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Two Parametric Non-Linear Dynamic Analysis Of Rigid Pavement

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***Abstract* - Highway and airport designers have always shown keen interest in understanding the pavement behavior and continuously improving it. In the early stages of development, the evaluation of pavement stress was generally based on a simplified static-elastic analysis. This has proved to be quite satisfactory for vehicles and airplanes that are light or travel at low speeds. However, as loading and speed requirements have continued to increase, improved models for evaluating stress and deflection in pavement have become necessary. The implication of such studies is that pavements should be analyzed dynamically. The objective of this study is to carry out dynamic analysis of pavements by using two parametric foundation models. By carrying out the regression analysis based on the results obtained from dynamic analysis using various soil and pavement parameters an empirical equation is developed for the prediction of critical velocity and maximum deflection.**

***Index Terms*–Two parameter model, Non-linear, Dynamic analysis, Soil structure interaction.**

I. Introduction

The main objective of the paper is to include the material nonlinearity of the supporting soil medium to study the dynamic response of rigid pavement resting on two- parameter soil medium which is subjected to moving load by considering vehicle as a force, as in most of earlier works on such topics, the material nonlinearity of the associate soil medium was not considered. The effects of moving, dynamic loads was firstly considered by Brian et al. (1990) and reviewed a mathematical description of vehicle-pavement interaction for different vehicle loadings. Later on Wu (1995) introduced a three-dimensional (3D) finite-element method in combination with Newmark integration scheme to investigate dynamic response of concrete pavements subjected to moving loads. The moving vehicle loads was modelled as lumped masses each supported by a spring-dashpot suspension system. Xiang et al. (1996) investigated the buckling and vibration behavior of moderately thick, simply supported symmetric cross-ply laminates resting on Pasternak foundations, subjected to uniformly distributed in-plane loads, Bhatti et al. (1998) had presented use of available field data to develop non-linear dynamic simulation techniques for predicting pavement performance. Huang et al. (2002) indicated that the foundation stiffness and the velocity and frequency of the moving load have significant effects on the dynamic response of the plate and on resonant velocities.

Higher deflections on top of subgrade can cause the pavement sections to fail before the end of design life examined by Hadi et al. (2003) by using mechanistic methods in the analysis of layered pavement systems under traffic load. Hadi et al. (2003) also noted that, if pavement designs are carried out assuming static loading and linear pavement materials, the deflections at top of subgrade are higher than the expected values, when pavement sections with non-linear materials are subjected to the moving load. Rahman et al. (2004) investigated some significant aspects of the dynamic behavior of rigid concrete pavements based on a Finite Element formulation of thick plate on elastic foundation, including the difference between static and dynamic response. In recent studies Patil et al. (2012) presented an improved solution algorithm based on Finite Element Method for dynamic analysis of rigid pavements under moving loads incorporating vehicle–pavement interaction which bears significant effect on the response, which shows that the critical velocity range gets compressed in case of infinite length pavements.

Most of earlier studies used the well-known model of elastic foundation developed by Winkler. The pavement model consists of discrete plates joined at the discontinuities by vertical springs and supported by uniformly distributed springs and dashpots representing the viscoelastic soil foundation. Many of the past studies did not considered nonlinearity of the supporting foundation material. To determine pavement response for vehicle loading static analysis was widely used with combination of FEM and other numerical methods by considering vehicle as static point load. To analyses the response of pavements to moving aircraft or vehicle loads based on the FEM it is necessary to carryout parametric study considering nonlinearity of material and dynamic nature of the moving vehicle.

II. Introduction to Soil Structure Interaction

A highway pavement is a structure consisting of superimposed layers of processed materials above the natural soil sub-grade, whose primary function is to distribute the applied vehicle loads to the sub-grade. The pavements can be classified based on the structural performance into flexible pavements and rigid pavements. In flexible pavements, wheel loads are transferred by grain-to-grain contact of the aggregate through the granular structure. The flexible pavement, having less flexural strength, acts like a flexible sheet (e.g. bituminous road). On the contrary, in rigid pavements, wheel loads are transferred to sub-grade soil by flexural strength of the pavement and the pavement acts like a rigid plate (e.g. cement concrete roads).

The foundation designer must consider the behavior of both structure and soil and their interaction with each other. The interaction problem is of importance to many civil engineering situations and it covers a wide spectrum of problems. To get solution of any interaction problem on the basis of all the above factors is very difficult and lengthy. Achieving realistic and purposeful solutions can only be done by idealizing the behavior of the soil by considering specific features of soil behavior. The simplest idealization of response naturally occurring soils is to assume linear elastic behavior of the supporting soil medium. Though these assumptions are not always strictly satisfied by in-situ soils.

The natural complexity in the behavior of in-situ soils has led to the development of many idealized models of soil behavior based on the classical theories of elasticity and plasticity for the analysis of Soil-Foundation Interaction problems. Although, the generalized stress-strain relations for soils don't represent even the gross physical properties of a soil mass, the idealized models are observed to provide a useful description of certain features of soil media under limited boundary conditions.

Elastic Models of soil behavior which exhibit purely elastic characteristic. The simplest type of idealized soil response is to assume the behavior of supporting soil medium as a linear elastic continuum. The deformations are thus assumed as linear and reversible. The various models are The Winkler's Model, Elastic Half-Space (Elastic continuum) Models and Two Parameter Elastic Models. The deficiency of the Winkler's Model in describing the continuous behavior of real soil masses and the mathematical complexities of the elastic continuum has led to the development of many other simple soil behavior models. These models possess some of the characteristics features of continuous elastic solids. The term "Two Parameter" signifies that the model is defined by two independent elastic constant. The model proposed by Pasternak assumes the existence of shear interaction between the spring elements as shown in Fig.1. This condition is accomplished by connecting the spring elements to a layer of incompressible vertical elements, which deform in transverse shear only. The response function for this model is,

$$q(x,y) = k\,x(x,y) - G\,\nabla^2\; w(x,y)$$

The continuity in this model is characterized by the consideration of the shear layer. A comparison of this model with that of Filonenko–Borodich implies their physical equivalency.

III. Formulation Of Problem And Methodology

For nonlinear analysis we have considered here a finite beam resting on an elastic foundation divided in to ten segments over which a load is travelling from one end to other. At every cycle of loading the deflections are calculated at each node and one of the maximum is noted for the corresponding propagating speed. The all results are to be summarized in tabulated form and same are to be presented in graphical form also. From the derived results some peak values are considered to obtain desired results. Analysis of pavement-subgrade system under traffic load has been traditionally done by static method, such as multilayer elastic theory. Although, in some examples, the static analysis method is satisfactory and conventional for the regular practice, actually dynamic analysis is needed in some distinct cases, such as shallow bedrock with greater vehicle speed. The nonlinear relationship for supporting soil medium will be assumed in hyperbolic relationship and the iterative approach will be used for nonlinear analysis. To carry out this nonlinear analysis the computer code generated in FORTRAN-90. Before using this computer code for actual analysis it will be validated by computing few parametric results with the classical theory results.

Figure.2 presents the vehicle-pavement-foundation model used in the present study. The moving vehicle is represented by the force supported over a concrete pavement. The pavement is modelled by the finite beam with thickness t. The compacted base course is modelled by the Pasternak's shear layer of thickness H with shear modulus while the underlying soft soil is modelled by Winkler's independent springs with constant k (simulating the subgrade modulus of soil).

The equilibrium condition of the beam resting on Pasternak's soil medium is expressed in the form of a differential equation for transverse displacement w.

$$EI\,\frac{d^4w}{d\,x^4} - G_p b\,H\,\frac{d^2w}{d\,x^2} + \;kbw = p(x) \qquad \ldots(1)$$

Where p(x) is the externally applied distributed load on beam; b is the width of beam; El is the flexural rigidity of the beam.
Strain energy U of the system can be expressed by the following equation:

$$U = \frac{1}{2}\int_0^L \left(EI\frac{d^4w}{d\,x^4} - G_p bH\frac{d^2w}{d\,x^2} + kbw \times w \right) dx \qquad \ldots(2)$$

This equation may also be written in a matrix form for any appropriately developed beam element as,

$$\left[[K_1] + [K_2] + [K_3]\right]\{q\} = \{Q\} \qquad \ldots(3)$$

Applying numerical integration technique the stiffness matrices, $[k_1]$, $[k_2]$ and $[k_3]$ can be derived using the following expressions,

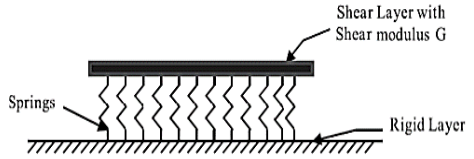


Figure.1 Pasternak model

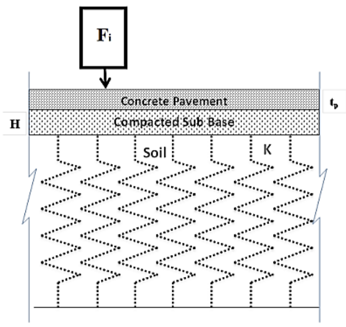


Figure.2 Vehicle Pavement Foundation model

$$[K_1] = \int_0^L \{[N^{xx}]^T EI [N^{xx}]\}\, dx \qquad [K_2] = \int_0^L \{[N^x]^T G_p bH [N^x]\}\, dx \qquad [K_3] = \int_0^L \{[N]^T kb [N]\}\, dx$$

Where

$$[N^{xx}] = \frac{d^2N}{dx^2} \quad \text{and} \quad [N^x] = \frac{dN}{dx} \qquad \text{... (4)}$$

the shape function matrix for a finite beam element will be expressed as

$$= \begin{bmatrix} (1 - 3\xi^2 + 2\xi^3) \\ L(\xi - 2\xi^2 + \xi^3) \\ (3\xi^2 + 2\xi^3) \\ L(-\xi^2 + 2\xi^3) \end{bmatrix} \qquad \text{... (5)}$$

After assemblage, the overall stiffness matrix for the total system is symbolized by capital-letters as below.

$$[[K_1] + [K_2] + [K_3]]\{d\} = \{F\} \text{ or } [K^*]\{d\} = \{F\} \qquad \text{... (7)}$$

Now the dynamic equilibrium equation is

$$[M]\frac{\partial^2[d]}{\partial t^2} + [C]\frac{\partial[d]}{\partial t} + [K^*]\{d\} = \{F\} \qquad \text{... (8)}$$

With $[M] = \sum \int_0^L \{[N]^T \rho A\ [N]\}\, dx$ And $[C] = \sum \int_0^L \{[N]^T c\ [N]\}\, dx$

ρ is the mass density, A is cross sectional area and c is damping constant for soil medium, From the above equations, the dynamic force equilibrium can be expressed as,

$$[[K_1] + [K_2] + [K_3]]\{d\} = \int_{-1}^{1} [N]^T [mg - m\ddot{w} - c_s\dot{w}]\, dx \qquad \text{... (09)}$$

Where Jacobian |J| relates conversion between global coordinates x (integration limits -1 to 1) in the mapping. By expressing the derivative of pavement deflection w in the right hand side in terms of nodal variable {d}, and by assembling the individual element matrices, the dynamic equilibrium equation can be expressed in the form,

$$[K^*]\{d\} = [W] - [M]\frac{\partial^2}{\partial t^2}\{d\} - [C]\frac{\partial}{\partial t}\{d\}$$

$$\text{Here, } [K^*] = \sum[[K_1] + [K_2] + [K_3]] \qquad \text{... (10)}$$

The tilde above [N] denotes that the shape functions are evaluated for a specific element where the mass is located. In which, 'v_m' and 'a_m' are the velocity and acceleration of moving load. The displacement at each time step 'h' can be evaluated by applying Newmark-Beta integration method. α_k and β_k are constants of Newmark-Beta integration method. The unknown nodal displacements $\{d_i\}$ and $\{u_i\}$ are compute by matrix inversion. Then nodal velocities and acceleration are calculated which are used in the computation of next time step.

Validation

Analysis of pavement-subgrade system under traffic load has been traditionally done by static method, such as multilayer elastic theory. Although, in some instances, the static analysis method is sufficient and conservative for the daily practice, truly dynamic analysis is needed in some special cases, such as shallow bedrock with high vehicle speed. The nonlinear relationship for supporting soil medium will be assumed in hyperbolic relationship and the iterative approach will be used for nonlinear analysis. To carry out this nonlinear analysis the computer code generated in FORTRAN-90. Before using this computer code for actual analysis it will be validated by computing few parametric results with the classical theory results. To check the accuracy of the finite element formulation and developed computer program, the response of the beam in the form of central deflection, end deflection and central moment is compared with standard closed form solutions available. Response for the condition of load P applied at the central position is considered for comparison. The governing differential equation for a beam supported by two parameter soil medium is given by,

$$EI\frac{d^4w}{dx^4} - G_p b\, H\frac{d^2w}{dx^2} + kbw = 0 \qquad \text{... (11)}$$

For a finite beam the solution of the above differential equation yields,

$$w = e^{-\lambda\mu x}(C'_1 \cos\theta_x + C'_2 \sin\theta_x) \qquad \text{... (12)}$$

Where

$$\theta_x = \lambda\beta' x; \quad C'_1 = \frac{P}{8EI\mu\lambda^3}; \text{ and } C'_2 = \frac{\mu}{\beta'}C'_1 \text{ also } \mu = \sqrt{1 + \frac{G_{pH}}{k}\lambda^2}; \quad \beta' = \sqrt{1 - \frac{G_{pH}}{k}\lambda^2}; \quad \lambda = \sqrt[4]{\frac{kb}{4EI}}$$

The constants C'_1 and C'_2 can be obtained by applying the boundary conditions for slope and shear at the center of the beam. Substituting these values in the equations for displacement and moment (Patil et al, 2010) can be written in terms of central deflection w_0 and central moment M_0 as follows

$$w = w_0 e^{-\lambda\mu x}(\cos\theta_x + \mu_R \sin\theta_x) \quad \text{and} \quad M = M_0 e^{-\lambda\mu x}(-\mu\sin\theta_x + \beta'\cos\theta_x) \qquad \text{... (13)}$$

Where

$$w_0 = \frac{P}{4EI\mu\lambda^3\mu(\beta'^2 + \mu^2)}; \quad M_0 = \frac{P}{4\lambda\beta\mu}; \quad \text{and} \quad \mu_R = \frac{\mu}{\beta'}$$

Deflection and moment at the central point are used for comparison. The beam response in the form of a deflection curve and bending moment diagram derived from the analytical procedure are compared. Table.1 presents the comparison of the FEM results with the corresponding analytical results. A good agreement is observed between the two solutions.

Table.1 Validation of beam

	Central Deflection (mm)	**Central Bending moment (kN-m)**
Analytical	1.506813	89.27447
FEM	1.506800	89.274
% error	0.0009	0.0005

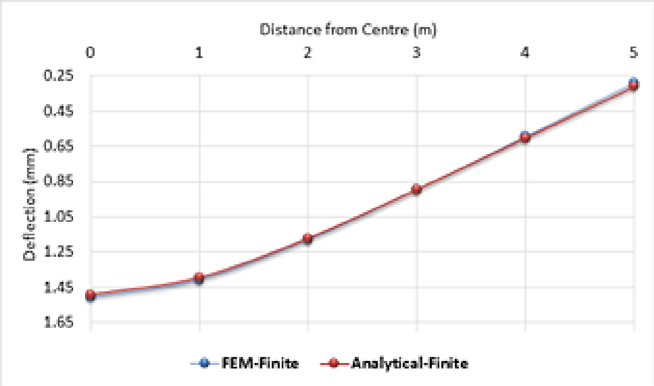


Figure.4.a Comparison of Deflected shape of beam

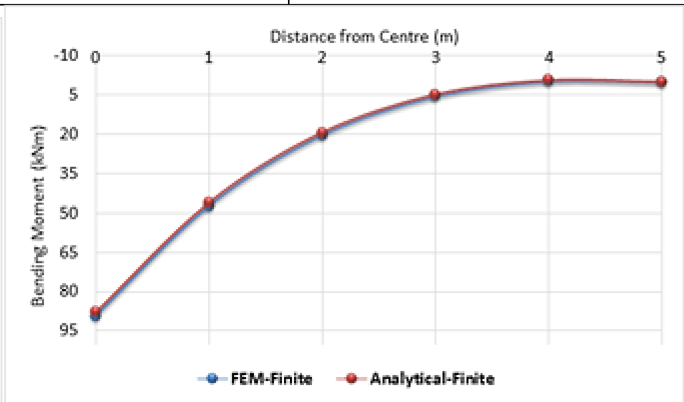


Figure.4.b Comparison of bending moment diagram

The closed form solutions for beams resting on an elastic foundation with two parameter model are available. The following material properties are adopted in the analysis for validation and parametric study.

- Beam L = 10.00 m, b = 1.00 m, Eb = 0.3605 × 108 kN/m2
- Soil Gp = 7000 kN/m2, k = 10000 kN/m3, H = 1.00 m
- Load P = 100.0 kN

IV. Results and Discussion

The data assumed in the parametric study is presented in Table.2 the analysis was carried out by incorporating the material nonlinearity of the soil medium. For each subgrade modulus the shear modulus (G) was varied between 10000 kPa to 70000 kPa at an interval of 10000. The analysis was carried out for soil modulus between 30000 kPa to 60000 kPa and considering typical finite beam element with thickness 1.0 m and width 0.5m. In each analysis the velocity of moving load was varied from 0.05 to 40 m/sec at an interval of 0.05 m/sec. The variations in maximum deflection with velocity for some typical combinations of subgrade modulus 'k', shear modulus 'G' and pavement thickness for a finite beam are presented in Figure.5a and 5b the plot shows a behavior similar to resonance phenomenon in dynamic problems. Several peaks are observed for each combination of soil moduli and pavement thickness. The velocity corresponding to peak value of deflection is called critical velocity. Two prominent peaks are being identified and the same are being reported in table.3 in terms of the critical velocities and the corresponding maximum deflections.

The fig.7 shows that at any peak the critical velocity is increasing while the corresponding maximum deflection is decreasing with the increase in the value of subgrade modulus for a constant shear modulus. Also at any peak the critical velocity decreases with increasing pavement thickness for any constant subgrade modulus and shear modulus. No definite trend is observed for the change in critical velocity with shear modulus at a constant sub- grade modulus and pavement thickness. To quantify these observed trends the critical velocities and the corresponding maximum deflections are normalized with respect to v_0 and w_0 static central deflection which can be obtained from Equation (13).

Table.2 Soil and Beam Parameters

Foundation		**Pavement**	
Soil K(kN/m^2/m)	30000, 40000, 50000, 60000	Thickness t_p(m)	0.5
Soil G_p(kN/m^2)	10000, 20000,30000, 40000,50000,60000,70000	Width b (m)	1.0
Vehicle Load P (kN)	343	Density ρ (kg/m^3)	2548.4
Vehicle Velocity	0 to 40 at an interval of .05 m/s^2	E_p (kN/m^2)	3.605 x 10^7

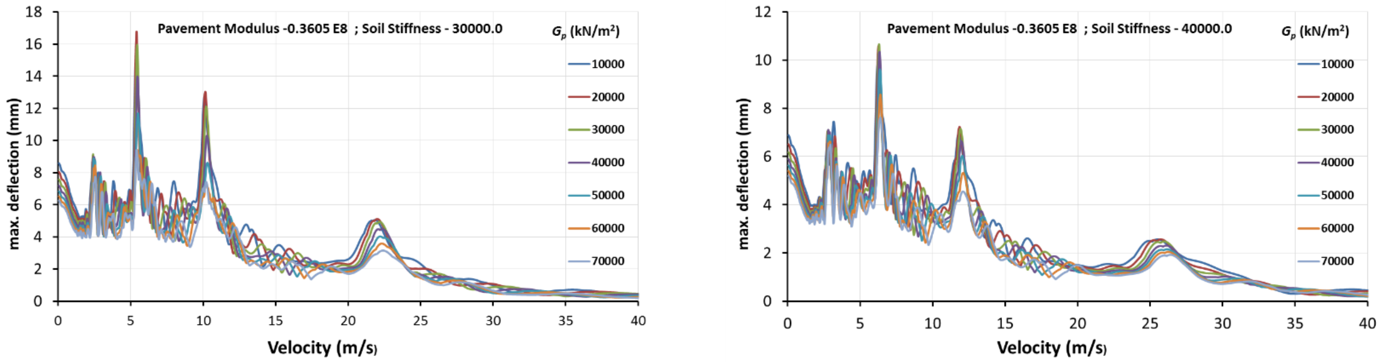
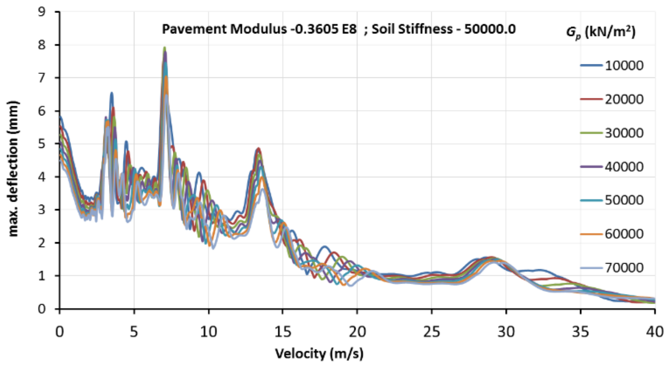


Figure.5.a Graph Obtained between Velocity and Deflection and Soil Stiffness = 30000.0 and 40000.0



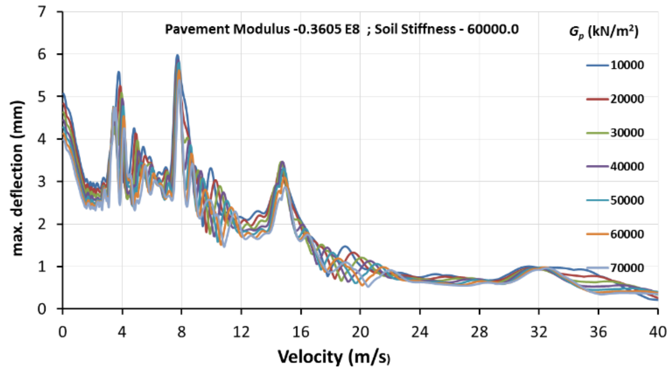


Figure.5.b Graph Obtained between Velocity and maximum Deflection for Soil Stiffness = 50000.0 and 60000.0

$$w_0 = \frac{P}{8EI\mu\,\lambda^3}\ ; \qquad \lambda = \sqrt[4]{\frac{kb}{4EI}}\ ; \qquad v_0 = \sqrt{\left(\frac{\sqrt{4EIkb} + G_p bH}{\rho}\right)}$$

The data corresponding to the two prominent peaks selected from the results of this non-linear analysis is presented in Table 7.2. Results obtained from the nonlinear analyses show that the nonlinearity of the soil has decreased the critical velocities while increased the maximum deflections of the pavement. By carrying out the regression analysis for this cases the following relationships are suggested to predict the critical velocities and the corresponding maximum deflections at the prominent peaks.

$$\frac{v_{cr}}{v_o} = \phi_0 + \phi_1\lambda \quad \text{and} \quad \frac{w_{max}}{w_o} = \psi_0 + \psi_1\lambda + \psi_2\lambda^2 + \psi_3\lambda^3 \qquad \ldots(25b)$$

The regression scatters of normalized critical velocity and normalized maximum deflections for both the analysis are presented in Figure 6.a and Figure 6.b respectively. The regression constants derived from the regression analysis are presented in Table 7.6.Thus, if subgrade has low value of ultimate bearing or stress carrying capacity and loading intensity is high, the soil behaviour is nonlinear and in this situation higher maximum defections are observed. It is observed that, at any peak, the critical velocity is increasing while the corresponding maximum deflection is decreasing with the increase in the value of soil modulus (E_{soil}) for any constant subbase modulus (E_{sub}) fig.7. It is also noted that the material nonlinearity has also affected the occurrence of the critical velocity. It is being observed that the maximum deflection have occurred at smaller velocity in case of nonlinear soil medium as compared to the linear soil medium.

V. Conclusions

The objective of this analysis is to study the dynamic response of rigid pavement resting on two-parametric soil medium considering the material nonlinearity of the supporting soil medium subjected to moving load. The pavement is modelled by finite beam elements

The supporting foundation is modelled by Pasternak's two parameter soil medium. The critical velocity increases and the corresponding maximum deflection decreases with increasing sub- grade modulus and shear modulus. It is observed that the material nonlinearity of the soil medium has increased the magnitude of maximum deflection of the pavements and decreased the magnitude of the critical velocity. Thus, the material nonlinearity has affected the occurrence of the critical velocity. It is observed that the maximum deflection have occurred at smaller velocity in case of nonlinear soil medium. In general at low value of ultimate bearing or stress carrying capacity of the subgrade material and higher magnitude forces, the material nonlinearity plays a predominant role in the pavement analysis and prominently affects the deflections of the pavements. By carrying out the regression analysis based on the results obtained from dynamic analysis using various soil and pavement parameters an empirical equation is developed for the prediction of critical velocity and maximum deflection. The design Engineers can effectively use the suggested empirical equations for the nonlinear analysis case to predict the critical velocities and the corresponding maximum deflections at the prominent peaks for analyzing the airport pavements.

Table.3 V_{cr} and W_{max} for corresponding G_p and K

	G_p (kN/m²)													
K (kN/m²)	**10000**		**20000**		**30000**		**40000**		**50000**		**60000**		**70000**	
	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)	V_{cr} (m/s)	W_{max} (mm)
							PEAK- I							
30000	2.35	8.65	2.4	9.07	2.4	9.10	2.45	8.97	2.45	8.87	2.55	8.453	2.55	7.749
40000	3.15	7.43	2.75	7.04	2.8	7.09	2.8	7.10	2.8	6.87	2.9	6.623	2.95	6.489
50000	3.5	6.54	3.6	6.09	3.65	5.81	3.1	5.80	3.15	5.73	3.25	5.689	3.25	5.516
60000	3.75	5.58	3.85	5.25	3.35	4.45	3.4	4.6	4.05	4.78	3.4	4.74	3.55	4.739
							PEAK- II							
30000	5.35	14.82	5.4	16.77	5.45	15.95	5.45	13.96	5.5	11.69	5.45	9.415	5.4	9.398
40000	6.25	9.98	6.25	10.57	6.3	10.65	6.35	10.34	6.35	9.613	6.4	8.57	6.4	7.608
50000	7.0	7.08	7.0	7.61	7.05	7.92	7.1	7.786	7.1	7.456	7.15	7.048	7.15	6.475
60000	7.7	5.97	7.7	5.79	7.75	5.76	7.75	5.858	7.8	5.794	7.8	5.619	7.85	5.388

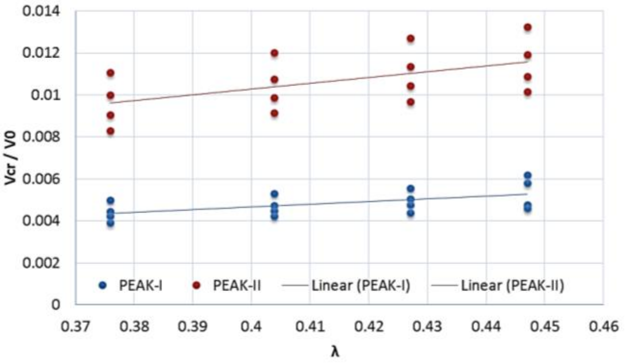


Figure.6.a Normalized Critical Velocity regressions Scatters

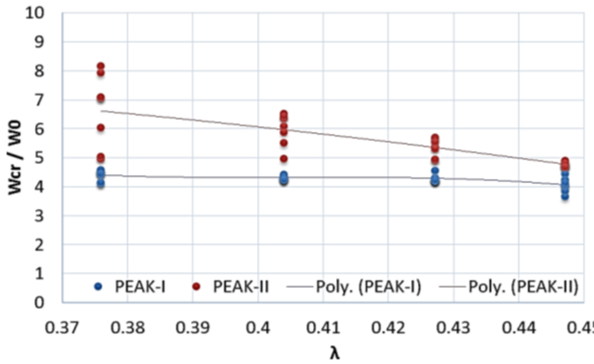


Figure.6.b Normalized Maximum Deflection regression Scatter

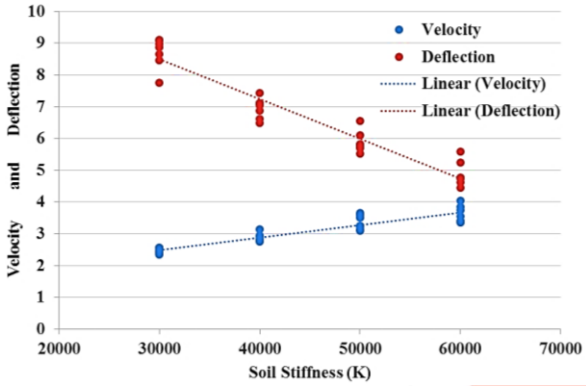


Figure.7 Relation between Velocity and Deflection

Table .4 Regression Analysis constants

Constant	Nonlinear	
	Peak—I	**Peak—II**
ϕ_0	-0.0006	-0.0005
ϕ_1	0.0272	0.0128
ψ_0	-4.476	263.11
ψ_1	100.41	-1916.7
ψ_2	-243.24	4730.7
ψ_3	145.28	-3890.9

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