Enhanced Newton Method Based Radial Distribution System Load Flow Analysis with Extrapolation Techniques

Asst. Prof. Dr. Hassan Kuhba
Electrical Engineering Department, Engineering College/Baghdad University, Baghdad, Iraq

***Abstract* - This paper presents a modified Newton method of load flow analysis for radial distribution systems. It is derived with the Jacobian matrix is in UDU^T form, where U is a constant upper triangular matrix depending solely on system topology and D is a block diagonal matrix. With this formulation, the conventional steps of forming the Jacobian matrix, LU factorization and forward/back substitution are replaced by back/forward sweeps on radial feeders with equivalent impedances. The method has advantages over Newton's method in terms of speed of solution (no. of iterations), and reliability of convergence by inserting a minimization technique (Linear and/or Geometric Extrapolation Techniques) as well as a cubic interpolation technique can be used. The algorithm exhibits a quadratic convergence as well as a control of the convergence. As such the method converges for cases when conventional Newton's method and some other popular methods diverge. Two large distribution systems of 490 nodes and 722 nodes with different R/X ratio in line impedance are used to examine the performance of the method. These tests have shown that the proposed method is as robust and efficient as the forward/back sweep method. The proposed method can be extended to the solution of three phase unbalanced representation.**

***Keywords* - Extrapolation Techniques, Load Flow Analysis, Newton's Method, Radial Distribution System, Successive Over Relaxation**

I. INTRODUCTION

A load flow study involves the determination of voltages, currents, powers, and losses at various points in an electrical network under existing or contemplated condition of normal operation, subject to the regulating capability of generators, condensers, and tap changing under load transformers as well as specified net interchange between individual operating systems. In 1967, Tinney and Hart developed the Newton based power flow solution method [1]. Later work by Stott and Alsac [2] made the fast decoupled Newton method as well as its alternatives; standard methods for load flow studies. The Fast decoupled Newton method works well for transmission systems, though its convergence performance is poor for most distribution systems due to their high R/X ratio which deteriorates the diagonal dominance of the Jacobian matrix. For this reason several non-Newton type methods have been proposed [3, 5]. Their algorithms all consist of back/forward sweeps on a ladder system. The formulation and the algorithm of these methods are different from the Newton's power flow method, which makes these methods hard to be extended to other applications, such as the state estimation and the optimal power flow, in which the Newton method seems more appropriate.

Recently, a fast decoupled load flow method has been proposed in [6]. This method orders the laterals instead of busses into layers, thus reduces the problem size to the number of laterals, and then assumes initial end voltage for all laterals. The iteration starts from the first lateral using the method proposed in [4]. The voltage mismatch obtained from this lateral is applied to correct not only the end voltage of this lateral but also the end voltage of laterals of the next layer. The algorithm converges when all voltage mismatches are within a certain tolerance. Using lateral variables instead of bus variables makes this method efficient for a given system topology, but it may add some overhead if the system topology is changed regularly, which is common in distribution systems due to switching operations. The purpose of this paper is to derive a modified Newton method for radial distribution systems without reducing the problem size, yet, capable of achieving robust convergence and high efficiency by inserting an accelerating and controlling factor using the extrapolation techniques [7]. Specifically, this paper aim to derive a Newton algorithm in which the Jacobian matrix is in ($\mathbf{UDU^T}$) form, where **U** is a constant upper triangular matrix depending solely on system topology and **D** is a block diagonal matrix resulting from the radial structure and special properties of the distribution system. With this formulation, the conventional Newton algorithm of forming the Jacobian matrix, **LU** factorization and forward /back substitution can be replaced by back/forward sweeps on radial feeders with equivalent impedances. To assist in presenting the main difference between the proposed method and other methods, the following definitions are used: (i) Conventional Newton method: a method in which the partial derivative of the power flow equation, i.e., the Jacobian matrix elements are used to determine the search direction, and forward/back substitutions on the **LU** factors of the Jacobian matrix are used to calculate the incremental correction of the state variables. (ii) Back/forward sweep method: a method in which the derivative of the power flow equation is not used, instead, basic circuit laws, i.e. Ohm' s law, KVL, and KCL (or the generalized KCL for power summation) are used as basis for back/forward sweeps on a radial network to calculate the incremental correction of the state variables. (iii) Enhanced Newton method: the proposed method in which an approximate Jacobian matrix in $\mathbf{UDU^T}$ form is used to determine the search direction, the linearized power flow equation based

on this Jacobian matrix is used as basis for back/forward sweeps on a radial network to calculate the incremental correction of the state variables. By means of extrapolation and/or cubic interpolation minimization techniques, the algorithm exhibits an accelerating factor (ß) to control the convergence process.

II. Basic Circuit Theory in Distribution System

For a linear, time invariant RLC circuit with a sinusoidal voltage source, the basic circuit theory can be expressed as (see Appendix I for details) [8]:

Ohm's law: $I_b = Y_b V_b$ (1)

KCL: $A I_b = I_n$ (2)

KVL: $b V_b = 0$ (3)

For fundamental frequency power flow calculations, a distribution system is always modeled as a linear, time-invariant RLC circuit. Earth is always treated as a reference node. For a radial distribution system with (n) nodes and without shunt branches, the number of branches is (n-1). Therefore, the dimension of matrix A is nx(n-1).

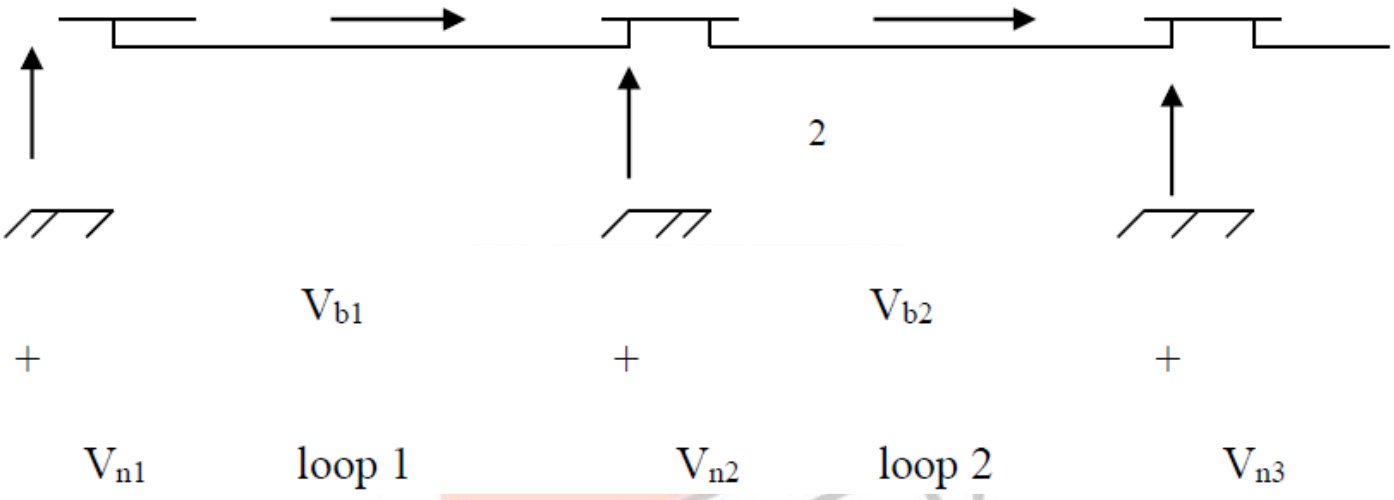


Figure 1: Distribution lines with a fictitious shunt branch

The independent loop for radial distribution systems can always be formed by a branch with its two shunt branches. Since shunt branches are usually neglected in modeling distribution lines, a fictitious shunt branch can be placed with branch voltage to be the nodal voltage as shown in Figure (1), and eqn.(3) can be written as:

$$V_b = A^T V_n \quad (4)$$

Combining (1), (2), and (4), we have:

$$A Y_b A^T V_n = I_n \quad (5)$$

By knowing the nodal voltage at one node, assuming it is the first node(slack bus "s") for convenience, and nodal current injections at the other (n-1) nodes, eqn. (6) can be derived from eqn. (5) for solving the remaining (n-1) unknown nodal voltages:

$$A_{n-1}\, Y_b\, (A_s^T\, A_{n-1}^T) \begin{bmatrix} V_s \\ V_{n-1} \end{bmatrix} = I_{n-1} \qquad (6)$$

Where $A = \begin{bmatrix} A_s \\ A_{n-1} \end{bmatrix}$, $V_n = \begin{bmatrix} V_s \\ V_{n-1} \end{bmatrix}$, and $I_n = \begin{bmatrix} I_s \\ I_{n-1} \end{bmatrix}$

Note matrix A_{n-1} is a square matrix. Since every branch is always directed away from one node and towards the other node, we have:

$$A^T e_n = 0 \qquad (7a)$$

or

$$A_s^T + A_{n-1}^T e_{n-1} = 0 \qquad (7b)$$

Where, e_n and e_{n-1} are unity column vectors with dimensions n and n-1 respectively. Hence, (6) can be simplified as:

$$A_{n-1} Y_b A_{n-1}^T (V_{n-1} - V_s\, e_{n-1}) = I_{n-1} \qquad (8)$$

$A_{n-1}\, Y_b\, A_{n-1}^T$ is the Nodal Admittance Matrix. In other words, for a radial system without shunt branches, the nodal admittance matrix is formed as the product of three square matrices. If we organize (8) as follows:

$$A_{n-1}\, I_L = I_{n-1} \qquad (8a)$$

$$Y_b\, A_{n-1}^T (V_{n-1} - V_s\, e_{n-1}) = I_L \qquad (8b)$$

Solving for I_L from (8a) is equivalent to the "Backward sweep", and solving for V_{n-1} from (8b) is equivalent to the "Forward sweep". This observation is very important as it motivated us to derive a Jacobian matrix in UDU^T form and a back/forward sweep algorithm for the Newton method.

III. The Enhanced Newton Method

Under the following assumptions:

Small voltage difference between two adjacent nodes, no shunt branches, the Jacobian matrix for a radial system is formed as UDU^T, where U is a constant upper triangular matrix depending solely on system topology and D is a block diagonal matrix. The first assumption above is valid, since typical distribution lines are short and power flows are not high. The second assumption is not valid if there exist shunt capacitor banks, constant impedance loads, as well as non-negligible shunt admittance of distribution line models (π-model). However, all the shunt branches can be converted to node power injections using initial and updated node voltages. In the conventional Newton method [7,9, 10], the power flow problem is to solve eqn.(9) for $\Delta\theta$ and ΔV :

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V/V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (9)$$

$$H_{km} = -V_k V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}) \qquad m \neq k \qquad (10)$$

$$H_{kk} = V_k \Sigma\, V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}) \qquad m=1,..n,\ m\neq k \qquad (11)$$

$$N_{km} = -V_k V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}) \qquad m \neq k \qquad (12)$$

$$N_{km} = -V_k\, \Sigma\, V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}) - 2V_k^2 G_{kk} \qquad m=1,..n\ ,m\neq k \qquad (13)$$

$$j_{km} = V_k V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}) \qquad m\neq k \qquad (14)$$

$$j_{kk} = -V_k\, \Sigma\, V_m(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}) \qquad m=1,..n\ ,m\neq k \qquad (15)$$

$$L_{km} = -V_k V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}) \qquad m\neq k \qquad (16)$$

$$L_{kk} = -V_k\, \Sigma\, V_m(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}) + 2V_k^2 B_{kk} \qquad m=1,..n\ ,m\neq k \qquad (17)$$

$G_{km} + jB_{km}$ is the entry of nodal admittance matrix. Since the voltage difference between two adjacent nodes is small as well as: $G_{kk}+jB_{kk}=-\Sigma(G_{km}+jB_{km})$,

For systems without shunt branches, the Jacobian matrix can be approximated as:

$$H_{km}=V_k V_m B_{km} \cos\theta_{km} \qquad m\neq k \qquad (18)$$

$$H_{kk}=-V_k \Sigma V_m B_{km} \cos\theta_{km} \qquad m=1,..n \ ,m\neq k \qquad (19)$$

$$N_{km}= - V_k V_m G_{km} \cos\theta_{km} \qquad m\neq k \qquad (20)$$

$$N_{kk} = V_k \ \Sigma V_m G_{km} \cos\theta_{km} \qquad m=1,..n \ ,m\neq k \qquad (21)$$

$$j_{km} = V_k V_m G_{km} \cos\theta_{km} \qquad m\neq k \qquad (22)$$

$$j_{kk} = -V_k \Sigma V_m G_{km} \cos\theta_{km} \qquad m=1,..n \ ,m\neq k \qquad (23)$$

$$L_{km} = V_k V_m B_{km} \cos\theta_{km} \qquad m \neq k \qquad (24)$$

$$L_{kk} = - V_k \Sigma V_m B_{km} \cos\theta_{km} \qquad m=1,..n \ ,m\neq k \qquad (25)$$

Equations (18) to (25) shows that matrices H, N, j, L all have the same properties (symmetry, sparsity pattern) as those of the nodal admittance matrix, hence they can be formed as:

$$H = L = A_{n-1} D_B A_{n-1}^T \qquad (26)$$

$$j = -N = A_{n-1} D_G A_{n-1}^T \qquad (27)$$

where D_B, and D_G are diagonal matrices with diagonal entries to be $V_k V_m B_{km} \cos\theta_{km}$ and $V_k V_m G_{km} \cos\theta_{km}$ respectively. Therefore (9) can be rewritten as:

$$\begin{bmatrix} A_{n-1} & \\ & A_{n-1} \end{bmatrix} \begin{bmatrix} D_B & -D_G \\ D_G & D_B \end{bmatrix} \begin{bmatrix} A_{n-1}^T & \\ & A_{n-1}^T \end{bmatrix} \begin{bmatrix} \Delta\Theta \\ \Delta V/V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (28)$$

It is also noted that if nodes and branches are ordered properly, An-1 is an upper triangular matrix with all diagonal entries to be 1 and all non-zero off- diagonal entries to be -1. One way to achieve such an An-1 is ordering branches by layers away from the root node (source bus) [3]. This ordering scheme is adopted here. The direction of each branch is towards the root node. The node ordering is proceeded simultaneously with the branch ordering. The branch "from side" node number is the same as the branch number, as illustrated in (2). The node to branch incidence matrix of Figure (2) is given in (29).

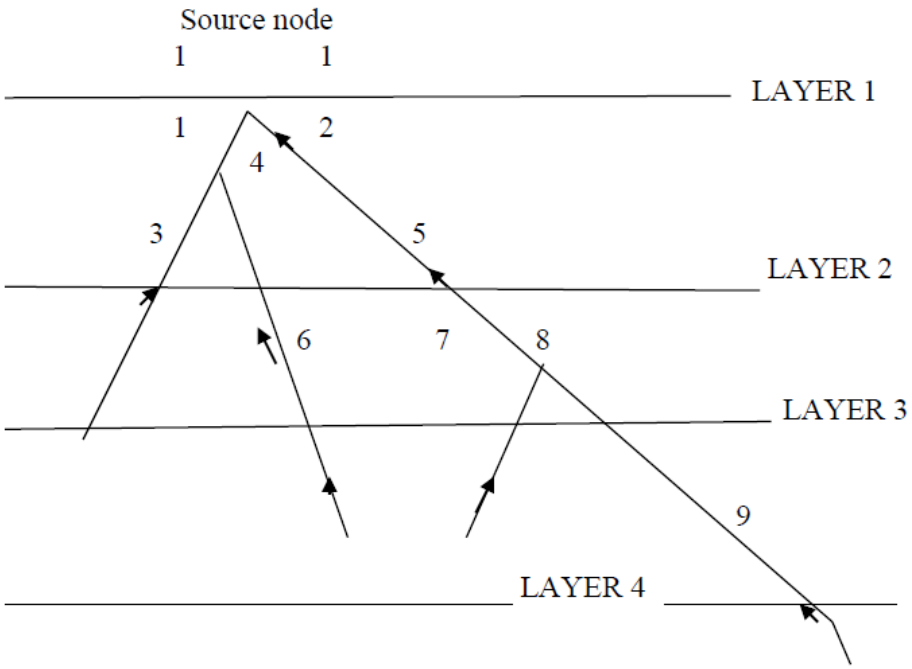


Figure 2. A simple radial distribution system

$$A_{n-1} = \begin{bmatrix} 1 & -1 & -1 & & & & & & \\ & 1 & & & -1 & & & & \\ & & 1 & & & & & & \\ & & & 1 & & -1 & & & \\ & & & & & & & & \\ & & & & 1 & & -1 & -1 & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & -1 \\ & & & & & & & & 1 \end{bmatrix} \quad (29)$$

By now we have shown that the Jacobian matrix can be formed as the product of three square matrices eqn. (28), same as the nodal admittance matrix in eqn. (8). Next, we will show that eqn. (28) can be solved by back/forward sweeps as well. Let's define:

$$E = \Delta\theta + j\,\Delta V/V \quad (30)$$

$$S = \Delta P + j\,\Delta Q \quad (31)$$

$$W = D_B + j\,D_G \quad (32)$$

Then eqn. (28) can be written as:

$$A_{n-1}\, W\, A_{n-1}^{T}\, E = S \tag{33}$$

$$\text{or:}\quad A_{n-1}\, S_L = S \tag{34}$$

$$W\, A_{n-1}^{T}\, E = S_L \tag{35}$$

Where (34) is the back sweep, the diagonal matrix W can be inverted for each line. The diagonal in W^{-1} is denoted as the equivalent line impedance:

$$Z_{eq} = R_{eq} + jX_{eq} \tag{36}$$

$$\text{Where}\qquad R_{eq} = X_{km}/(V_k V_m \cos\theta_{km}) \tag{37}$$

$$X_{eq} = R_{km}/(V_k V_m \cos\theta_{km}) \tag{38}$$

R_{km} and X_{km} are resistance and reactance of line k-m respectively. Note that matrix A_{n-1} of nonzero entries is either -1 or 1.

IV. Linear and Geometric Extrapolation Techniques

It is well-known that the load flow calculation can be regarded as a nonlinear programming problem [7, 9] which determines the direction and magnitude of the solution so that a certain function F(x) may be minimized. The F(x) is the squares of the active and reactive mismatch power. By employing this formulation, the valuable property can be obtained that the solution never diverges. The value of the function F(x) becomes eventually zero if there is a solution from the initial estimate, and stays at a positive value if no solution exists. In nonlinear programming approach (Fletcher-Powell method), (Δx) is modified by a correction factor (ß) which can be considered as an acceleration factor. The computation of (ß) is made such that F(x) is minimized F(x). The function to be minimized is

$$F(V, \Theta) = \sum_{k\in PQ,PV}^{n} \Delta P_k^2 + \sum_{k\in PQ}^{n} \Delta Q_k^2 \tag{39}$$

The minimization of F(x) with respect to (ß) in the direction of (Δx) is a one-dimensional problem. The object is to determine the correction factor (ß) given (Δx) and the point (x). The problem can be stated as that of finding a value of (ß) that will minimize F(V, Θ).

With this type of process, convergence can be accelerated. The most popular method is the small linear extrapolation technique known as "Successive over Relaxation" (S.O.R.).

With (S.O.R.), the value of each voltage variable calculated after each iteration is projected by a constant factor ß in the direction in which it is changing, at every iteration. Hence if (E_k^{p+1}) is the normal unaccelerated voltage, it is updated immediately by:

$$E_k^{p+1}\ (\text{accelerated}) = E_k^{p} + ß\,(E_k^{p+1} - E_k^{p}) \tag{40}$$

Where $1 < ß < 2$. A different value of (ß) can be applied to the real and imaginary parts of (E_k) or a complex (ß) can be used in (40). Normally a real (ß) in (40) is used. The optimum value can be determined by experiment on a given problem, but with experience on a particular system, or can be calculated by cubic interpolation technique [9], the load flow user can estimate a near-optimum value.

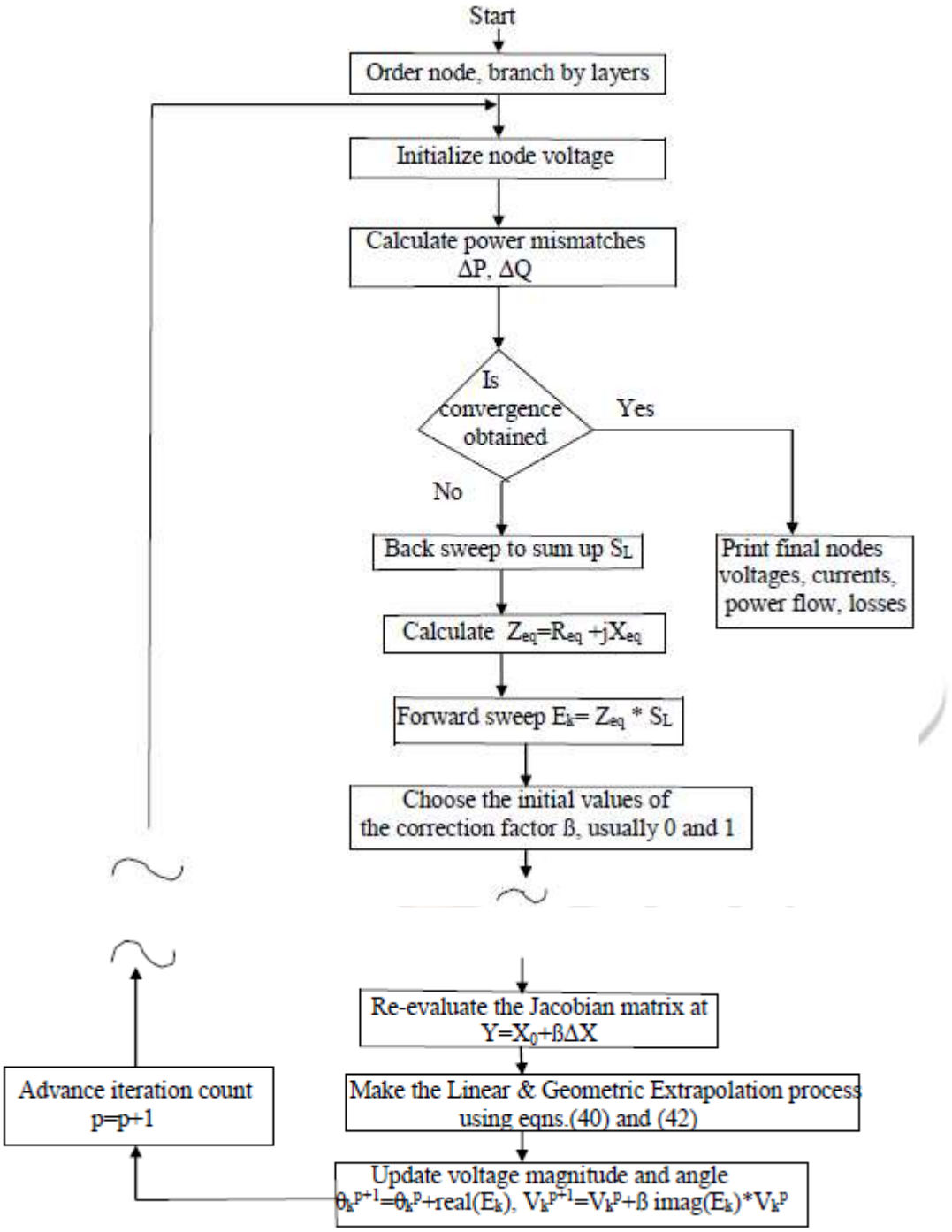
Another form of acceleration that can be used in combination with (S.O.R.) is the "geometric-extrapolation-to infinity" method. At any stage in the solution where all the voltages are seen to be converging smoothly (monotonically), it is assumed that convergence is geometric, that is, for each variable E_k:

$$\frac{E_k^{p+1}}{E_k^{p}} = \frac{E_k^{p}}{E_k^{p-1}} \tag{41}$$

On this assumption, the solution at an infinite number of iterations for the monotonic asymptotically-converging curve can be obtained by assuming a geometric series:

$$E_k^{p+1}\ (\text{accelerated}) = E_k^{p+1} - \frac{(E_k^{p+1} - E_k^{p})^2}{E_k^{p+1} - 2E_k^{p} + E_k^{p-1}} \tag{42}$$

Since convergence is only approximately (not exactly geometric), this does not give the true solution in one application. Algorithm (42) is applied to each variable a number of times during the solution, whenever monotonic convergence is detected. It is time-consuming to have to detect this, and previous values of the variable need to store. Figure (3) is the flow chart of the proposed algorithm. The matrix An-1 has never been formed in the program since its non-zero entries are either -1 or 1 [9].



V. Analysis Based on Numerical Examples

A 490-node and 722--node typical distribution systems of various sizes are taken from the pacific Gas and Electric distribution system [5]. Both capacitors and regulators are included in the test systems, but the automatic controls for switching capacitors on/off, and the automatic tap adjustment function for regulators are not modeled in the test. All the loads are modeled as lumped constant power load. Table (1) lists the attributes of these test systems. It is seen that the two test systems have wide range of R/X ratio and line impedance.

Table1. Attributes of Test systems

Test system	No. of nodes	Voltage level (Kv)	R/X ratio Max	R/X ratio Min	$\|Z_{km}\|$ Ω Max	$\|Z_{km}\|$ Ω Min
1	490	12	5.06	0.15	3.07	0.0012
2	722	12	5.06	0.26	2.43	0.0004

Table (2) shows that the load flow problem was solved by back/forward sweep conventional Newton's method in 4 iterations to an accuracy of 10-4 for each individual power mismatch (ΔPk, ΔQk). In the proposed method 3iterations were required with the same accuracy. The values of optimum (ß) obtained for each iteration in the proposed method were close to one. My experience has shown that optimum (ß) is either close to 1 or very close to 0. The linear and geometric extrapolation formula will produce an appropriate value of (ß) even in the case where the optimum is near, but outside the extrapolating limits. So optimum (ß) may sometimes be slightly greater than one. If the extrapolation is performed between zero and one, the correction (ß) value would be determined for all cases without any extra Jacobian calculation per iteration, thus saving computation time. From table (2), it is seen that the proposed method is as robust as the back/forward sweep method. All the cases tested have reached convergence regardless the wide range of R/X ratio and line impedance. The principal value of the proposed method lies in the control of the convergence process for both weakly Meshed networks and data error cases, whereas using the conventional Newton's method alone during the iterations of a load flow problem may result in poor solution or divergence.

Table 2. Comparison of the proposed method with back/forward sweep method with the value of optimum (ß).

Test distribution system	No. of iterations Back/Forward sweep method	No. of iterations Proposed method	Optimum ß/iteration Proposed method 1	2	3	4
Test sys. 1	4	3	1.022	1.011	1.001	
Test sys. 2	5	4	0.994	1.013	1.01	1.001

Table (3) shows the active and reactive power mismatches at each iteration for both methods.

Table 3. Convergence pattern (power mismatches at each iteration) of the proposed method and the back/forward sweep method

Iteration	Test system 1		Test system 2	
	Proposed	B/F sweep	Proposed	B/F sweep
1 ΔP_k	0.30000551	0.30000881	0.14004100	0.14005200
ΔQ_k	0.14322081	0.14323091	0.05243800	0.05244800
2 ΔP_k	0.00988422	0.00643219	0.01708456	0.00982139
ΔQ_k	0.00139278	0.01113158	0.01412962	0.02180736
3 ΔP_k	0.00008821	0.00058982	0.00012250	0.00415120
ΔQ_k	0.00002741	0.00027818	0.00011821	0.00148164
4 ΔP_k	0.00001153	0.00004853	0.00024887	0.00015401
ΔQ_k	0.00001554	0.00003041	0.00075424	0.00005526
5 ΔP_k	0.00000059	0.00005526	0.00015501	0.00066501
ΔQ_k	0.00000043	0.00009652	0.00001753	0.00054223

Note: All algorithms programs were executed using MATLAB version 7.4

VI. CONCLUSIONS

This paper presents an enhanced Newton method for solving the load flow problem of radial distribution systems. The derivation of this method has revealed that under certain assumptions, the Jacobian matrix of a radial system can be formed as UDU^T, with identical topology to that of the nodal admittance matrix. The attractive characteristic of such a Jacobian matrix is to allow a back/forward iteration algorithm instead of the conventional LU factorization. Thus, the proposed method is more reliable in convergence with ill-conditioned radial distribution systems. Also, a more rapid convergence and a non-divergent characteristic for both well-conditioned and ill-conditioned systems by using an optimum correction factor (ß) through the linear and geometric extrapolation techniques as well as a cubic interpolation technique. Tests of the proposed method on large distribution feeders have shown that it is a robust and more efficient than the back/forward sweep method.

VII. REFERENCES

[1] W.F. Tinney and C. E. Hart, "Power Flow solution by Newton's method", IEEE Trans. Power App. System, Vol.PAS-86, Nov. 1967, pp. 1449-1460.

[2] B. Stott and O. Alsac, "Fast Decoupled Load Flow", IEEE Trans. Power App. System, Vol.PAS-93, May-June 1974, pp. 859-869.

[3] D. Shirmohammadi, H.W. Hang, A.semlyen, and G. X. Luo, "A compensation-based power flow method for weakly Meshed distribution networks", IEEE Trans. on power systems, Vol.3,No.2, May 1988, pp. 753-762.

[4] G. X. Luo and A. Semlyen, "Efficient load flow for large weakly meshed networks", IEEE Trans. on power systems, Vol.5, No.4, Nov. 1990, pp. 1309-1316.

[5] C. S. Cheng and D. Shirmohammadi, "A three-phase power flow method for real-time distribution system analysis", IEEE Trans. on power systems, Vol.10, No.2, May 1995, pp. 671-679.

[6] R. D. Zimmerman and H. D. Chiang,"Fast decoupled power flow for unbalanced radial distribution system", 1995, IEEE PES Winter meeting, New York, Paper No. 95 WM 219-6 PWRS.

[7] H. A. Kubba, "A rapid and more reliable load flow solution method for ill-conditioned power system", Eng. And Technology. Vol.17, No.5, 1998, pp. 550-568.

[8] H. N. Dhar, "Computer aided power system operation and analysis", 1982, McGraw-Hill company

[9] Kubba H. A., ″An Efficient and more Reliable Second Order Load Flow Solution Method″, Journal of International Association for the Advancement of Modelling and Simulation Techniques in Enterprises, Vol. 46, Iss. 1, No. 1-2, 2009, pp. 1-19.

[10] H. A. Kubba,"An improved method of automatic adjustment of transformer and phase shifter taps for constrained load flow", AL-Muhandis, No.4, Dec.2001, pp. 34-50.

APPENDIX I

For a linear, time-invariant RLC circuit with sinusoidal voltage, which a typical distribution system is modeled as, Ohm's law has the following familiar form [8]:

where Ib : a vector of complex branch current, Yb: a diagonal matrix with diagonal element to be branch admittance, Vb: a vector of complex branch voltage. The directions of Ib and Vb must be consistent. For KCL, we have:

where In : vector of nodal injection current, A : node to branch incidence matrix, defined as:
1, if branch m is directed away from node k
Akm = -1, if branch m is directed towards node k
0, if branch m is not incident to node k

Each row of (A.2) corresponds to KCL for a node. Since each branch is always directed away from one node and towards one other node, the summation of all rows will end with zero. Hence, for a system with n nodes, only (n-1) rows in (A.2) are independent. An arbitrary row can be removed from (A.2) so that the remaining (n-1) rows are independent. The node corresponding to the removed row is usually called the reference node. To express KVL in matrix form, a loop matrix B is defined as:

1, if branch m is in loop k, and their direction agree
bkm = -1, if branch m is in loop k, and their direction oppose
0, if branch m is not in loop k
Then KVL can be expressed as:
b Vb = 0 (A.3)

Each row of (A.3) corresponds to KVL for a loop. It is not necessary to include all loops in (A.3) since independent loops are sufficient for Vb to obey. The independent loop is defined as that not all the branches of the independent loop can be found in other independent loops.