

# Response Analysis of Two Parallel Structures Connected by Friction Damper using Viscoplasticity Model

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**Abstract** – Earthquake causes damage to many lives and properties. To decrease the effect of earthquake, energy dissipating devices are used for the dissipation of earthquake energy, due to which effect of earthquake on lives and properties get reduced. This study presents the Numerical investigation on the use of friction damper to connect two parallel structures. A numerical model (Viscoplasticity model) is used to model the friction forces in the damper. Seismic responses of single structure (SDOF) equipped with friction damper and two parallel structures (SDOF) connected with friction damper is calculated. The base excitation considered is Imperial Valley 1940 earthquake recorded excitation. A numerical model (Viscoplasticity model) is used to model the friction forces in the damper. The equations of motion are solved by Newmark's step-by-step method. MATLAB computer program are generated to calculate the seismic response of connected structure under earthquake excitation. Result shows that when friction damper is used to connect two parallel structures, there is reduction in earthquake response.

**IndexTerms** –Energy dissipating devices, friction damper, parallel structures, seismic response.

## I. INTRODUCTION

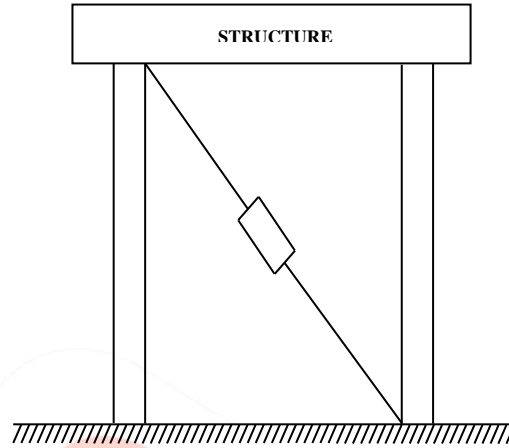
Many earthquakes are experienced all over the world leading to the loss of thousand of live and billion of economy. This is due the bad seismic performance of structure, which indicates that the majority of existing structures is vulnerable to significant seismic damage. An important aspect of mitigating seismic risk in existing as well in newly constructed structure is through the proper implementation of structural control strategies. In view of this research in the direction of developing new and innovative structural control methodologies, those are economically attractive and structurally sound, is of vital importance for enhancing seismic performances of structure. One of advanced technology in engineering is known as a structural vibration control, it simply consist implementing energy dissipation devices or control system into structures, which dissipate the energy or reduce excessive vibration in a structure. Structural control technology increases human comfort. This structure control technology is used for retrofiting of historical structures which are affected by the earthquakes. On the basis of energy consumption there are different types of structural control system, passive control system, active control system, hybrid control system, semi-active control system. Passive control systems have different types of damper, which are used in structure for the dissipation of earthquake energy. Seismic damper are used in place of structural elements, like diagonal braces, for controlling seismic damage in structure. Viscous damper (energy is absorbed by silicon-base fluid passing between piston-cylinder arrangement.), friction damper (energy is absorbed by surface with friction between them rubbing against each other,) metallic yielding damper (energy is absorbed by metallic components that yield.), viscoelastic damper (energy is absorb by layer of copolymer bonded with steel). Passive control system has two basic technologies. These are base isolation device and seismic damper. The idea behind base isolation is to isolate the building from the ground in such way that earthquake movement are not transmitted up though the building, or at least greatly reduced and may not cause any damage in the structure. Various dampers are used in the building to dissipate the energy which is produced by earthquake excitation. It is important for reducing seismic risk in an existing structure as well as in newly constructed structures there should be a proper implementation of structural control system. Structural control for seismic load is rapidly expanding field and the family of control systems, also known as earthquake protective system, now embraces passive, active and hybrid, semi-active system. Structural control system provides an alternative to conventional design methods, which are based on ductile yielding response. Passive seismic control system do not require additional energy source to operate and are activated by the earthquake in put motion only. Passive control system consists of one or more devices, attached to the structure, designed to modify the stiffness or damping of structure in an appropriate manner without requiring an external power source to operate. Control forces are developed opposite to the motion of controlled structural system. Energy dissipating devices and base isolation technique are two types of passive control system. Energy dissipating devices are the specially designed mechanical system to dissipate a large portion of earthquake input energy. In general, energy dissipating device are characterized by its capability to enhance energy dissipation in structural system to which they are installed.

## II. MATHEMATICAL FORMULATION

For the present study, a mathematical formulation is planned to investigate the effect of earthquake energy on the structure during earthquake. The main objective of this investigation is to find out the seismic response of structure during earthquake. These seismic responses are calculated by using Newmark's  $\gamma, \beta$  method.

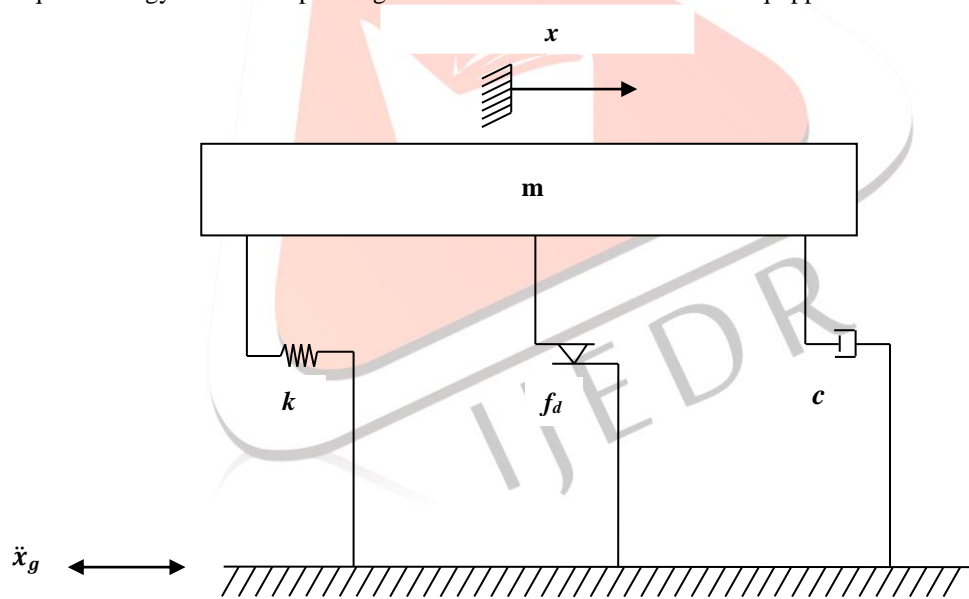
### *SDOF Structure Equipped with a friction damper*

Consider a single structure which is equipped with a friction damper as shown in the Fig.1 (a).



(a) Friction damper equipped in a SDOF Structure

This structure is a single-degree-of-freedom system. Friction damper is a passive energy dissipating devices that is used to dissipate the earthquake energy. The corresponding mathematical model of structure equipped with the friction damper is shown in Fig. 1 (b).



(b) Mathematical model

**Fig. 1** Structural model of SDOF structure equipped with friction damper. (a) Friction damper equipped in a SDOF Structure. (b) Mathematical model.

### *Equation of motion for SDOF structure*

Let  $m$ ,  $c$  and  $k$  be the mass, damping co-efficient and stiffness of structure. The system is subject to the earthquake motion which is given by

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g + f_d \quad (1)$$

where  $x$  is displacement,  $\dot{x}$  is velocity and  $\ddot{x}$  is acceleration.

Michalakos Constantinous et al. (1990) proposed frictional forces using a Wen's equation (1976).

$$f_d = \mu NZ \quad (2)$$

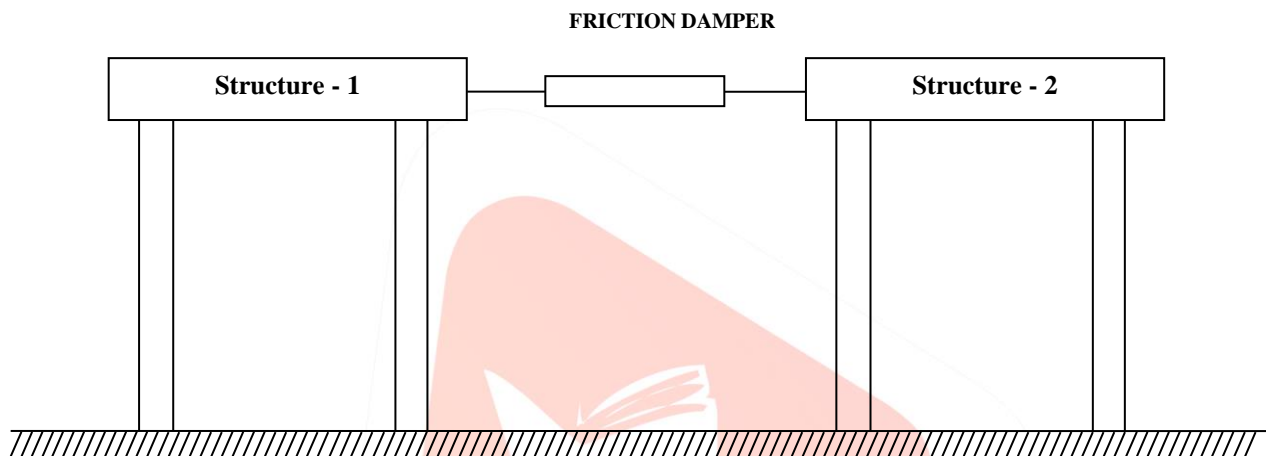
where  $f_d$  is a damping force,  $\mu$  is friction coefficient and  $N$  is clamping force,  $Z$  is Hysteretic dimensionless quantity. The value of friction coefficient is constant. Non-linear first order differential equation is expressed as

$$q \frac{dZ}{dt} = A(\dot{x}) + \beta_1 |(\dot{x})| Z |Z|^{n-1} - \tau(\dot{x}) |Z|^n \quad (3)$$

where  $\dot{x}$  is a initial velocity. At initial stage it is taken equal to zero and  $q$  is the yield displacement.  $\beta_1, \tau, n$  and  $A$  are the dimensionless parameter of hysteresis loop. Shape of loop will depend upon  $\beta_1, \tau, n$  and  $A$ . The recommended values for the above parameters are given as;  $q = 0.25\text{mm}$ ,  $A=1$ ,  $\beta_1=0.9$ ,  $\tau=0.1$  and  $n=2$ . As a result, the governing equations of motion are solved in the incremental form using Newmark's  $\gamma, \beta$  method assuming linear variation of acceleration over a small time interval,  $\Delta t$ . The iteration is required due to the dependence of  $Z$  on the response of the system at the end of each time step.

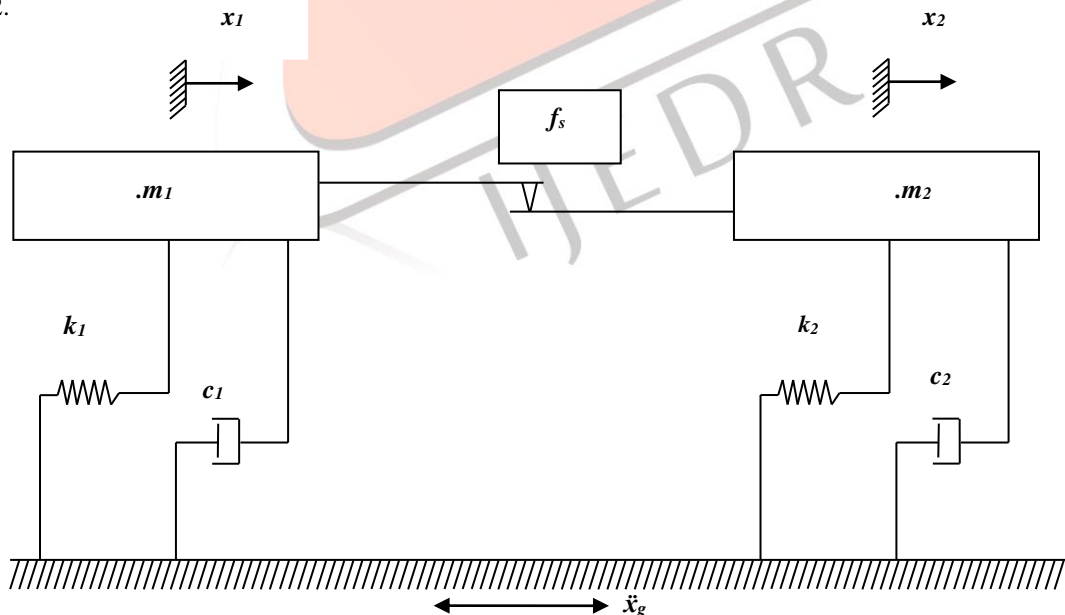
### Two SDOF structure connected with friction damper

Consider two parallel structures which are connected with friction damper as shown in Fig. 2 (a).



(a) Parallel structures connected with friction damper

These parallel structures are idealized as a single-degree-of-freedom system. These two structures are indicated as structure 1 and structure 2.



(b) Mathematical model

**Fig. 2** Structural model of two parallel SDOF structures connected with a friction damper. (a) Parallel structures connected with friction damper. (b) Mathematical model.

These two parallel structures are connected by friction damper and friction damper is represented by a viscoplasticity model. The frictional force mobilized in the damper has typical viscoplasticity characteristics. The space between these two parallel

structures is occupied by the friction damper. The corresponding mathematical model of structures connected with the friction damper is shown in Fig. 2 (b).

Let  $m_1$ ,  $c_1$  and  $k_1$  be the mass, damping co-efficient and stiffness respectively of structure 1. Similarly  $m_2$ ,  $c_2$  and  $k_2$  denote the corresponding parameters of structure 2. The system is subjected to the earthquake motion and depends upon the system parameters and excitation level, when these parallel connected structures vibrate together, no slip will occur in the friction damper.

Let  $\eta$  be the mass ratio and  $\lambda$  be the frequency ratio of two structures, respectively, expressed as

$$\text{Mass ratio } (\eta) = \frac{m_1}{m_2} \quad (4)$$

$$\text{Frequency ratio } (\lambda) = \frac{\omega_2}{\omega_1} \quad (5)$$

On the other hand, the numerical models are continuous and the required continuity is automatically maintained by the hysteretic displacement component used in the viscoplasticity model. Due to continuity of numerical model, they are preferred over the analytical model.

**Non-slide mode-** Both the structure vibrates together as a single-degree-of-freedom system during non slip mode under ground excitation. When slippage does not occurs the structure is said to be in a non-slide mode. The equation of motion for the combined system is given by

$$m_c \ddot{x}_c + c_c \dot{x}_c + k_c x_c = -m_c \ddot{x}_g \quad (6)$$

where  $m_c = m_1 + m_2$ ,  $c_c = c_1 + c_2$ ,  $k_c = k_1 + k_2$  are the mass, damping co-efficient and stiffness of the combined system, respectively and  $x_c$ ,  $\dot{x}_c$ ,  $\ddot{x}_c$  are the displacement, velocity and acceleration of the combined system respectively; and  $\ddot{x}_g$  is the ground acceleration respectively. By considering the dynamic equilibrium of structure 1 and structure 2 the frictional force in the damper can be obtained. Thus the non slip mode of the damper is valid until the following inequalities hold good.

$$|m_1 (\ddot{x}_c + \ddot{x}_g) + c_1 \dot{x}_c + k_1 x_c| \leq f_s \quad (7)$$

$$|m_2 (\ddot{x}_c + \ddot{x}_g) + c_2 \dot{x}_c + k_2 x_c| \leq f_s \quad (8)$$

where  $f_s$  is a slip force.

**Slip mode-** When slip occurs between the two structures, the system is said to be in slip mode. The condition for initiation of slippage is written as

$$|m_1 (\ddot{x}_1 + \ddot{x}_g) + c_1 \dot{x}_1 + k_1 x_1| > f_s \quad (9)$$

$$|m_2 (\ddot{x}_2 + \ddot{x}_g) + c_2 \dot{x}_2 + k_2 x_2| > f_s \quad (10)$$

where  $x_1$ ,  $x_2$  are the displacements of structure 1 and structure 2. The equations of motion of two connected structures in the slip mode are given by

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_g + f_s Z \quad (11)$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = -m_2 \ddot{x}_g - f_s Z \quad (12)$$

where  $f_s$  is a slip force. Till the relative velocities in the friction damper become zero then coupling system of structure will remain in the slip mode. Thus velocity at structure 1 is equal to velocity at structure 2, i.e  $\dot{x}_1 = \dot{x}_2$ . It should be noted that during slip-slip mode,  $Z$  is taken value of  $\pm 1$ . During stick-stick mode the absolute value of  $Z$  is taken as less than unity.

$$\{Z = \pm 1\} \quad \text{slip-slip mode}$$

$$\{Z = \text{less than unity}\} \quad \text{stick-stick mode}$$

### Governing equations of motion for two parallel SDOF structures

The two SDOF structure are symmetric with their symmetric planes in the alignment. The equations of motion for this system are expressed as

From Eqn. (11) and (12)

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_g + f_s Z$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = -m_2 \ddot{x}_g - f_s Z$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} -m_1 & 0 \\ 0 & -m_2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \ddot{x}_g + \begin{bmatrix} f_s Z \\ -f_s Z \end{bmatrix} \quad (13)$$

$$M\ddot{X} + C\dot{X} + KX = -MI\ddot{x}_g + F_D \quad (14)$$

where M, C and K are the mass, damping and stiffness matrices of the combined system, respectively and these are expressed as

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \quad I = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad F_D = \begin{bmatrix} f_s Z \\ -f_s Z \end{bmatrix}$$

$F_D$  is a vector consisting of the forces in the friction dampers;  $X$  is the floor displacement vector.  $\dot{X}$  is velocity vector or  $\ddot{X}$  is acceleration vector.

Two parallel structures which are connected with a friction damper by using Viscoplasticity model. Viscoplasticity model is proposed to model the force in the friction damper connecting two parallel structures. The equation of motion for the two connected structure with is written as.

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_g + f_d \quad (15)$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = -m_2 \ddot{x}_g - f_d \quad (16)$$

where  $f_d$  denote a damping force. This is calculated on the basis of mode of vibration. This viscoplasticity model is based on the principal of theory of viscoplasticity. It is considered as a modified viscoplasticity model.

From Eq. (2) 
$$f_d = \mu NZ$$

Damping force is taken equal to  $f_d = \mu NZ$ . Where  $\mu$  is friction coefficient and N is clamping forces. Z is Hysteretic dimensionless quantity. The non linear first order equation for connected structure-

$$q \frac{dZ}{dt} = A(\dot{x}_2 - \dot{x}_1) + \beta I |(\dot{x}_2 - \dot{x}_1)| Z |Z|^{n-1} - \tau (\dot{x}_2 - \dot{x}_1) |Z|^n \quad (17)$$

All the parameters are similar to the Eq. (3).  $(\dot{x}_2 - \dot{x}_1)$  is relative velocity.

### Solution for equations of motion

The frictional force mobilized in the friction damper is a non-linear function of displacement and velocity of the system. As a result, the governing equations of motion are solved in the incremental form using Newmark's  $\gamma, \beta$  method assuming variation of acceleration over a small time interval,  $\Delta t$ .

$$\dot{x}_{i+1} = \dot{x}_i + [(1 - \gamma)\Delta t]\ddot{x}_i + (\gamma\Delta t)\ddot{x}_{i+1} \quad (18)$$

$$x_{i+1} = x_i + (\Delta t)\dot{x}_i + [(0.5 - \beta)(\Delta t)^2]\ddot{x}_i + [\beta(\Delta t)^2]\ddot{x}_{i+1} \quad (19)$$

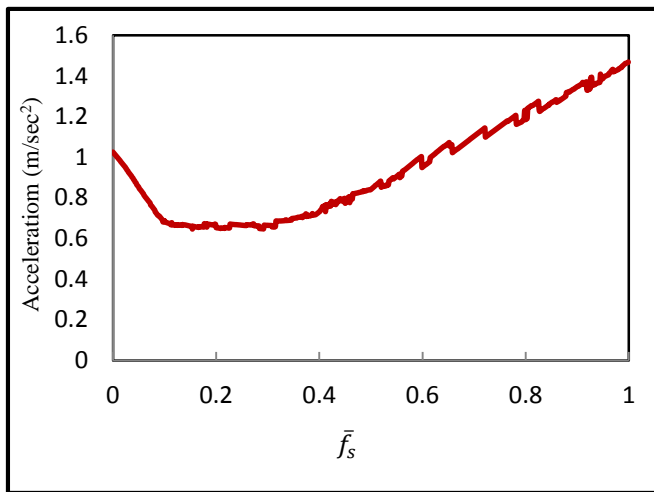
The parameters  $\beta$  and  $\gamma$  define the variation of acceleration over a time step and determine the stability and accuracy characteristics of the method. Typical selection for  $\gamma$  is  $\frac{1}{2}$  and  $\frac{1}{6} \leq \beta \leq \frac{1}{4}$  is satisfactory from all points of view, including that of accuracy. These two equations, combined with the equation of motion at the end of the time step, provide the basis for computing  $x_{i+1}$ ,  $\dot{x}_{i+1}$  and  $\ddot{x}_{i+1}$  at time  $i + 1$  from the known  $x_i$ ,  $\dot{x}_i$  and  $\ddot{x}_i$  at time  $i$ . For viscoplasticity model, an iterative procedure is used for evaluating the incremental and dependence of Z on the response of the system at the end of each time step. Thus a fourth order Runge-kutta method is employed for the solution of the first-order differential equation (referred Eq. 17).

## III. RESULTS AND DISCUSSION

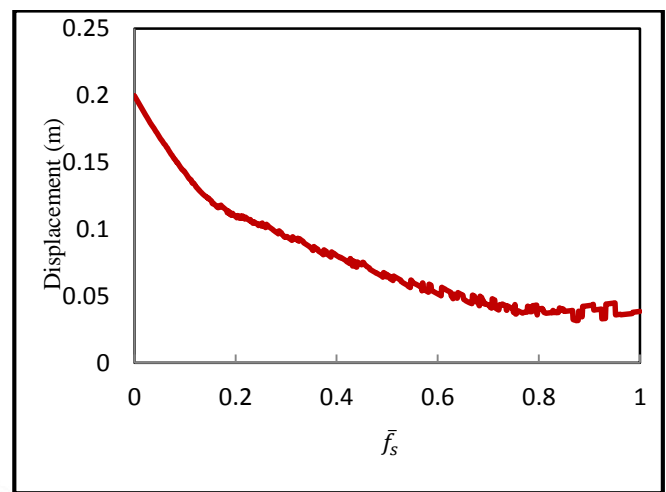
For the present study, SDOF structure and two parallel structures are considered. The floor mass and stiffness are considered to be uniform for both the cases. The masses of two building are assumed to be same and the damping ratio in each structure is taken as 5% (Concrete structure). Frequency ratio  $\lambda$  is considered equal to 2 ( $\omega_1 = 0.908\pi$  rad/sec,  $\omega_2 = 1.816\pi$  rad/sec) and mass ratio is considered equal to 1. In case of single structure, to arrive at the optimum slip force in the friction damper, the variation of peak displacement, peak acceleration of SDOF structure are plotted with the normalized slip forces and are shown in Fig. 3 and Fig. 4 for Imperial Valley earthquake motion. When normalized slip force increase there is decrease in the displacement. On the

other hand, when slip force increases the acceleration get decrease up to certain value after that when there is decrease in the slip force the acceleration get increased. Thus value where acceleration starts to increase is selected as optimum slip force.

### Discussion and results for SDOF Structure



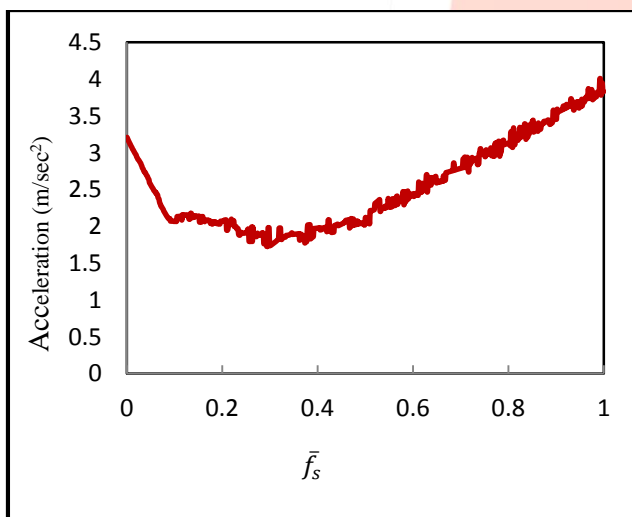
(a)



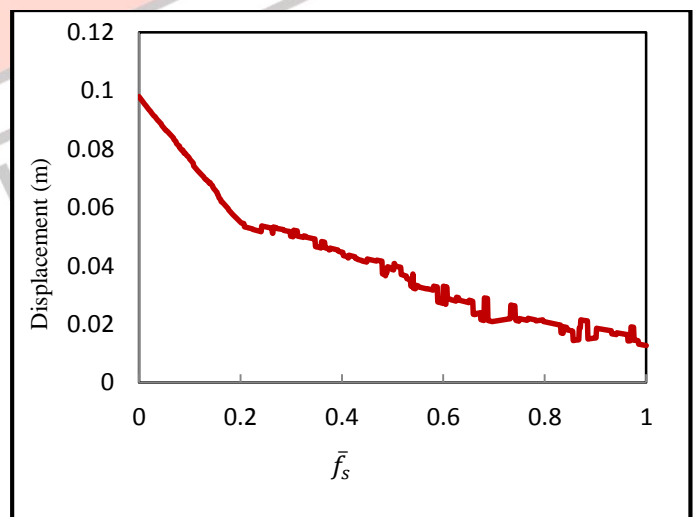
(b)

**Fig. 3** (a) Variation of peak acceleration against normalized slip force  $\bar{f}_s$  for SDOF structure equipped with friction damper ( $\omega_1 = 0.908\pi$  rad/sec). (b) Variation of peak displacement against normalized slip force for SDOF structure equipped with friction damper ( $\omega_1 = 0.908\pi$  rad/sec).

The optimum value which is selected for SDOF Structure equipped with friction damper is 0.249.



(a)

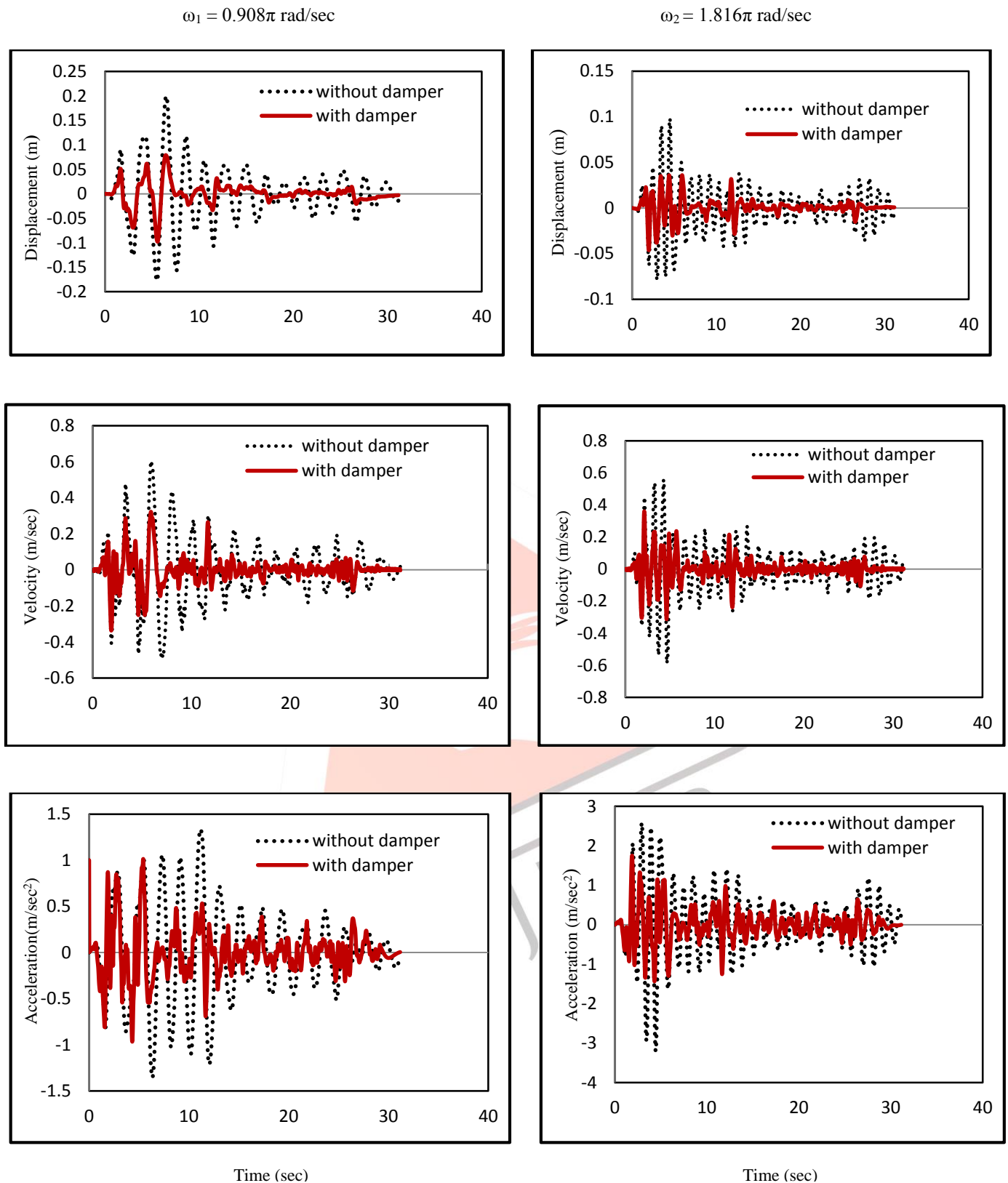


(b)

**Fig. 4** (a) Variation of peak acceleration against normalized slip force  $\bar{f}_s$  for SDOF structure equipped with friction damper ( $\omega = 1.816\pi$  rad/sec). (b) Variation of peak displacement against normalized slip force for SDOF structure equipped with friction damper ( $\omega = 1.816\pi$  rad/sec).

The optimum value which is selected for SDOF Structure equipped with friction damper is 0.318.





**Fig. 5** Time histories of displacement, velocity and acceleration of SDOF Structure equipped with friction damper for two different natural frequencies.

**Table1.** Seismic responses of SDOF structure equipped with damper ( $\omega_1 = 0.908\pi$  rad/sec).

Earthquake	Displacement (m)		Velocity (m/sec)		Acceleration (m/sec <sup>2</sup> )	
	Unconnected	Connected	Unconnected	Connected	Unconnected	Connected
Imperial valley, 1940	0.19999	0.09798 (51.00) <sup>#</sup>	0.60382	0.33650 (44.27) <sup>#</sup>	1.63515	1.03474 (36.71) <sup>#</sup>

# Quantity within the parenthesis denotes the percentage reduction.

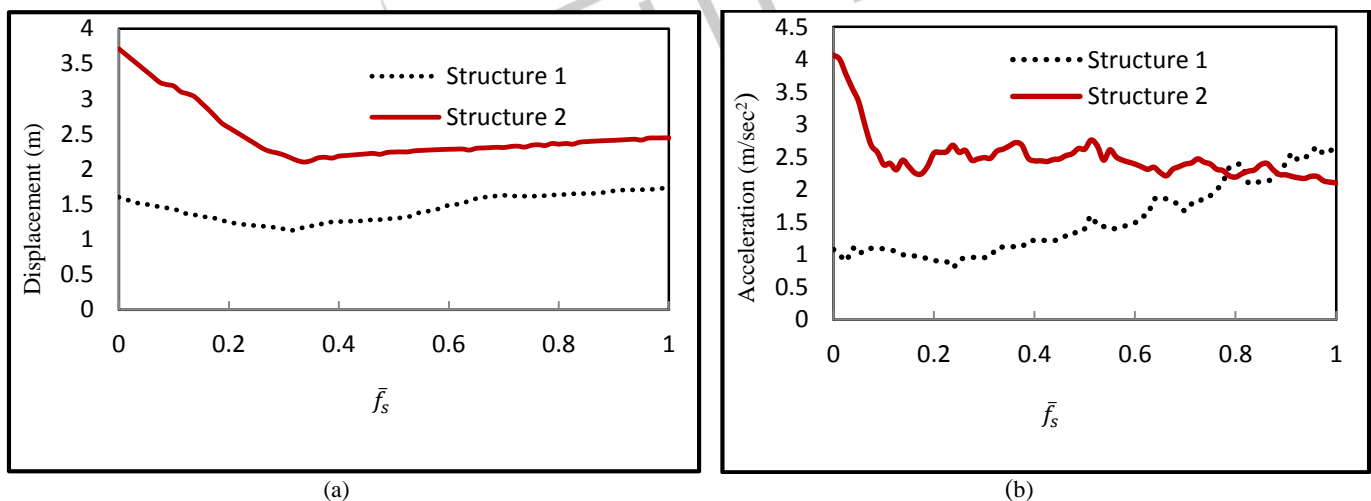
**Table2.** Seismic responses of SDOF structure equipped with damper ( $\omega_2 = 1.816\pi$  rad/sec).

Earthquake	Displacement (m)		Velocity (m/sec)		Acceleration (m/sec <sup>2</sup> )	
	Unconnected	Connected	Unconnected	Connected	Unconnected	Connected
Imperial valley, 1940	0.09800	0.04539 (53.37) <sup>#</sup>	0.58832	0.32014 (45.55) <sup>#</sup>	3.20800	1.98501 (38.12) <sup>#</sup>

# Quantity within the parenthesis denotes the percentage reduction.

**Discussion and results for two SDOF Structures connected by friction damper**

In case of two structures which are connected by friction damper, to arrive at the optimum slip force in the friction damper, the variation of peak displacement, peak acceleration are plotted with normalized slip force are shown in Fig. 6 for Imperial Valley earthquake motion.



**Fig. 6** (a) Variation of peak displacement against normalized slip force  $\bar{f}_s$  for two SDOF structures connected with friction damper ( $\eta = 2$ ,  $\lambda = 1$ ). (b) Variation of peak acceleration against normalized slip force for two SDOF structures connected with friction damper ( $\eta = 2$ ,  $\lambda = 1$ ).

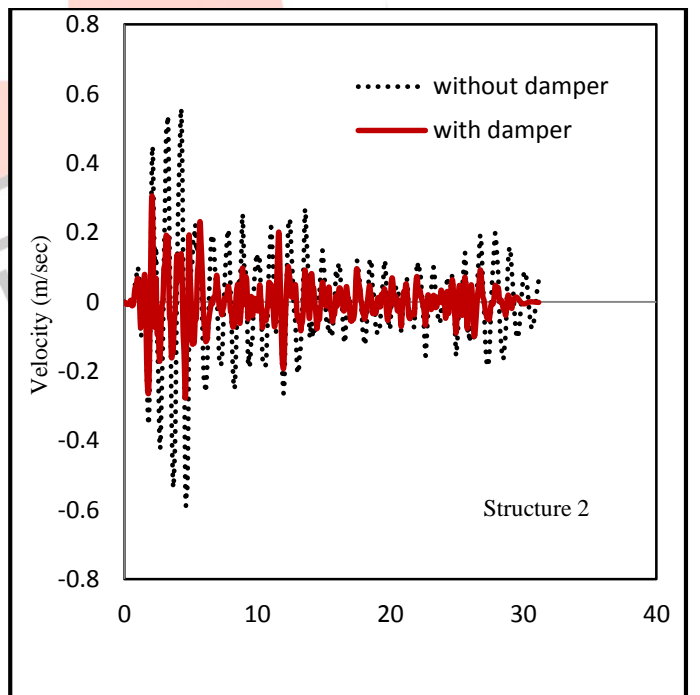
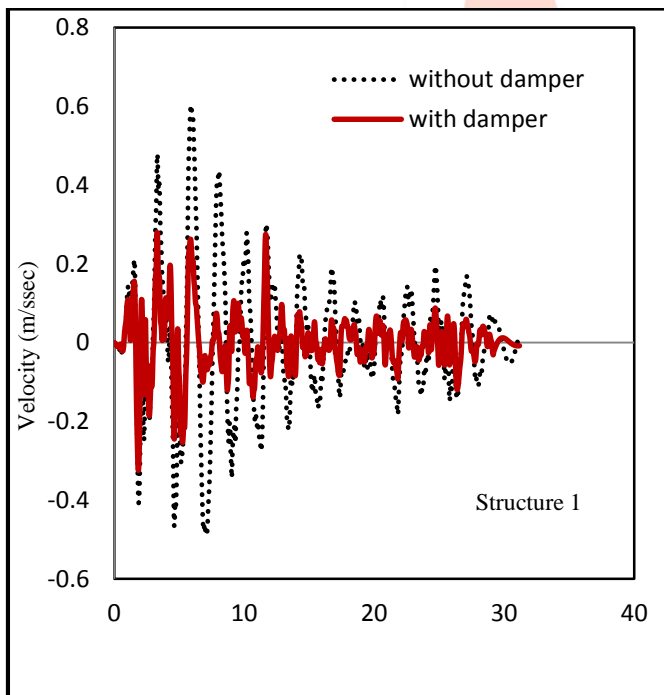
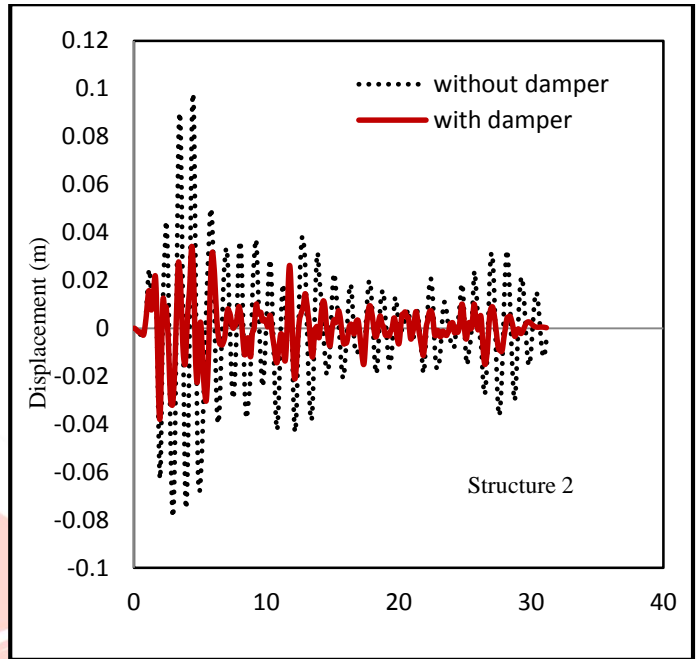
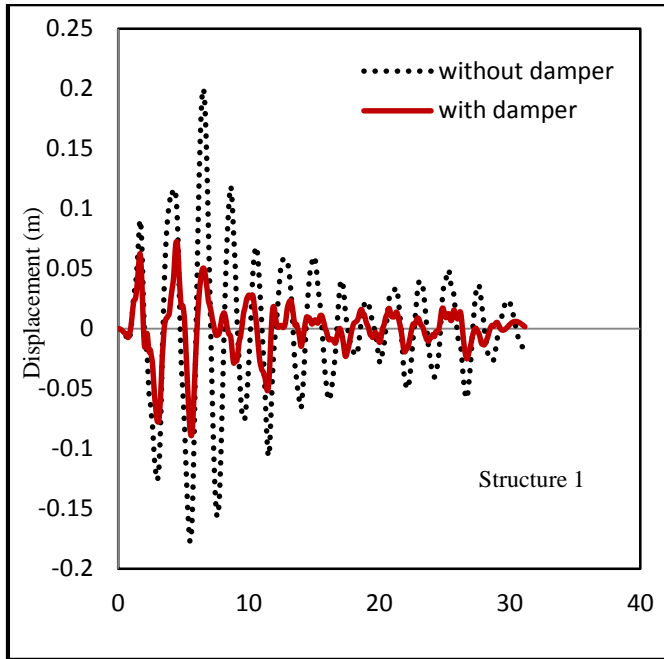
It is observed that the response of both the structure are reduced up to a certain increase in the value of slip forces and with further increase in the value of the slip force they again increased.

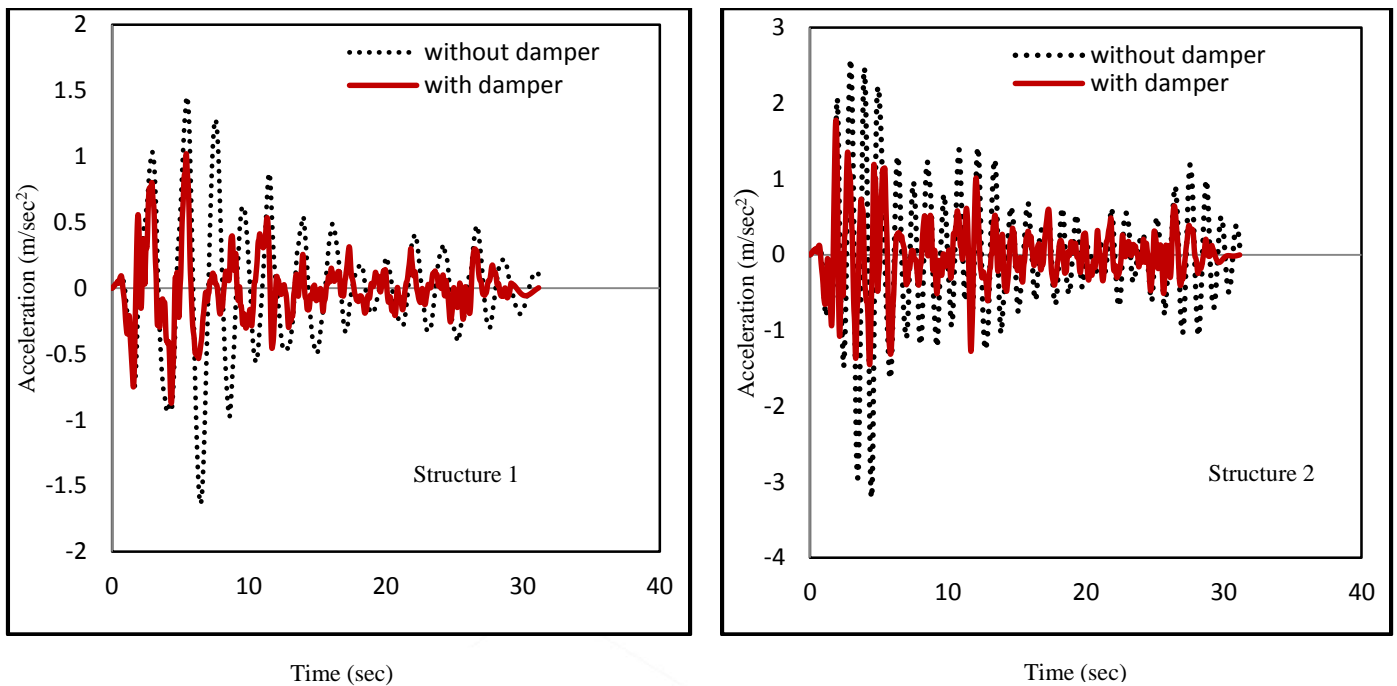


Therefore it is clear from the figures that optimum slip force exists to attain the minimum responses in both the structures. The optimum slip force is not exactly the same for both the structures; the optimum value is taken as the one, which gives the minimum sum of the responses of two structures.

Structure 1

Structure 2





**Fig. 7** Time histories of displacement, velocity and acceleration of two SDOF Structures connected with friction damper.

**Table3.** Seismic responses of two SDOF structures connected with friction damper.

Earthquake	Structure	Displacement (m)		Velocity (m/sec)		Acceleration (m/sec <sup>2</sup> )	
		Unconnected	Connected	Unconnected	Connected	Unconnected	Connected
Imperial Valley, 1940	1	0.19999	0.08906 (55.46) <sup>#</sup>	0.60382	0.32471 (46.22) <sup>#</sup>	1.63515	1.02103 (37.55) <sup>#</sup>
	2	0.09800	0.03807 (61.15) <sup>#</sup>	0.58823	0.30570 (48.03) <sup>#</sup>	3.20800	1.77728 (44.59) <sup>#</sup>

<sup>#</sup> Quantity within the parenthesis denotes the percentage reduction.

#### IV. CONCLUSION

Under the Imperial Valley earthquake excitation, the behavior of SDOF structure equipped friction damper and two parallel structures connected with a friction damper is worked out. The governing equations of motion are formulated for SDOF structure and two SDOF structures during non-slip and slip modes of damper. A numerical model (Viscoplasticity Model) is used to evaluate the friction forces in the damper. The result obtained in this study provides information towards the design of coupling friction damper. It is found that friction damper is more effective in reducing the earthquake response of two parallel structures connected by it. There exists an optimum slip force of friction dampers and minimize the earthquake response of parallel connected structures. When there exist slight variation in the optimum slip force of damper than it does not have much effect on the optimum responses and hence, small variations in the slip load over life of the structure does not warrant any adjustments of replacement of friction damper.

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