

Analysis of Equalization in Digital Communication

¹Pratibha Mishra, ²Mohd. Ayub Khan, ³Waris Junaid

¹Shivdaan Singh Institute of Technology and Management Aligarh

²Anand Engineering College, Agra

³Shivdaan Singh Institute of Technology and Management Aligarh

Abstract - Equalization Goes Parallel With Intersymbol Interference (ISI) Which Is Been Created By Multipath Within Its Respected Time Dispersive Channels. Suppose If The Bandwidth Of The Modulation Is Greater Than Bandwidth Of The Coherence Of The Radio Channel And At The Same Time Modulation Pulses Are Spread In Time, This Is The Situation For ISI To Occur. Receiver Having An Equalizer Combats An Average Range Of Delay And The Channel Amplitude Which Is Expected. Equalizers Must Be Adaptive In Nature Since The Channel Is Generally Time Varying. To Cancel The Interference While Providing Diversity, A Variety Of Adaptive Equalizers Can Be Used In Radio Channels. The Time Varying Characteristics Of The Mobile Channel Is Tracked By Them Because The Mobile Fading Channel Is Time Varying, Equalizer Must Be Adaptive In Nature.

Keywords - Intersymbol Interference, Dispersive channel, Equalizer, Radio channel, Fading channel

I. INTRODUCTION

Equalization plays a vital role in many modern communication systems. Adaptive equalization is a particular example of adaptive signal processing. It is an important technique to combat distortion and interference in communication links. The approach which is conventional to communication channel equalization is based on adaptive linear system theory. Channel equalization is an important subsystem in a communication receiver. A technique used to remove inter-symbol interference (ISI) produced due to the limited bandwidth of the transmission channel is called Equalization [1]. When the channel is band limited, symbols transmitted through will be dispersed. This causes previous symbols to interfere with the next symbols, yielding the ISI. Also, multipath reception in wireless communications causes ISI at the receiver. Thus, equalizers are used to make the frequency response of the combined channel equalizer system flat. The main motive of an equalizer is to reduce the ISI as much as possible to maximize the probability of correct decisions. Channel equalization is a simple way of mitigating the detrimental effects caused by a frequency-selective and/or dispersive communication link between sender and receiver. For this demonstration, all signals are assumed to have a digital baseband representation. During the training phase of channel equalization, a digital signal $s[n]$ that is known to both the transmitter and receiver is sent by the transmitter to the receiver. The received signal $x[n]$ contains two signals: the signal $s[n]$ filtered by the channel impulse response, and an unknown broadband noise signal $v[n]$. The goal is to filter $x[n]$ to remove the inter-symbol interference (ISI) caused by the dispersive channel and to minimize the effect of the additive noise $v[n]$. Ideally, the output signal would closely follow a delayed version of the transmitted signal $s[n]$. There are three classic equalizer algorithms namely Zero Forcing (ZF) Algorithm, Least Mean Square (LMS) Algorithm, Recursive Least Square (RLS) Algorithm.

1.1 Zero Forcing (ZF) Algorithms

The equalizer coefficients are chosen such that they force the samples of the combined channel and equalizer impulse response to zero at all except one of the NT spaced sample points in the tapped delay line filter. By allowing the number of coefficients increase without limit, an infinite length equalizer with zero ISI at the output can be obtained. If the frequency response of the channel is $H(s)$, then the input is multiplied with the reciprocal of it, in order to cancel all ISI from the received signal. Thus, an infinite length, zero, ISI equalizer is as simple as an inverse filter that inverts the folded frequency response of the channel. Such an infinite length equalizer is usually implemented by a truncated length version.

$$H_{ch}(s)H_{eq}(s) = 1, |f| < \frac{1}{2T} \quad (\text{Eq.4.1})$$

When each of the delay elements provides a time delay equal to the symbol duration T , the frequency response $H_{eq}(s)$ of the equalizer is periodic with a period equal to the symbol rate $1/T$. The combined response of the channel with the equalizer satisfies the Nyquist Criterion as shown in the above equation.

$$e_k = d_k - \tilde{d}_k = x_k - \tilde{d}_k \quad (\text{Eq.4.2})$$

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1.2 Least Mean Square (LMS) Algorithm

The Least Mean Square (LMS) algorithm, given by Widrow and Hoff in 1959 is an adaptive algorithm, that uses gradient-based method of steepest decent. Estimates of the gradient vector are used from the available data. LMS involves an iterative procedure

that successively corrects the weight vector along the negative direction of the gradient vector that ultimately leads to minimum mean square error. The principle used is the minimization of the mean square error (MSE) between the desired output and the actual output. The prediction error is given by

$$e_k = d_k - \tilde{d}_k = x_k - \tilde{d}_k \text{ (Eq.4.2)}$$

To compute the mean square error at time instant k, we use

$$\mathcal{E} = E [e_k * e_k] \text{ (Eq.4.3)}$$

The LMS algorithm minimizes the mean square error given in the above equation. Compared to other algorithms LMS algorithm is relatively simple; it neither requires correlation function calculation nor does it require matrix inversions.

1.3 Recursive Least square (RLS) Algorithm

The Recursive least squares (RLS) adaptive filter is an algorithm which recursively finds the filter coefficients to minimize a weighted linear least squares cost function that relates to the input signals. The convergence rate of the gradient-based LMS algorithm is very slow, especially when the eigen values of the input covariance matrix have a very large spread. In order to achieve faster convergence, complex algorithms involving additional parameters are used. Faster converging algorithms are based least squares approach, in contrast with the statistical approach used in the LMS algorithm. That is, rapid convergence relies on error measures expressed in terms of a time average of the actual received signal instead of a statistical average. This leads to the family of powerful, complex, adaptive signal processing techniques known as recursive least squares (RLS), which aims at improving the convergence of adaptive equalizers. The least square error based on the time average is given as

$$J(n) = \sum_{i=1}^n \lambda_{n-1} e^*(i, n) e(i, n) \text{ (Eq. 4.4)}$$

To obtain the minimum of least square error J (n) , the gradient of J (n) in equation (4.4) is set to zero,

$$\frac{\partial J(n)}{\partial w} = 0 \text{ (Eq. 4.5)}$$

II. COMPARISON OF EQUALIZATION ALGORITHMS

RLS algorithms have similar convergence and tracking performances, much better than the LMS algorithm. However, these RLS algorithms usually have high computational requirement and complex program structures. Also, some RLS algorithms tend to be unstable.

Algorithm	Number of Multiply Operations	Advantages	Disadvantages
LMS	2N+1	Low computational complexity, simple Program	Slow convergence, poor Tracking
RLS	2.5N ² + 4.5N	Fast convergence, good tracking ability	High computational complexity

Table 2: Comparison of Various Algorithms for Adaptive Equalization

$$\mathcal{E} = E [e_k * e_k] \text{ (Eq.4.3)}$$

2.1 Equalization problem of WPM

In OFDM, if the delay spread of the channel is shorter than the cyclic prefix, a pre-equalizer is not required. For longer delay spread, the pre-detection equalizer aims at shortening the apparent channel impulse response to a value lower or equal to the cyclic prefix duration. If it succeeds, the symbol at the input of the DFT is free from inter-symbol interference. For a channel with long impulse response, the length of the prefix required leads to a significant loss in capacity and transmit power. For WPM however, the use of a cyclic prefix is not possible as the successive symbols overlap. Hence it gives rise to both inter-symbol interference (ISI) and inter-symbols inter-carriers interference (ISCI) that have to be cancelled by the equalization scheme.

$$ISl_j(n) = \sum_{r=-\infty}^{\infty} x_j(r) h_{jip}(n_j(n-r)) \tag{Eq. 4.6}$$

$$ICl_j(n) = \sum_{\substack{k=0 \\ k \neq j}}^{M-1} \sum_{r=-\infty}^{\infty} x_j(r) h_{jip}(n_j n - nkr) \tag{Eq. 4.7}$$

Where $h_{jip}(n) = h_k(n-p) * h_j^*(-n) = \sum_{m=-\infty}^{\infty} h_k(m-p)h_j^*(m-n)$ and h_k is the sub channel impulse response. The power of IS1 and ICI, σ_{IS1}^2 , σ_{ICI}^2 are determined, respectively, as

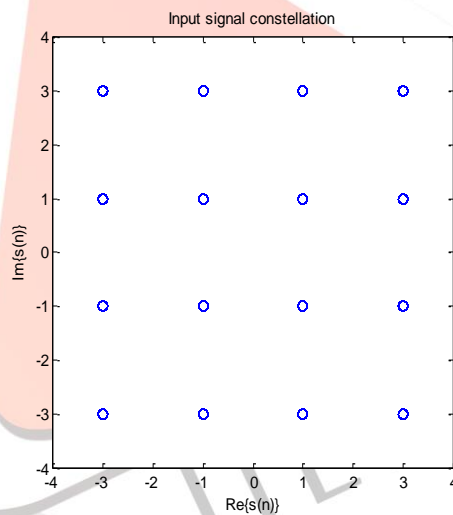
$$\sigma_{IS1j}^2 = \sum_{m=-\infty}^{\infty} |h_{jip}(n,m)|^2 \tag{Eq. 4.8}$$

$$\sigma_{ICIj}^2 = \sum_{\substack{k=0 \\ k \neq j}}^{M-1} \sum_{m=-\infty}^{\infty} |h_{jip}(n_k m)|^2 \tag{Eq. 4.9}$$

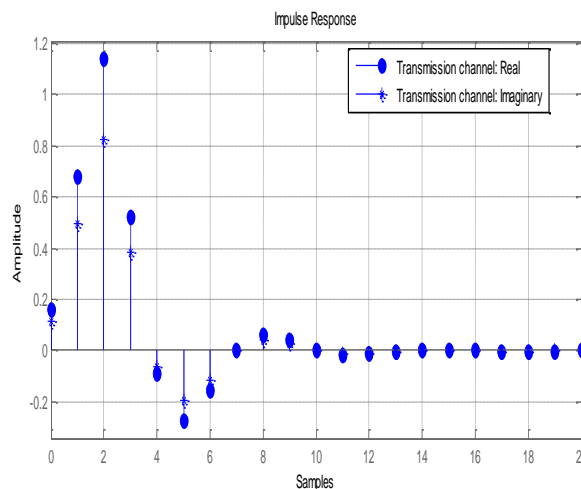
So in multipath channels, equalization of WPM is more complex than that of OFDM. This is essentially because of the use of cyclic prefix that gives an edge to OFDM, when compared to overlapping multicarrier schemes such as WPM.

III. RESULTS

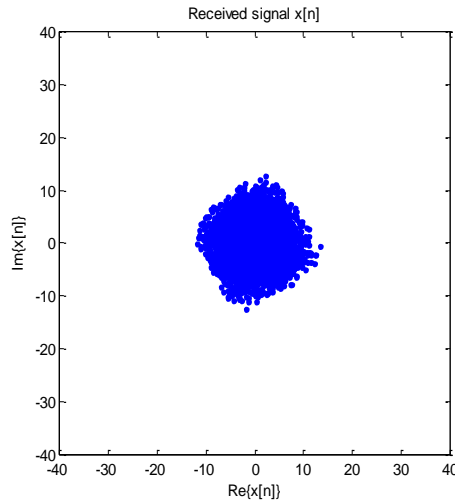
A digital signal carries information through its discrete structure. There are several common baseband signaling methods. We shall use a 16-QAM complex-valued symbol set, in which the input signal takes one of sixteen different values given by all possible combinations of $\{-3, -1, 1, 3\} + j\{-3, -1, 1, 3\}$, where $j = \text{sqrt}(-1)$. Let's generate a sequence of 5000 such symbols, where each one is equiprobable.



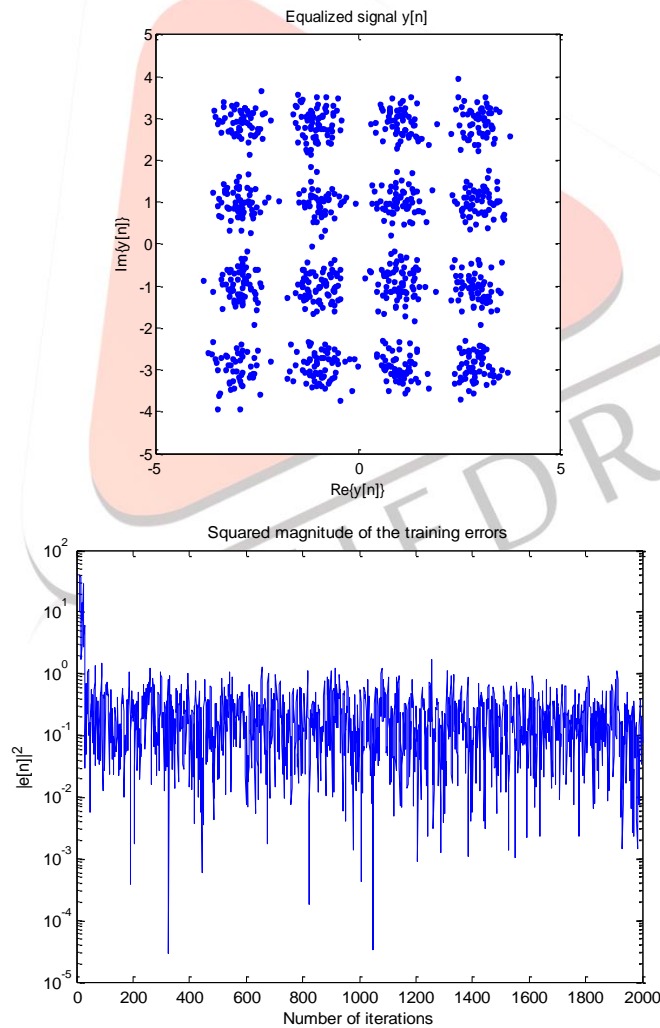
The transmission channel is defined by the channel impulse response and the noise characteristics. We shall choose a particular channel that exhibits both frequency selectivity and dispersion. The noise variance is chosen so that the received signal-to-noise ratio is 30 dB



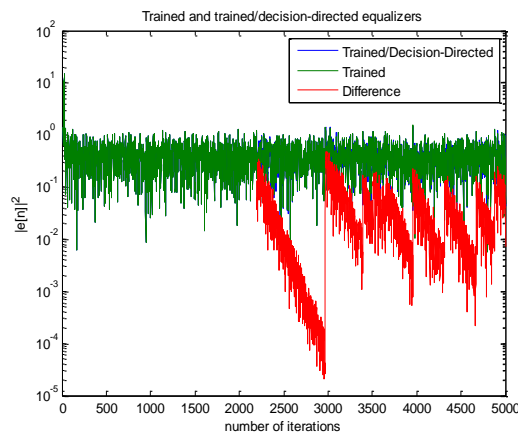
The received signal $x[n]$ is generated by the transmitted signal $s[n]$ filtered by the channel impulse response with additive noise $v[n]$. We shall assume a complex Gaussian noise signal for the additive n



The training signal is a shifted version of the original transmitted signal $s[n]$. This signal would be known to both the transmitter and receiver. To obtain the fastest convergence, we shall use the conventional version of a recursive least-squares estimator. Only the first 2000 samples are used for training. The output signal constellation shows clusters of values centered on the sixteen different symbol values--an indication that equalization has been achieved.



Plotting the squared magnitude of the error signal $e[n]$, we see that convergence with the RLS algorithm is fast. It occurs in about 60 samples with the equalizer settings chosen. Once the equalizer has converged, we can use decision-directed adaptation to continue adaptation during periods where no training data are available. In such cases, the desired signal $d[n]$ is replaced by a quantized version of the output signal $y[n]$ that is nearest to a valid symbol in the transmitted signal. We can use the RLS adaptive algorithm to implement this decision-directed algorithm in a sample-by-sample mode.



If the symbol decisions are correct, then decision- directed adaptation produces identical performance to trained adaptation. We can compare the error sequence from the combined training/decision-directed adaptive equalizer with one that uses training data over the whole received signal. A sudden jump in the difference in the error signals indicates an incorrect symbol decision was used in the decision- directed algorithm. So long as these errors are infrequent enough, the effects of these errors decay away, and the decision-directed equalizer's performance remains similar to that of the trained equalizer.

IV. CONCLUSION

Adaptive equalization and the more general field of adaptive filtering have been areas of active research and development for more than two decades. It is, therefore, tempting to state that no substantial further work remains to be done. However, this has not been the case in the last decade despite how mature the field appeared in 1973 [59]. In fact, a number of the topics covered in this paper; e.g., fractionally spaced equalizers, decision-aided ISI cancellation, and fast recursive least squares algorithms, were not yet fully understood or were yet to be discovered. Of course, tremendous strides have since been made in implementation technology which have spawned new applications, e.g., digital subscriber loops, and pushed existing applications toward their limits, e.g., 256-QAM digital radios and voice-band modems with rates approaching 19.2 kbits/s. Programmable digital signal processors now permit implementation of ever more sophisticated and computationally complex algorithms; and so the study and research must continue in new directions. There is still more work to be done in adaptive equalization of nonlinearities with memory and in equalizer algorithms for coded modulation systems. However, the emphasis has already shifted from adaptive equalization theory toward the more general theory and applications of adaptive filters, and toward structures and implementation technologies which are uniquely suited to particular applications.

V. REFERENCES

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