Optimal Redundancy Allocation in Complex Systems

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***Abstract*— A simple computational procedure has been developed by using Monte Carlo for allocating redundancy among subsystems so as to achieve maximum reliability of a complex systems subjected to multiple constraints which may be linear, non linear separable or non separable. Two examples of linear, non linear separable and non separable constraints with having twenty problems are solved.**

***Index Terms*— Active Redundancy, System Reliability, Constraints, Problem formulation, Monte Carlo Algorithm.**

I. INTRODUCTION

Reliability of an overall system can be increased by introducing redundancies in subsystems. In order to ensure that factors such as cost, weight, and volume remain within resources available, system reliability is optimized with respect to these constraints.
The redundancy optimization problem is a classical problem that has attracted considerable attention from the research community. It has been solved by using optimization approaches and techniques such as dynamic programming[1], integer programming[3], mixed-integer linear programming[2], heuristics[4] and meta-heuristics[5] which gives either optimal or very good solutions.
This paper presents a Monte Carlo Method [6] that solves the problems of constrained redundancy optimization in complex system which gives the local optimal system reliability. Our aim is to show how to maximize system reliability subjected to the given constraints. For optimizing the system reliability we prefer simulation because it gives the result fast & accurate.
For solving this problem (or optimizing system reliability) many authors have used cumbersome approaches. However in this paper the proposed Monte Carlo Method is applied for all systems and to all type of constraints and also it has been tried on many problems, with satisfactory results.

II. PROBLEM FORMULATION AND COMPUTATION PROCEDURE

A. *Notations*

- $Ri(x_i)$, $Q_i(x_i)$ reliability, unreliability of subsystem-i with x_i components.
- $Rs(x)$ system reliability
- x_i number of components in subsystem-i
- x $(x_1,.............. x_n)$
- $x^*(r_u)$ optimum value of x in iteration u
- $G_{ir}(x_i)$ resource-r consumed in subsystem-i with x_i components.
- b_r maximum of resource-r
- k number of constraints
- n number of subsystems
- f(.) function that yields the system reliability, based on n unique, subsystems and which depends on the configuration of the subsystems.
- $\alpha_l(x)$ Sensitivity-factor
- $b_i(x_i)$ Selection-factor
- p1 Minimal path Set-l of the system
- c_i Cost of the x_i subsystem

B. *Assumptions*

- The system and all of its subsystems are coherent.
- There are n subsystems in the system. Subsystem structure (other than coherence) is not restricted.
- All component states are mutually statistically in dependent.
- All constraints are separable and additive among components. Each constraint is an increasing function of x_i for each subsystem.
- Redundant components can not cross subsystem boundaries.

C. *Problem Formulation*

- The problem of constrained redundancy optimization in complex systems can be reduced to the following integer programming problem:
- Maximize: $Rs(x) = f\,(R_1(x_1),..........,R_n(x_n))$
- Subject to multiple linear or nonlinear constraints :

$$\sum_{i=1}^{k} G_{i\,r}(x) \le b_r; \quad r = 1,2,3.........s \text{ where the } x_i \text{ are positive integers, for all i}$$

III. MONTE CARLO ALGORITHM

Steps of Algorithm

1. Let x_i =1 for all i
2. Calculate Rs(x)
3. Repeat 100 times

   x_i^* = random number between 1 to 10 for all i.

   Both linear and nonlinear ($\sum_{i=1}^{k} G_{i\,r}(x_i) \le b_r$) are violated. If no constraint are violated

   Calculate Rs(x*)

   If Rs(x*) ≥ Rs(x)

   Then Rs(x) = Rs(x*) at x = x*

   Calculate the output x_i^*, G(x) and Rs(x)
4. The simulation is limited to 3000 iteration only

IV. FIRST NUMERICAL EXAMPLE [7]

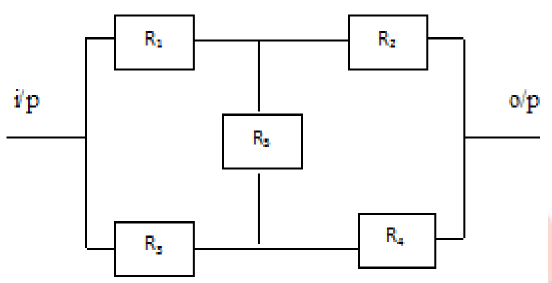

Fig.1 A Bridge System

The expression of system reliability (R_s) is

$R_s(x) = R_1(x_1)R_2(x_2)Q_3(x_3)Q_5(x_5) + Q_1(x_1) R_3(x_3) R_4(x_4) Q_5(x_5) + [R_1(x_1)R_3(x_3) + R_3(x_3)R_5(x_5) + R_5(x_5)R_1(x_1) - 2R_1(x_1)R_3(x_3)R_5(x_5)] *[R_2(x_2) + R_4(x_4) - R_2(x_2)R_4(x_4)$

A. *Problem*

TABLE 1
Shows subsystem's data

i	1	2	3	4	5
R_i	0.70	0.85	0.75	0.80	0.90
Q_i	0.30	0.15	0.25	0.20	0.10
c_i	2	3	2	3	1

Maximize $R_s(x)$

Subjected to linear constraints $\sum_{i=1}^{5} \mathbf{c_i x_i} \le 20$ Where x_i, i= 0,......,5 are positive integers

B. *Solution*

TABLE 2
Shows the optimal solution after particular Iteration

R_i	0.70	0.85	0.75	0.80	0.90	$\sum c_i x_i$	$R_s(x)$	Iteration (u)
Q_i	0.30	0.15	0.25	0.20	0.10	20	0.9949	2400
c_i	2	3	2	3	1			
x_i	1	2	4	1	1			
$c_i x_i$	2	6	8	3	1			

The above table shows the result of Monte Carlo Simulation. At 2400 iteration we get best combination of redundancy (x_1,............x_5), which satisfy the given constraints, It means the optimal solution is x*= {1,2,4,1,1} and $R_s(x^*)$ = 0.994

V. TWENTY DIFFERENT SETS OF PROBLEM

Twenty different data sets are considered and are solved by the proposed algorithm to obtain the best combination of redundancy (x_i) and system reliability (R_s).

A. *Problems*

TABLE 3
Shows the cost of the subsystems

Cost (c_i)	c_1	c_2	c_3	c_4	c_5
	2	3	2	3	1

TABLE 4
Shows the subsystem's reliability and consumed resources

Problem No.	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$	$R_4(x_4)$	$R_5(x_5)$	$\sum_{i=1}^{5} c_i x_i \leq$
1	0.70	0.85	0.75	0.80	0.90	20
2	0.60	0.70	0.80	0.90	0.60	30
3	0.70	0.85	0.90	0.80	0.75	30
4	0.70	0.85	0.90	0.80	0.75	35
5	0.80	0.80	0.90	0.90	0.90	25
6	0.80	0.80	0.90	0.90	0.90	30
7	0.80	0.80	0.90	0.90	0.90	35
8	0.75	0.65	0.85	0.95	0.55	20
9	0.75	0.65	0.85	0.95	0.55	25
10	0.75	0.65	0.85	0.95	0.55	30
11	0.80	0.65	0.72	0.78	0.60	25
12	0.80	0.65	0.72	0.78	0.60	30
13	0.80	0.65	0.72	0.78	0.60	34
14	0.90	0.90	0.75	0.75	0.75	25
15	0.90	0.90	0.75	0.80	0.75	25
16	0.90	0.90	0.75	0.80	0.75	30
17	0.72	0.62	0.59	0.93	0.67	25
18	0.72	0.62	0.59	0.93	0.67	30
19	0.72	0.62	0.59	0.93	0.67	35
20	0.95	0.67	0.59	0.93	0.67	30

B. *Solutions*

TABLE 5
Shows Optimal Redundancy and system reliability for the given problems

Problem no.	Redundancy {x_i}	Consumed Resources ($\sum c_i x_i$)	Reliability ($R_S(x_i)$)	Iteration (u)
1	1,2,4,1,1	20	0.9949	2600
2	1,2,5,3,3	30	0.9998	1900
3	4,2,1,4,2	30	0.9991	2100
4	5,3,3,3,1	35	0.99996	1700
5	3,1,1,4,2	25	0.9990	2700
6	2,3,3,3,2	30	0.9999	1600
7	4,1,4,4,4	35	0.99996	800
8	3,1,2,2,1	20	0.9953	2600
9	4,2,1,2,3	25	0.9974	2300
10	2,3,5,2,1	30	0.9998	1500
11	3,1,2,3,3	25	0.9939	2300
12	2,2,3,3,5	30	0.9978	1100
13	1,2,5,5,1	34	0.9995	900
14	3,3,2,1,1	23	0.9997	2800

15	3,3,1,1,5	25	0.9995	2100
16	3,3,1,4,1	30	0.9997	1700
17	5,1,1,2,4	25	0.9956	1900
18	3,2,4,2,4	30	0.9986	1200
19	4,1,5,3,5	35	0.9998	900
20	2,1,5,3,4	30	0.9998	1400

VI. SECOND NUMERICAL EXAMPLE

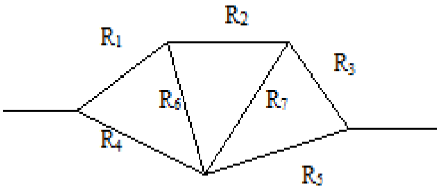

Fig.2 A Complex System (5 nodes, 7edges)

The system reliability (R_S) equation is

$R_S(X) = (Q_6^{x_6})[(1-Q_1^{x_1})(1-Q_2^{x_2})(1-Q_3^{x_3})(Q_7^{x_7})(Q_4^{x_4}) + (Q_1^{x_1})(Q_2^{x_2})(1-Q_4^{x_4})(1-Q_5^{x_5})(Q_7^{x_7}) + [(1-Q_1^{x_1})(1-Q_2^{x_2})(1-Q_4^{x_4}) + (1-Q_4^{x_4})(1-Q_7^{x_7}) + (1-Q_7^{x_7}(1-Q_1^{x_1})(1-Q_2^{x_2}) - 2(1-Q_1^{x_1})(1-Q_2^{x_2})(1-Q_4^{x_4})(1-Q_7^{x_7})][(1-Q_3^{x_3}) + (1-Q_5^{x_5}) - (1-Q_3^{x_3})(1-Q_5^{x_5})]] + (1-Q_6^{x_6})(1-Q_1^{x_1}Q_4^{x_4})[1-((1-Q_2^{x_2}Q_7^{x_7})(1-Q_1^{x_1}))Q_5^{x_5}]$

A. Problem

Maximize the System Reliability $R_s(x)$

Subject to the given constraints $\sum_{i=1}^{7} c_i x_i \le 35$ where x_i, i = 1, ..., 7 are positive integers.

B. Solution

TABLE 6
Shows the optimal solution after particular Iteration

R_i	0.60	0.55	0.75	0.90	0.90	0.75	0.60	$\sum c_i x_i$	$R_s(x)$	Iteration (u)
Q_i	0.40	0.45	0.25	0.10	0.10	0.25	0.40			
c_i	2	3	3	2	2	1	1	35	0.9899	1700
x_i	1	2	3	3	2	3	5			
$c_i x_i$	2	6	9	6	4	3	5			

The above table shows the result of Monte Carlo Simulation; at 1700 iteration we get best combination of redundancy (x_1............x_7). It means the optimal solution is x*= {1,2,3,3,2,3,5} and Rs(x*) = 0.9899 .

VII. APPLICATIONS

Monte Carlo is also applicable for both linear and nonlinear Constraints. Same bridge systems as shown in fig.1 are taken and new twenty different problems are framed by changing the each subsystem reliability, we can obtain the optimal system reliability under the subjected non linear constraints.

Maximize the system reliability $R_s(x) = R_1(x_1)R_2(x_2)Q_3(x_3)Q_5(x_5) + Q_1(x_1)R_3(x_3)R_4(x_4)Q_5(x_5) + [R_1(x_1)R_3(x_3) + R_3(x_3)R_5(x_5) + R_5(x_5)R_1(x_1) - 2R_1(x_1)R_3(x_3)R_5(x_5)]*[R_2(x_2) + R_4(x_4) - R_2(x_2)R_4(x_4)]$

A. *Subjected to the Non Linear (Separable) Constraints [8]*

$g_1(x) = x_1^2 + 2x_2^2 + 3x_3^2 + 4x_4^2 + 2x_5^2 \le C_1$

$g_2(x) = 7(x_1+\exp(x_1/4)) + 7(x_2+\exp(x_2/4)) + 5(x_3+\exp(x_3/4)) + 9(x_4+\exp(x_4/4) + 4(x_5+\exp(x_5/4)) \le C_2$

$g_3(x) = 7x_1\exp(x_1/4) + 8x_2\exp(x_2/4) + 8x_3\exp(x_3/4) + 6x_4\exp(x_4/4) + 9x_5\exp(x_5/4) \le C_3$

B. *Twenty Different Problems*

TABLE 7
Shows 20 problems containing subsystem's reliability

Problem No.	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$	$R_4(x_4)$	$R_5(x_5)$	C_1	C_2	C_3
1	0.80	0.85	0.90	0.65	0.75	110	175	200
2	0.70	0.90	0.55	0.65	0.85	110	175	200
3	0.90	0.95	0.55	0.85	0.55	110	175	220
4	0.80	0.70	0.60	0.80	0.70	115	140	220
5	0.60	0.78	0.65	0.82	0.70	115	140	220
6	0.60	0.78	0.65	0.82	0.70	120	130	180

7	0.90	0.88	0.82	0.83	0.50	120	130	180
8	0.90	0.88	0.82	0.83	0.50	150	140	210
9	0.86	0.79	0.78	0.88	0.78	150	140	210
10	0.65	0.75	0.55	0.45	0.85	150	140	210
11	0.65	0.75	0.55	0.45	0.85	125	155	215
12	0.70	0.80	0.90	0.90	0.90	125	155	215
13	0.78	0.88	0.91	0.81	0.76	125	155	215
14	0.78	0.88	0.91	0.81	0.76	150	150	250
15	0.90	0.80	0.75	0.65	0.83	150	150	250
16	0.9	0.6	0.76	0.79	0.81	110	175	200
17	0.9	0.6	0.76	0.79	0.81	113	177	210
18	0.77	0.68	0.59	0.91	0.86	54	138	90
19	0.77	0.68	0.59	0.91	0.86	61	142	187
20	0.58	0.61	0.93	0.74	0.9	112	147	187

C. *Solutions of Twenty Different Problems*

TABLE 8

Shows Optimal Redundancy, system reliability and iteration.

Problem No.	Redundancy	Constrains			Reliability	Iteration
	x_1,x_2,x_3,x_4,x_5	C_1	C_2	C_3	$R_s(x_i)$	u
1	3,3,3,2,1	110	175	200	0.9993	2800
2	2,1,4,3,1	110	175	200	0.9910	1800
3	4,2,2,1,3	110	175	220	0.9996	900
4	3,2,2,2,2	115	140	220	0.9939	2500
5	3,2,3,2,1	115	140	220	0.9933	1900
6	2,2,4,2,1	120	130	180	0.9944	2600
7	1,1,2,3,3	120	130	180	0.9957	1900
8	1,3,3,2,2	150	140	210	0.9986	1100
9	2,1,3,3,1	150	140	210	0.9989	1700
10	2,4,3,1,1	150	140	210	0.9775	1600
11	2,4,3,3,1	125	155	215	0.9854	2100
12	3,2,3,2,1	125	155	215	0.9995	1200
13	3,2,2,3,3	125	155	215	0.9998	1000
14	2,3,3,3,2	150	150	250	0.9858	1300
15	3,4,3,1,1	150	150	250	0.9994	1600
16	1,2,3,3,3	110	175	200	0.9971	1100
17	3,2,3,4,1	113	177	210	0.9993	1400
18	5,2,1,2,1	54	138	90	0.9931	1700
19	4,3,1,2,2	61	142	187	0.9983	1900
20	1,3,3,4,1	112	147	187	0.9994	2400

D. Subjected to Non Linear (Non Separable) Constraints

$$g_1(x) = 2x_1x_2^2 + 3x_3x_4^2 + 4x_4x_5^2 \leq C_1$$

$$g_2(x) = 7(x_1+\exp(x_1/4))+7(x_2+\exp(x_2/4))+5(x_3+\exp(x_3/4))+9(x_4+\exp(x_4/4) \leq C_2$$

$$g_3(x) = 7x_1\exp(x_2/4) + 8x_2\exp(x_1/4) + 8x_3\exp(x_4/4) + 6x_4\exp(x_3/4) + 9x_4\exp(x_5/4) \leq C_3$$

E. *Twenty Different Problems*

TABLE 9

Contains the subsystem's reliability and consumed resources

	Reliabilities R_i (x_i)					Constraints		
Problem No.	R_1	R_2	R_3	R_4	R_5	C_1	C_2	C_3
1	0.80	0.85	0.90	0.65	0.75	110	175	200
2	0.70	0.90	0.55	0.65	0.85	110	175	200
3	0.90	0.95	0.55	0.85	0.55	110	175	220
4	0.80	0.70	0.60	0.80	0.70	115	140	220
5	0.60	0.78	0.65	0.82	0.70	115	140	220
6	0.60	0.78	0.65	0.82	0.70	120	130	180
7	0.90	0.88	0.82	0.83	0.50	120	130	180
8	0.90	0.88	0.82	0.83	0.50	150	140	210
9	0.86	0.79	0.78	0.88	0.78	150	140	210
10	0.65	0.75	0.55	0.45	0.85	150	140	210
11	0.65	0.75	0.55	0.45	0.85	125	155	215
12	0.70	0.80	0.90	0.90	0.90	125	155	215
13	0.78	0.88	0.91	0.81	0.76	130	155	215
14	0.78	0.88	0.91	0.81	0.76	150	150	250
15	0.90	0.80	0.75	0.65	0.83	150	150	250
16	0.90	0.60	0.76	0.79	0.81	110	175	200
17	0.90	0.60	0.76	0.79	0.81	113	177	210

18	0.77	0.68	0.59	0.91	0.86	70	145	150
19	0.77	0.68	0.59	0.91	0.86	70	138	140
20	0.58	0.61	0.93	0.74	0.90	170	160	210

F. *Solution of Twenty Different Problems*

TABLE 10
Shows Optimal Redundancy and system reliability and Constrains

Problem No.	Redundancy	Constraints			Reliability	Iteration
	$\{x_i\}$	C_1	C_2	C_3	$R_S(x_i)$	u
1	1,3,4,1,3	66	126	122	0.9977	1600
2	5,2,4,1,2	68	154	186	0.9964	2600
3	3,2,2,1,2	46	115	114	0.9993	600
4	2,3,3,2,2	104	134	164	0.9961	2400
5	3,3,2,2,1	86	132	165	0.9909	2700
6	1,2,4,2,2	88	123	147	0.9913	1800
7	2,3,1,2,2	80	120	127	0.9980	2800
8	3,3,2,2,2	110	137	171	0.9999	1400
9	1,3,3,3,1	111	133	169	0.9984	1700
10	3,3,4,1,1	70	135	164	0.9863	2500
11	3,4,3,1,1	109	138	180	0.9907	2800
12	4,3,1,2,1	116	142	183	0.9991	1400
13	2,3,3,3,1	129	142	193	0.9999	1800
14	1,3,3,3,2	147	138	179	0.9997	1900
15	2,3,3,3,1	129	142	193	0.9994	2400
16	3,3,3,2,1	98	139	183	0.9970	1200
17	1,3,3,3,1	111	133	169	0.9977	2600
18	5,1,2,2,2	66	141	149	0.9963	2100
19	1,3,5,1,3	69	135	137	0.9943	2800
20	4,3,2,2,3	168	168	209	0.9992	1800

VIII. RESULTS AND CONCLUSIONS

In this paper, the Proposed Monte Carlo method is applied to different problems of bridge and complex (5 nodes,7edges) system to optimize the system reliability under the given constraints.

This leads to the conclusion that the system reliability of Monte Carlo is much more optimized and we can get better result. The proposed method is also not affected by any kind or number of constraints which is shown by applying different problems. Its simplicity and suitability for computer solution makes this method highly useful to reliability engineers. It has not been rigorously proved that this method is exact, but no faulty solution has yet been found. Even if the method is not exact, it appears to be very close and we get better optimal solution for any complex system.

REFERENCES

1. D.K. Kulshrestha, M.C. Gupta, "Use of dynamic programming for reliability engineers, IEEE Trans. Reliability, vol R-22, pp 240- 241 (Oct 1973).
2. P.J. Kolesar, "Linear programming and the reliability of multi component systems," Naval Research Logistics Quarterly, vol 15, pp 317-327 (Sep 1967).
3. K.B. Misra, "A method of solving redundancy optimization problems," IEEE Trans. Reliability, vol R-20, pp 1 17-120 (Aug 1971).
4. K.K. Aggarwal, "Redundancy optimization in general systems," IEEE Trans. Reliability, vol R-25, pp 330-332 (Dec 1976).
5. F. Altiparmak, B. Dengiz, and A. E. Smith, “Optimal design of reliable computer networks: A comparison of metaheuristics,” *J. Heuristics* vol. 9, no. 6, pp. 471–487, Dec. 2003.
6. J. Y. Lin and C. E. Donaghey, “Monte Carlo simulation to determine minimal cut sets and system reliability,” in *Proceedings of the 1993 Annual Reliability and Maintainability Symposium*, Atlanta, GA, USA, 1993.
7. Dinghua, Shi; “A New Heuristic Algorithm for Constrained Redundancy-Optimization in Complex Systems,” IEEE Trans.Reliability, Volume: R-36, pp 621 – 623(Dec 1987)
8. J. Sharma, K.V. Venkateswaran, "A direct method for maximizing the system reliability", IEEE Trans Reliability, vol R-20, 1971 Nov, pp 256- 259.