MATHEMATICAL MODELING OF OPEN LOOP PMDC MOTOR USING MATLAB/SIMULINK

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ABSTRACT: Linear Dynamical models of physical systems One of the most used actuators in control systems is a direct current (DC) motor. The general output variable of this actuator can be angular speed or angular displacement motion, but, coupled with wheels or drums and cables, can provide translation motion. This paper proposes a state space model of the DC motor built for constant flux and considering two inputs: supply voltage and resistive torque. The two states of the resulting model are represented by angular speed and step response any of these states can be an output variable for a simulation model. Consequently, the system's model has two inputs and two outputs. For the system's simulation a VI is built where the most important element is a Matlab script which contains the matrices A, B, C, D of the state-space model, the independent variable time and the Matlab simulation function. The motor's parameters are given by digital controls on the panel so that these parameters can be interactively modified. To generate inputs, two case structures are used where the input variables can be set: impulse, step and ramp. Here it is also possible to set the signal amplitude and duration, either by step input or torque N.m. The output signals are live display, either one by one or together, on the WAVEFORM GRAPH.

KEYWORDS: DC Motor, Out Put Response Analysis, MATLAB/SIMULINK.

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1.0 INTRODUCTION

Direct-current (de) motors are one of the most widely used prime movers in industry today. Years ago, the majority of the small servomotors used for control purposes were ac. In reality, ac motors are more difficult to control, especially for position control, and their characteristics are quite nonlinear, which makes the analytical task more difficult. DC motors, on the other hand, are more expensive, because of their brushes and commutators, and variable-flux de motors are suitable only for certain types of control applications.

Before permanent-magnet technology was fully developed, the torque per unit volume or weight of a de motor with a permanent-magnet (PM) field was far from desirable. Today, with the development of the rare-earth magnet, it is possible to achieve very high torque-to-volume PM de motors at reasonable cost. Furthermore, the advance made in brush-and-commutator technology have made these wearable parts practically maintenance-free. The advancements made in power electronics have made brushless dc motors quite popular in high-performance control systems. Advanced manufacture techniques have also produced dc motors with ironless rotors that have very low inertia thus achieving very high torque-to-inertia ratio. Low-time-constant properties have opened new applications for dc motors in computer peripheral equipment such as tape drive printers, disk drives, and word processors, as well as in the automation and machine-t industries.

1.1 DC MOTORS IN CONTROL SYSTEMS

In laboratory for academician, the model and the Voltage(V) and current (I) in the classroom to simulate the running of a DC motor and also to learn how to incorporate the latter into a control loop. To start with, based on a functionality structure of the DC motor and on the laws of physics and electricity that rule the motor's operation, students build a mathematical model. Afterwards, using the mathematical model in a series of simulation experiments in Direct-current (dc) motors are one of the most widely used prime movers in industry today. Years ago, the majority of the small servomotors used for control purposes were ac. In reality, ac motors are more difficult to control, especially for position control, and their characteristics are quite nonlinear, which makes the analytical task more difficult.

The dc motor is basically a torque transducer that converts electric energy into mechanical energy. The torque developed on the motor shaft is directly proportional to tie fie flux and the armature current. Since dc motor are used extensively in control system, it is necessary to establish mathematical model for analytical purpose dc motor for controls applications.

We use the equivalent circuit diagram as shown in below Fig. to represent a PM dc motor.
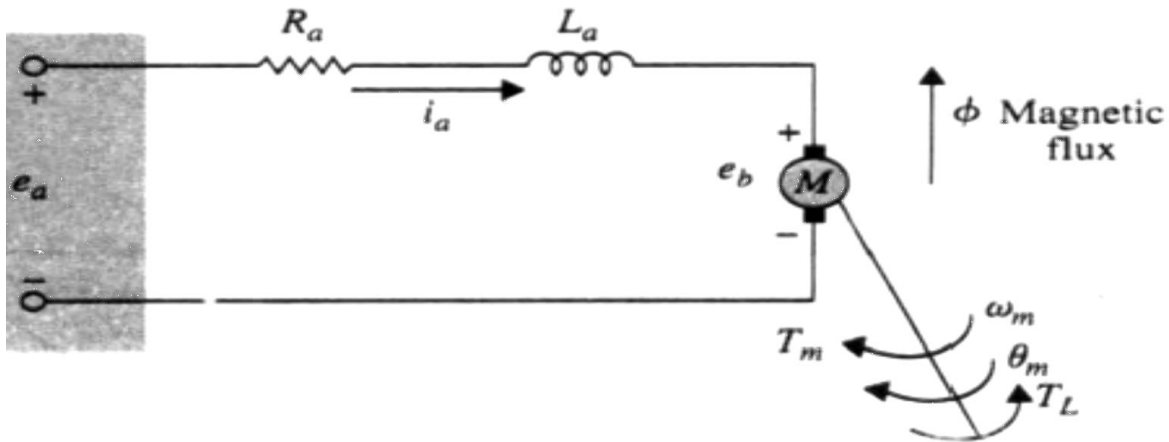
Figure 1. EQUIVALENT MODEL OF A SEPARATELY EXCITED DC MOTOR.

1.2 Mathematical Modeling of PM DC Motors Functionality Equations: -

Since dc motor are used extensively in control system, it is necessary to establish mathematical model for analytical purpose dc motor for controls applications. We use the equivalent circuit diagram in Fig. 1. to represent a PM dc motor.

The armature is modeled as a circuit with resistance R_a connected in series with an inductance L_a, and a voltage source e_a, e_b representing the Back-Emf (electromotive force) in the armature when the rotor rotates. The motor variables and parameters are defined as follows:

$e_a(t)$	= Applied voltage	$e_m(t)$	= Rotor displacement
R_a	= Armature resistance	K_t	= Torque constant
$i_a(t)$	= Armature current	K_b	= Back-emf constant
L_a	= Armature inductance	Φ	= Magnetic flux in the air gap
$e_b(t)$	= Back emf	$W_m(t)$	= Rotor angular velocity
$T_L(t)$	= Load torque	J	= Rotor inertia
$T_m(t)$	= Motor torque	B_m	= Viscous-friction coefficient

With reference to the circuit diagram of Fig. 1, the control of the dc motor is applied at the armature terminals in the form of the applied voltage $e_a(t)$.

For linear analysis, we assume that the torque developed by the motor is proportional to the air-gap flux and the armature current.

$$T_m(t) = K_m(t) \times \Phi \times i_a(t)$$
………(1)

Since Φ is constant, Eq. (1) is written as,

$$T_m(t) = K_t(t) \times i_a(t) \qquad \text{……. (2)}$$

Where, K_t is the **torque constant** in N-m/A, lb-ft/A, or oz-in/A.

Starting with the control input voltage $e_a(t)$, the cause-and-effect equations for the motor circuit in Fig.-1 are:

$$e_a(t) = R_a\, i_a(t) + L_a\frac{di_a}{dt} + e_b(t) \qquad \text{..……(3)}$$

$$T_m(t) = B_m\, w_m(t) + J\,\frac{dw_m(t)}{dt} + T_L(t) \qquad \text{..……(4)}$$

$$T_m(t) = K_t(t) \times i_a(t) \qquad \text{…….. (5)}$$

$$e_b(t) = K_b(t)\,\frac{d\theta_m(t)}{dt} = K_b(t)\, w_m(t) \qquad \text{..……(6)}$$

Where $T_L(t)$ represents a load frictional torque such as Coulomb friction. Consider that the applied voltage $e_a(t)$ is the cause; Eqn. (3) considers that $\dfrac{di_a(t)}{dt}$ is the immediate effect due to $e_a(t)$; In Eqn. (2) $i_a(t)$ Causes the torque $T_m(t)$;

Eqn. (6) defines the back emf; and, finally, in Eqn. (8), the torque $T_m(t)$ causes the angular velocity $w_m(t)$ For modeling purposes it is often convenient to derive a state-space representation of a motor. Eqn. (3) and (4) are rewritten as,

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a}\, i_a(t) - \frac{1}{L_a} e_a(t) - \frac{1}{L_a} e_b(t)$$

$$= -\frac{R_a}{L_a}\, i_a(t) - \frac{1}{L_a} e_a(t) - \frac{K_b}{L_a}\, w_m(t) \qquad \text{……. (7)}$$

$$\frac{dw_m(t)}{dt} = \frac{1}{J}\, T_m(t) - \frac{1}{J}\, T_L(t) - \frac{B_m}{J}\, w_m(t)$$

$$= \frac{K_t}{J}\, i_a(t) - \frac{1}{J}\, T_L(t) - \frac{B_m}{J}\, w_m(t) \qquad \text{…… (8)}$$

And a matrix representation of Eqn. (7) and (8) follows directly the general mathematical model of the DC electrical motor. Now it can build the mathematical model in state space form so that it is possible to use it in a Matlab simulation. This equation will be used in MATLAB / SIMULINK to analyze speed response of the motor:

$$\begin{bmatrix} \dfrac{di_a(t)}{dt} \\ \dfrac{dw_m(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_a}{L_a} & -\dfrac{K_b}{L_a} \\ \dfrac{K_t}{J} & -\dfrac{B_m}{J} \end{bmatrix} \times \begin{bmatrix} i_a(t) \\ w_m(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_a} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix} \times \begin{bmatrix} e_a(t) \\ T_L(t) \end{bmatrix}$$

Following are data of motor:

R_a = 1Ω

L_a = 0.005H

K_b = 0.1

K_t = 10 N*M

J = 2

B_m = 0.5

2.0 MATLAB SIMULATION BLOCKS:
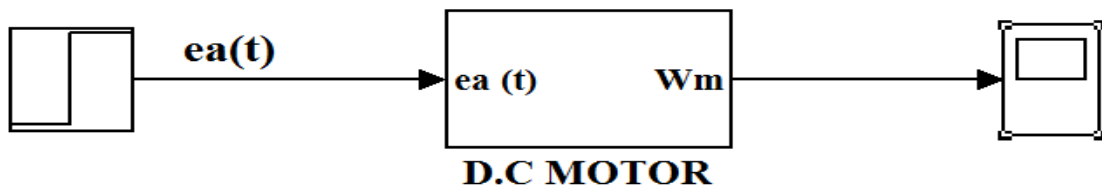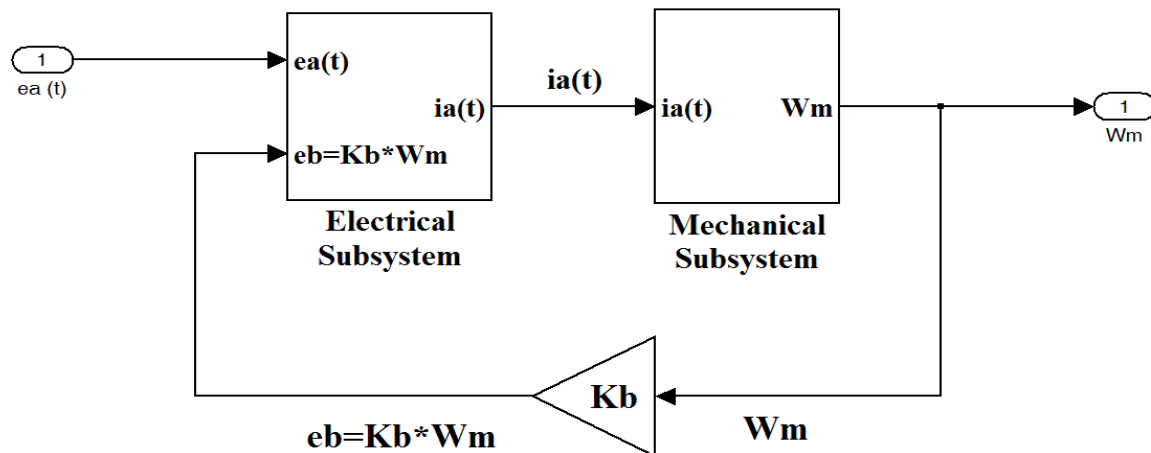

Figure 2. BLOCK DIAGRAM OF DC MOTOR

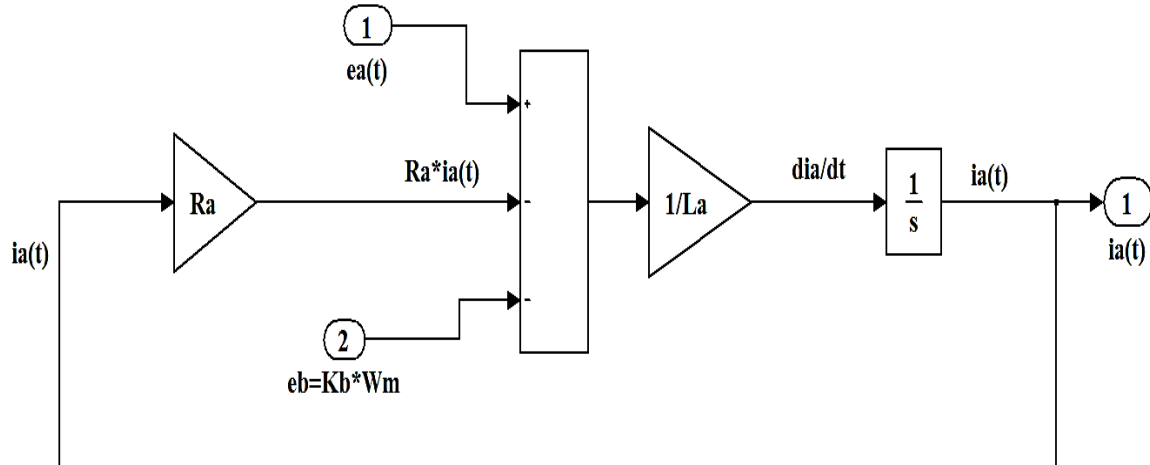Figure 3. CONTENTS OF DC MOTOR

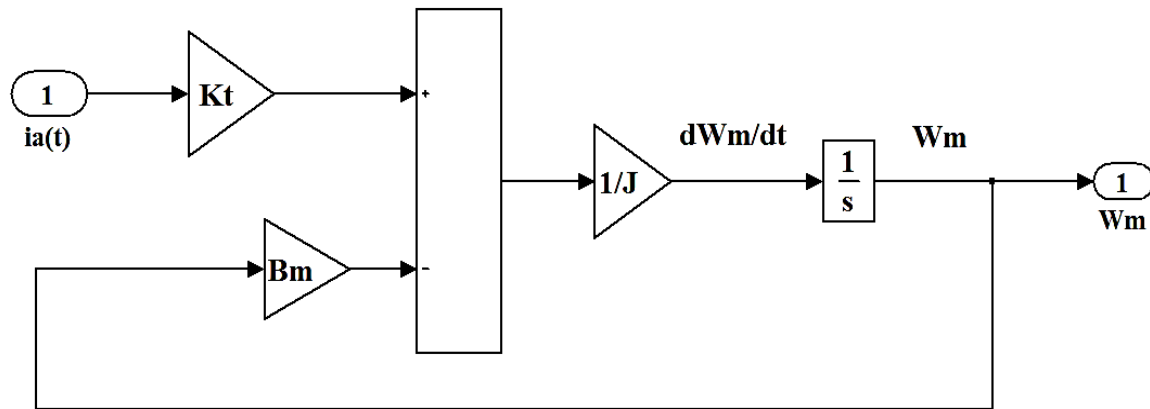**Figure 2 CONTENT OF ELECTRICAL SUBSYSTEM**

Figure 5. CONTENT OF MECHANICAL SUBSYSTEM

Although a de motor by itself is basically an open-loop system, the diagram of Fig. 4 and the block diagram of Fig. 5 show that the motor has a "built-in" feedback loop caused by the back emf. Physically, the back emf represents the feedback of a signal that is proportional to the negative of the speed of the motor. As seen from Eq. (7 & 8), the back emf constant K_b represents an added term to the resistance R_a and the viscous-friction coefficient B_m. Therefore, the back-emf effect is equivalent to an "**electric friction**," which tends to improve the stability of the motor; and in general, the stability of the system.

2.1 Relation between K_t and K_b .

Although functionally the torque constant K_t; and back-emf constant K_b are two separate parameters, for a given motor their values are closely related. To show the relationship.
we write the mechanical power developed in the armature as

$$P = e_b\,(t) \times i_a\,(t) \qquad\qquad \text{……... (9)}$$

The mechanical power is also expressed as,

$$P = T_m\,(t) \times w_m\,(t) \qquad\qquad \text{……... (10)}$$

where, in SI units, $T_m\,(t)$ is in N-m, and $w_m\,(t)$ is in rad/sec. Now substituting Eqns. (2) and (6) in Eq. (9)

we get,

$$P = T_m(t) \times w_m(t) = K_b\, w_m(t) \times \frac{T_m(t)}{K_t} \qquad \text{……... (11)}$$

from which we get,

$$K_b\, (\text{V/rad/sec}) = K_t\, (\text{N-m/A})$$

Thus, we see that in SI units, the values of K_b and K_t are identical if K_b is represented in V/rad/sec and K_t is in N-m/A.

3.0 Result

All The equation will be used in MATLAB / SIMULINK in to analyze speed response of the motor
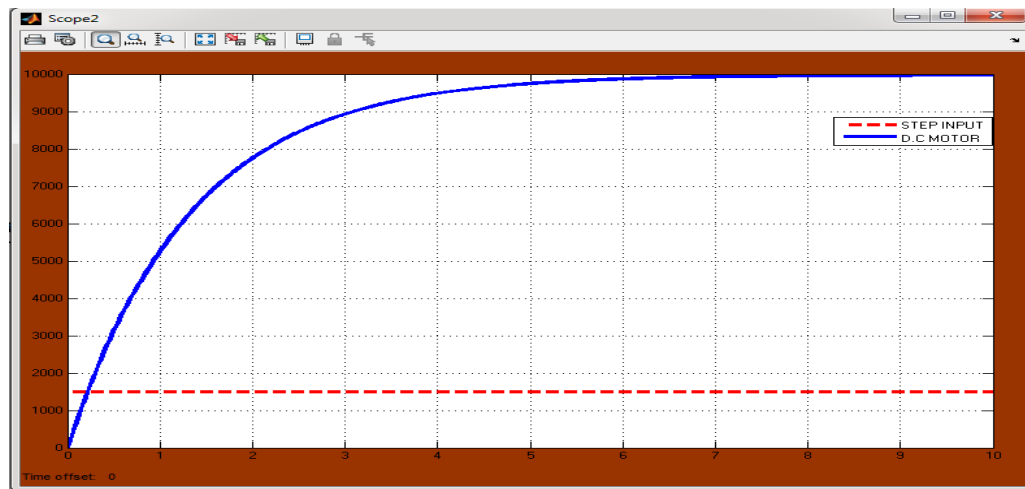

Figure 6. INPUT AND OUTPUT RESPONSE OF THE MOTOR

4.0 APPLICATION

- In many industries such as paper mills, rolling mills, printing machine tools, excavators and cranes etc   the dc motor is used for way a product from one place to another on the conveyer belt.
- Robotics arm for peak and place purpose.
- In electrical lab purpose.
SOFTWARE used is MATLAB

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