Normalized Cut based Image Segmentation

Jagruti Vadher
Student
MEFGI

***Abstract* - Image segmentation is a process of grouping image pixels into image regions i.e. regions corresponding to particular characteristics. There are many methods developed for image segmentation. This paper presents one of the graph based method of image segmentation to partition an image depending on global criterion. Proposed paper implements normalized cut algorithm and compares it with graph based image segmentation.**

***Keywords* - Segmentation, Affinity measure, Normalized cut**

I. INTRODUCTION

In computer vision and image processing research, the importance and difficulties in perceptual grouping has been well studied. The main problem in perceptual grouping is how to partition an image into image subsets. One of the possible ways is to partition based on prior knowledge. The difficulty in specifying the knowledge in useful way i.e. in defining descriptors of an image. Partition based on low level descriptors may result in incorrect segmentation, so partition based on high level descriptors is needed to obtain correct segmentation.

Several approaches to image segmentation are available that includes based on thresholding, region growing and region based split and merge[4]. To implement any algorithm, it is important to determine what criterion we want to optimize and what algorithms are available for that purpose.

The proposed normalized cut based image segmentation takes a global feature descriptor as a weighted graph and reduces image segmentation to optimal partitioning.

Image as graphs

An image can be represented by graph with node at each pixel location. Edges represents relationships within pixel contents. Edges can be further weighted as similarity criterion as defined by affinity measure.

Affinity measure includes two measures: distance measure and intensity measure.

Distance measure: One way to define similarity between pixels is to find spatial distance between them. For this, the measurement that quickly falls with distance is desired, so we can define affinity as an exponential function of distance.

$$\mathrm{aff}(\boldsymbol{x}, \boldsymbol{y}) = \exp\left\{-\left((\boldsymbol{x}-\boldsymbol{y})^t(\boldsymbol{x}-\boldsymbol{y})/2\sigma_d^2\right)\right\}$$

Where σ_d controls how far away pixels effects current location.

Intensity measure: Similar argument can be made regard to intensity. Pixels in near- by intensity place may be considered similar. Again exponential measure is needed to define how far away pixels iteract.

$$\mathrm{aff}(\boldsymbol{x}, \boldsymbol{y}) = \exp\left\{-\left((I(\boldsymbol{x})-I(\boldsymbol{y}))^t(I(\boldsymbol{x})-I(\boldsymbol{y}))/2\sigma_I^2\right)\right\}$$

II. GRAPH CUTS FOR IMAGE SEGMENTATION

Sorting pixel neighborhood based on affinity measure can produce good segmentation results. By formulating the problem as an optimization of affinity measure and solving using Lagrange multiplier[6], the following linear system needed to be solved:

$$Aw=\lambda w$$

Where w is the solution given by eigenvector of affinity matrix A.

III. NCUT BASED IMAGE SEGMENTATION

3.1.Graph Partitioning

The set of points in a feature space is presented by a weighted undirected graph G=(V,E). Nodes of a graph are points in a feature space. An edge is formed between every pair of nodes in a graph and weight on each edge is defined as a function of similarity between nodes i and j. A graph is divided into two complementary disjoint sets A and B=V-A and dissimilarity between sets is defined as total weight of edges that has been removed. This relates mathematical formulation of a cut:

$$\mathrm{cut}(A, B) = \sum_{u\in A, v\in B} w(u, v) \quad \ldots\ldots(1)$$

Related Approaches

Problem formulation is to find optimal bi partitioning of a graph. The problem reduces to find cost function that gives perceptually good results. Global optimum is achieved if we minimize the cut value[3]. If N is cardinality of V, then number of possible partitions is 2^N. Wu and Leahy proposed a clustering method based on minimum cut criterion [5]. They proposed the method to partition the graph into k subgraphs such that maximum cut across the partition is minimized. They introduced an efficient way to bi-partition the graph recursively and produces good segmentation results. But the results are affected by the graph outliers. The following example shows the case of bad partition.

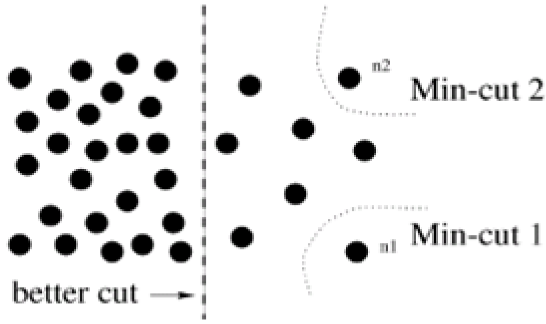

Figure 1 A case where minimum cut gives a bad partition.

Cox proposed a modified cost function (2), where weight is some function of A, i.e. sum of elements in A.

$\mathrm{Wcut}(A) = \frac{\mathrm{cut}(A, V-A)}{\mathrm{weight}(A)}$ ……(2)

3.2.Normalized Cut

Shi and Malik [1,2]introduced a modified cost function, called normalized cut to eliminate the problem of graph outliers. Instead of using the value of total edge weight connecting two partitions, proposed method computes the cut cost as a fraction of total edge connections to all nodes:

$\mathrm{Ncut}(A,B) = \frac{\mathrm{cut}(A,B)}{\mathrm{assoc}(A,V)} + \frac{\mathrm{cut}(A,B)}{\mathrm{assoc}(B,V)}$ ……(3)

where

$$\mathrm{assoc}(A,V) = \mathrm{vol}(A) = \sum_{u\in A, t\in V} w(u,t)$$

is the total connection from nodes in A to all nodes in the graph. Ncut value will not be small for isolating points in the graph. Normalized cut is an efficient algorithm for graph partitioning. If partition of the graph into two complementary sets is A and B and x is an N=|V| dimensional indicator vector, $x_i = 1$, if node i is in A, and -1 otherwise.

Let $d_i = \sum_j w(i,j)$ be the total connection from node i to all other nodes. Ncut can be rewritten as (4),

$$\mathrm{Ncut}(A,B) = \frac{\mathrm{cut}(A,B)}{\mathrm{assoc}(A,V)} + \frac{\mathrm{cut}(A,B)}{\mathrm{assoc}(B,V)}$$

$$= \frac{\sum_{x_i>0, x_j<0} -w_{ij} x_i x_j}{\sum_{x_i>0} d_i} + \frac{\sum_{x_i<0, x_j>0} -w_{ij} x_i x_j}{\sum_{x_i<0} d_i} \quad \text{……..(4)}$$

$$\min_x Ncut(x) = \min_y \frac{y^T(D-W)y}{y^T D y}$$

Let D= $\mathrm{diag}(d_1, d_2, \ldots, d_n)$ and w(i,j) =w_{ij}. Finding global optimum reduces to (5)

where $y_i \in \left\{1, \frac{-\sum_{x_i>0} d_i}{\sum_{x_i<0} d_i}\right.$, $y_i \in \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} = 0$ … (5)

If y is relaxed to take on real values, (5) can minimized by solving generalized eigen value system,

$(D - W)y = \lambda D y$ …(6)

Constrain on y comes from the corresponding indicator vector x. The second smallest eigen vector of eigen system (6) satisfies the normality condition. y_1 is the real valued solution to eigen value system.

$$y_1 = \min_{y^T \in \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix} = 0} \frac{y^T(D-W)y}{y^T D y}$$

3.3.Grouping algorithm

The eigenvector corresponding to second smallest eigen value is the real valued solution that optimally partitions the entire graph, third eigen value partitions the first into two etc. Thus entire graph can be partitioned into subgraphs using the eigen vector with the next eigen value. Since approximation error accumulates with every eigen vector taken, so solution based on higher eigen vector becomes unreliable. Eigenvalue computation is very expensive. So Lanczos method for computation of eigen vector of very sparse matrix is used where only couple of eigen vectors is needed. Computation cost of Lanczos algorithm is typically less than $O(n^{3/2})$, where n is the number of nodes in the graph

Graph is partitioned using the second smallest eigen vector. Eigen vector takes on continuous values, so we need to define a splitting point for every eigen vector. Number of evenly spaced spiltting points in the range of eigenvector is checked and point that gives minimum Ncut value is chosen as a splitting point. The other approach is to take 0 or median value as a splitting point, but it is not reliable because of approximation error. The algorithm is recursively applied to every subgraph until Ncut value exceeds certain limit.

3.4. The Ncut algorithm can be summarized as following

- Define a feature description matrix for an image and a weighting function.
- Define a weighted undirected graph G=(V,E).
- Solve $(D - W)x = \lambda Dx$ for eigenvectors with smallest eigenvalues.
- Use the eigenvector with second smallest eigenvalue to bipartition the graph by finding the splitting point such that Ncut value is minimized.
- Recursively partition the segmented parts until Ncut exceeds the certain limit.

IV. EXPERIMENTAL RESULTS

Two methods described above are implemented in Matlab on an image. Image was created to be 25×25 pixels.

Results for Graph Cut Method

The experiment involves 3-segment image, with added Gaussian noise with zero mean and small variance. Combination of distance+intensity affinity measure is used to obtain good segmentation result. The results plotted below shows various components of analysis along with final segmentation result.

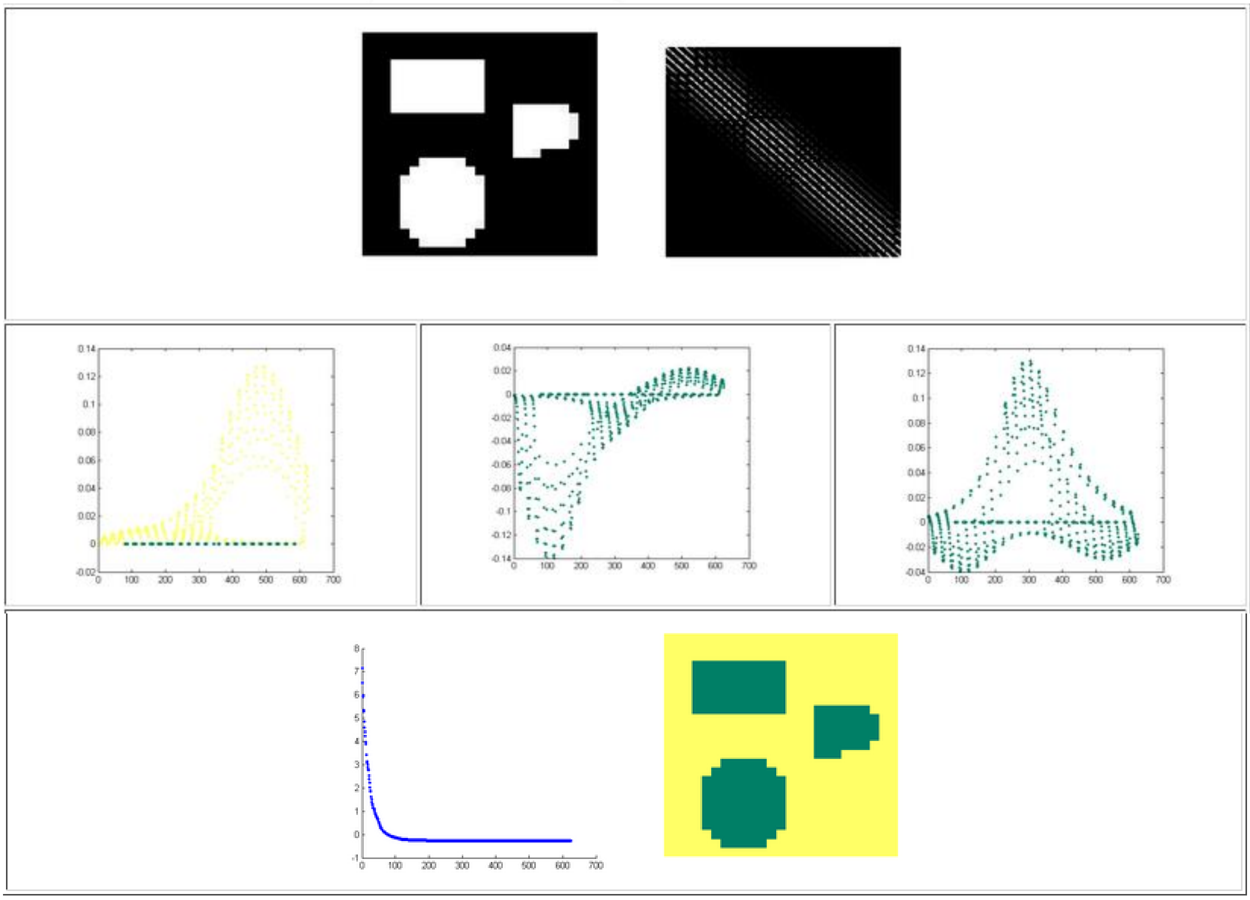

Fig 2 Results for synthetic image. Top - (1) original image (2) affinity matrix. Middle - (1,2,3) eigenvectors used for segmentation. Bottom - (1) eigenvalue distribution, (2) final segmentation

From the results, affinity matrix is not useful and components of segmentation cannot be determined from the affinity matrix. The eigenvalue distribution is also not useful to determine components of segmentation.

Results for normalized cut based method

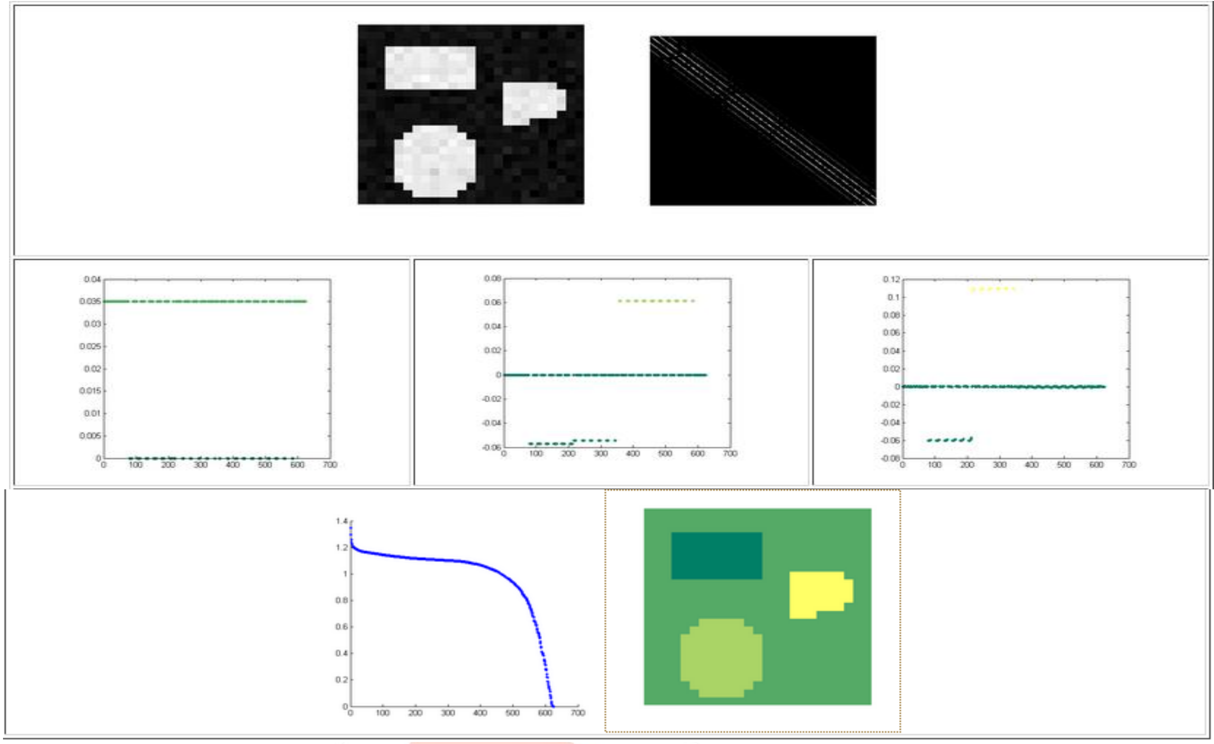

Fig 3 Results for synthetic image. Top - (1) original image, (2) affinity matrix.Middle-(1,2,3) eigenvectors used to segmen. Bottom - (1) eigenvalue distribution, (2) final segmentation

Here, distance+intensity based affinity matrix is used for segmentation and only positive weights are used to threshold the labels.

V. CONCLUSION

Discrimination is much more clear in Ncut method compared to graph cut method. From the results, graph cut method is not able to differentiate between the components similar in intensity even if they are spatially well separated. Prior knowledge of components is an advantage, since it is not clear from eigenvalue distribution. Hierarchical methods should be used to invlve more eigenvectors till segmentation accounts for a certain amount variation present in the image.

REFERENCES

[1] J. Shi and J. Malik, "Normalized Cuts and Image Segmentation," Int. Conf. Computer Vision and Pattern Recognition, San Juan, Puerto Rico, June 1997.

[2] J. Shi and J. Malik, "Normalized Cuts and Image Segmentation," Technical Report CSD-97-940, UC Berkley, 1997.

[3] A. Pothen and H. Simon, “Partitioning Sparse Matrices with Eigenvectors of Graphs”, IBM Journal of Research and Development, pp. 420-425, 1973.

[4] Gonzalez R. C., Richard E. Woods (2007), “Digital Image Processing”, 2nd ed., Beijing: Publishing House of Electronics Industry.

[5] Wu, Z., and Leahy, R., “An Optimal Graph Theoretic Approach to Data Clustering: Theory and Its Application to Image Segmentation”, PAM I (15), No. 11, pp. 1101-1113, 1993.

[6] Everett H. (1963), “Generalized Lagrange multiplier method for solving problems of optimum allocation of resources,” Oper. Res., vol. 11, pp.399–417