (6).

$\Psi_{B1}(x) = A_{B1} \exp(ik_{B1}x) + B_{B1} \exp(-ik_{B1}x)$ (5)

for $-(b-2c-2d)<x<-(b+2c)$ inside the first barrier

$\Psi_{W1}(x)=A_{W1}\exp(ikw_1x)+B_{W1}\exp(-ik_{W1}x)$ (6)

for $-(b+2c)<x<-b$ inside the first well

$\Psi_{B2}(x) = A_{B2} \exp(ik_{B2}x) + B_{B2} \exp(-ik_{B2}x)$ (7)

for $-b<x<+b$ inside the second barrier.

$\Psi_{W2}(x) = A_{W2} \exp(ikw_2x) + B_{W2}\exp(-ik_{W2}x)$ (8)

for $b<x<b+2a$ inside the second well

$\Psi_{B3}x) = A_{B3} \exp(ik_{B3}x)$

for $(b+2a) < x<(b+2a+2f)$

$\Psi_{B3}(x) = A_{B3} \exp(ik_{B3}x)$ (9)

for $(b+2a) < x<(b+2a+2f)$ inside the third barrier

Now we applied boundary conditions and we got the transfer matrix for the coefficients of the wave function at the leftmost slab to those of the right most slabs is given below:

$$\begin{bmatrix} A_{BI} \\ B_{BI} \end{bmatrix} = \begin{bmatrix} M_{11}[j] & M_{12}[j] \\ M_{21}[j] & M_{22}[j] \end{bmatrix} \begin{bmatrix} A_{BI+1} \\ B_{BI+1} \end{bmatrix} \quad (10)$$

Where j = 1, 2, 3.. etc. Using equations (2-6), we obtained the coefficients of the wave function at the leftmost slab to those of the right most slabs of the FMQWs.

$$\begin{bmatrix} A_{BI} \\ B_{BI} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -ik_{wI}^{-1} \\ 1 & ik_{wI}^{-1} \end{bmatrix} [M_j] \begin{bmatrix} 1 & 1 \\ ik_{w_I} & -ik_{wI} \end{bmatrix} \begin{bmatrix} A_{BI+1} \\ B_{BI+1} \end{bmatrix} \quad (11)$$

Where M_J is the j^{th} transfer matrix corresponding to the j^{th} junction written as:

$$M_j = M_{Bn}(b_j)M_{wI}(a_j)M_{Bn}(b_{j+1}) \quad (12)$$

Doing this for all the slices X_1……….X_N, we obtained the complete transfer matrix M that connects the wave function on the left side of the potential with the one on the right side, $U_1 = MU_{N+1}$ where $M=T^1T^2 \ldots T^N$ (13)

The transmission coefficient derived by TMM method for the most general case simplified to some extent and becomes:

$\tau= [4k_Bk_W/ (m_wm_B)]^4/[T_1+2T_2+2T_3+2T_4]$ (14)

The expression for the transmission coefficient simplifies and becomes:

$\tau = 16(k_B k_W)^2/[(m_B m_W)^2\{(k_B/m_B) +(k_W /m_W)\}^4$
$\{(k_W/m_W)(k_B/m_B)\}^4 2\{(k_W/m_W)^2(k_B/m_B)^2\}^2\cos(4ck_B)\}]$ (15)

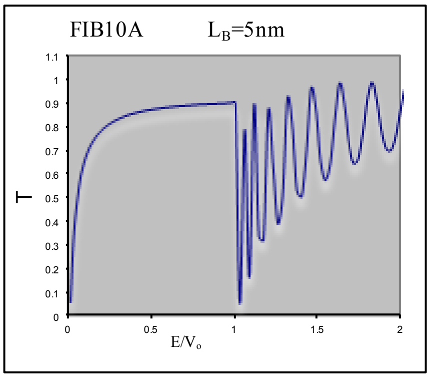
The Eigen value energy equation becomes

$(m_W k_B) /(m_Bk_W) = \tan(ck_B) /\tan(ck_W)$ (16)

where c is the well width. The coupling energies and tunneling probabilities in Fibonacci series multiple quantum wells are obtained from the above equations (15) and (16).The resonant tunneling across the Fibonacci multiple quantum wells (FMQW) system reached when τ =1. Here we have found the resonant tunneling energies in the FMQWs system from the τ vs E curve by a computer program using the search technique. Resonant tunneling across the MQWs system occurs for both the regions E < V0 and E.

3. Results and Discussions

In this section we present our results obtained numerically by using MATLAB programming for the transmission coefficient across Fibonacci chain multiple quantum wells containing asymmetric and aperiodic heterostructures. The materials chosen are AlGaAs/GaAs. The values of the parameters chosen are: Electron effective mass of m ($Al_xGa_{1-x}As$) = (0.063+0.083x) and the energy band gap is given $E_g(Al_xGa_{1-x}As$)= (1.9+0.125x+0.143x2).The mole fraction x= 0.47. The band gaps for AlGaAs and GaAs are respectively (AlGaAs) = 1.99eV and E_g (GaAs) = 1.42eV. The conduction band difference is ΔE_c =67% x=E_g(AlGaAs)–E_g (GaAs) = 0.38 eV. The electron effective mass of GaAs is m*(GaAs) = 0.067m_o; and that for AlGaAs is m*(AlGaAs) = 0.106m_o. Here we consider the wells are in Fibonacci series and barriers lengths are in same. The wells with are taken as 5nm, 5nm, 10nm, 15nm, 25nm….etc.



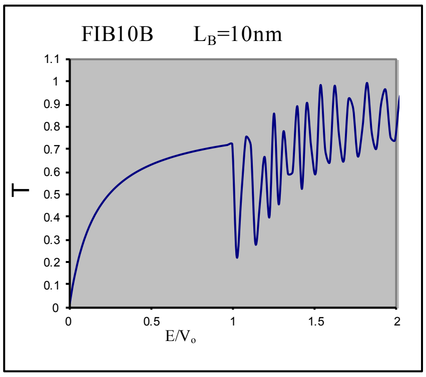
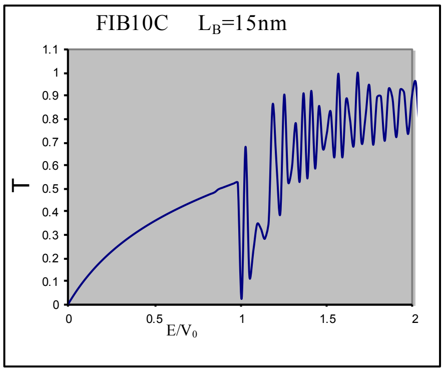


Fig.3 Fig. 4

Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for GaAs/AlGaAs/GaAs FMQWs for (a) barriers with L_B= 5 nm for FIB10A (Fig.3),
(b) with L_B= 10nm for FIB10A (Fig.3)



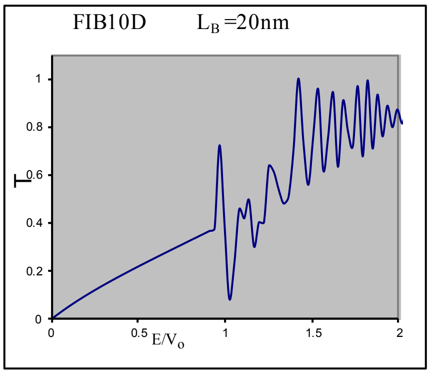


Fig.5 Fig.6

Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for

GaAs/AlGaAs/GaAs FMQWs for(C) barriers with L_B= 15 nm for FIB10C (Fig.5),
(d) barriers with L_B= 20 nm for FIB10D.

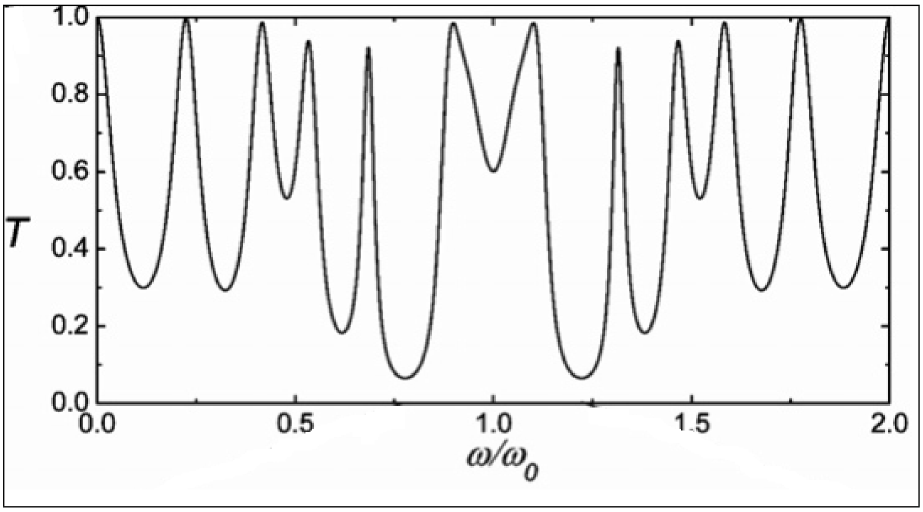


Fig.7

Fig.7: Variation of Transmission Coefficient (T) with reduced frequency (w/w_0).Normal-incidence transmission spectra of light beam into a aperiodic Fibonacci MQWs Photonic nanostructure.

Here we changed the barriers length of same Fibonacci structure FIB10 and found structures FIB10A, FIB10B, FIB10C and FIB10D for barriers lengths 5nm, 10nm, 15nm and 20nm respectively and the variation of transmission coefficient of electrons with normalized Eigen energies as shown above figures (Fig3 –Fig.6).Here we observed that when $E/V_o<1$, the transmission coefficient T increases from 0 to1 in a non-linear fashion. Beyond the normalized energy (E_{nor} = E/Vo) >1, there is resonance; i.e., there are quantized energy values where transmission reaches peak values sharply. When the barrier width decreases the peaks get more separated in energy and the normalized energy values vary continuously. If the widths of wells are reduced to the order of nanometres then the energy values inside the quantum wells will be quantized. This effect will be reflected in the nature of variation of the transmission coefficients and will have a crucial effect on carrier tunneling in FMQWs [12-14].The optical transmission spectrum for the FIB10 aperiodic Fibonacci MQWs sequence is shown as a function of the reduced frequency ($w/w_{0)}$ in Fig.7. The transmission spectrum presents a unique mirror symmetrical profile around the mid frequency (w/w_0) = 1. We found that the structure is transparent where T=1 at the reduced frequencies (w/w_0) = 0.898 and 1.101, forming two broad peaks, also distributed Symmetrically around (w/w_0) = 1.

4. Conclusions

The resonant energy states are obtained on the basis of the resonance condition T_N =1. During the resonance tunneling, the electron energy resonates at the bound states of the single quantum well [6],[8-9]. Here we see that the transmission coefficient exhibits a series of resonant peaks and valleys. The first series of resonant peaks are attributed to the resonant transmission tunneling through the fundamental quasibound state in the quantum well, while the second series is due to the tunneling through the first excited state. The width of the allowed band reduces significantly with increase in barrier width. Increase in barrier width causes decrease in the overlap interaction among the states of adjacent wells resulting in the decrease of band width. The energy-splitting phenomenon described here is in agreement with reported experimental results involving 1-D photonic crystals [8-10]. Our simulations have also shown that the total number of transmitted resonant states can be controlled by modifying the width of the photonic barriers in these nanostructures. Due to the enhanced light-matter coupling, the values of transmission coefficient in the FMQWs for higher generation orders are significantly stronger than those in the PQWs under the Bragg or anti Bragg conditions.

Here the excitation properties shown in Fibonacci tunneling produce exotic Fibonacci quasiparticles called anyons. The anyons tunneling in a double-layer system in Fibonacci MQWs, can transition to an exotic non-Abelian state that contains "Fibonacci" anyons which are powerful enough for universal topological quantum computation.

The resonant and excitation state describes here in proposed device might be useful for developing quantum computer. Fibonacci series MQWs could form the basis of future quantum computers.

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