Performance analysis of Cornish-Fisher corrected p chart and fuzzy based p chart

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Abstract—Control charts are one of the most powerful tools in statistical process control for measuring and improving products quality and productivity. **Now a days manufacturing process with very low rejection rate are regularly observed in practice. These type process are known as high quality process such situations normal p chart shows serious drawbacks. Traditionally, the study of the rate of nonconformities was carried out using the conventional 3-sigma p control chart (Shewhart), constructed by the normal approximation. But this p chart suffers a serious inaccuracy in the modelling process and in control limits specification when the true rate of nonconforming items is small. Silvia Joekes developed a Cornish-fisher corrected p chart for overcome these draw backs. In this paper fuzzy version of Cornish-Fisher p chart was developed and analyse the various performance measures of both the chart by the help of a connecting rod manufacturing process.**

Index Terms—**Cornish -Fisher quantile correction, Triangular α cut fuzzy control chart, performance analysis**

I. Introduction

Control charts are one of the most powerful tools in statistical process control for measuring and improving products quality and productivity. Now a days manufacturing process with very low rejection rate are regularly observed in practice. These type process are known as high quality process such situations normal p chart shows serious drawbacks. Traditionally, the study of the rate of nonconformities was carried out using the conventional 3-sigma p control chart (Shewhart), constructed by the normal approximation. But this p chart suffers a serious inaccuracy in the modelling process and in control limits specification when the true rate of nonconforming items is small.so [1] developed a Cornish-fisher corrected p chart for overcome these draw backs. Then a fuzzy version of Cornish fisher p chart was developed then analysis the various performance measures of both chart.

II. LITERATURER REVIEW

The rapid development of technology has led to processes improved to such an extent that many traditional control charts currently can show problems of performance or practical implementation. Moreover, as a result of technological developments and the application of modern statistical methods for monitoring and control critical attributes, many processes have been improved giving a very low non-conforming fraction. These processes are currently known as high quality processes. In such cases the traditional control charts shows lot of problems. Control charts can be improved in several ways. [1] shows that with simple adjustments to the control limits, performance of the charts can be improved. With simple adjustments to the control limits of the p-chart, achieving equal or even better improvement while still working on the original data scale, is feasible. When production processes reach high quality standards they are known as ‘‘high quality processes’’. In high quality processes the values of p are usually very small and the sample sizes are not large enough. This situation determines that conventional Shewhart p charts have serious drawbacks in detecting nonconforming products (excess of false alarm risk). The Cornish–Fisher expansion can directly determine adjustments on the control limits that improve probabilistic properties of p charts, in terms of putting false alarm risk under control. According to [1] developed a correction in p-chart based on the Cornish–Fisher quantile correction formula. Just including a new term, this modified p chart has some advantages especially in the sense that it allows monitoring lower values of p, as is the case of very high quality processes.

Fuzzy logic offers a systematic base to deal with situations, which are ambiguous or not well defined. [13] explains that the fuzzy control charts are inevitable to use when the statistical data in consideration are uncertain or vague; or available information about the process is incomplete or includes human subjectivity. The fuzzy set theory and fuzzy logics are playing very important role in Statistical Process control. In many industrial situations, we may come across situation where quality has to be defined using linguistic variables using subjective measures like rating on a scale. These are variables whose states are fuzzy numbers. [8] used fuzzy set theory as the basis for interpreting the representation of a grade degree of product conformance with a quality standard.[3] explained two approaches for constructing variable control chart based on linguistic data when the product quality is classified ‘perfect’, ‘good’, ‘poor’ etc. The representative fuzzy measures are obtained by using any of the four commonly used methods, namely, Fuzzy average, fuzzy mode, and fuzzy median and α- level fuzzy-midrange, to construct the control chart. The membership functions defined for the linguistic variables in the above method are chosen arbitrarily and hence decision for process control may change as per the user’s choice of values of decision parameter. [9] have developed Fuzzy Multinomial control chart for fixed Sample Size and [10] illustrated fuzzy multinomial control chart based on linguistic variable which is classified into more than two categories with variable sample size. And also develop a Triangular Fuzzy Multinomial control chart (TFM chart) with VSS for linguistics variables using fuzzy number with α –cut fuzzy midrange transform techniques is proposed. The proposed method is compared with regular p-chart and FM-chat with VSS.

III. METHODOLOGY

Step 1: Development of control chart using Cornish –fisher p chart
Step 2: Selection of an appropriate Fuzzyfication method.
Step 3: Development of new chart
Step 4: Numerical illustration
Step 5: Results and Discussions

Normal p-chart:

When the binomial distribution is adequately approximated by the normal distribution, control limits for monitoring the proportion of non-conforming units are easy to derive, namely, applying the Shewhart 3σ framework which calls for limits placed at $\mu_p \pm 3\sigma_{p.}$. If the true non-conforming proportion p (fraction defective) is known or is accurately estimated, then, the 3σ control chart model gives

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$CL = p$$

$$UCL = p - 3\sqrt{\frac{p(1-p)}{n}}$$

False alarm α risk: the evaluation of a p-chart performance is based on type I error. The type I error is the probability that p does not fall between the upper and the lower limits of the chart (when in fact the process is under control), called false alarm probability. This probability is denoted by α, and the reference value for α is the usual 0.0027 which is pre-fixed.

The p-chart with one adjustment

[1] proposed two types of control charts based on the Cornish–Fisher expansion, which is an improvement to the chart proposed by Winterbottom.

The control limits of the proposed chart with one adjustment are given below.

$UCL_1 = UCL + \frac{4}{3n}(1-2p)$

$LCL_1 = LCL + \frac{4}{3n}(1-2p)$

Where UCL_1 and LCL_1 are the improved limits with one correction term.

The p-chart with two adjustment

They explains about another extension by including new terms in an improved p chart. The control limits are,

$UCL_2 = UCL_1 - \frac{[p(1-p)+2]}{6n^2[\frac{p(1-p)}{n}]^{1/2}}$

$LCL_2 = LCL_1 - \frac{[p(1-p)+2]}{6n^2[\frac{p(1-p)}{n}]^{1/2}}$

Development of Fuzzy based control chart.

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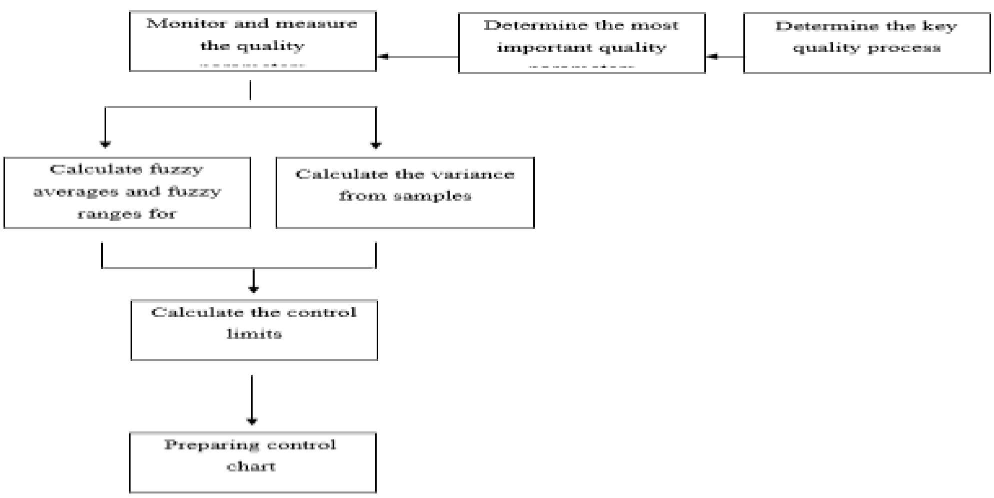

Figure 3.1: Flow chart indicating steps involved in development of fuzzy based control chart

Fuzzy numbers and fuzzy transformation methods

Based up on fuzzy set theory, a linguistics variable $\hat{L}$ which is classified by the set of k mutually exclusive members $\{l_1, l_2, l_{3,} \dots \dots . l_k\}$. We estimate the weight w_i to each term l_i and the fuzzy set is defined as

$\hat{L}$ = $\{(l_1, m_1)\,(l_2, m_2)\dots\dots\dots\dots\dots\dots(l_{k,} m_k)\}$

To monitor the out of control signal in the production process we are taking independent samples of different size and categorized as perfect, good, poor etc. form the $\{n_1, n_2, n_3 \dots \dots \dots n_s\}$

$\hat{L}$ is a linguistic variable which can categorize k mutually exclusive members $\{l_1, l_2, l_{3,} \dots \dots . l_k\}$. And each members are more skewed for each variable sample sizes. The weights of the membership degree are also assumed as 1, 0.75, 0.5, 0.25 and 0 in the Fuzzy Multinomial distribution control chart.

Fuzzy number construction

Step1: Let the observation for quality characteristics from samples of different sizes are assigned on a rank of 1 to k. a relative distance matrix

D = $[d_{ij}]_{k*k}$ where $d_{ij} = |R_1 - R_2|$ is evaluated

Step 2: The average of relative distance for each l_i is calculate by

$\bar{d_j} = \sum_{j=1}^{k} \frac{d_{ij}}{k-1}$ $\bar{d_i} = \sum_{i=1}^{k} \frac{d_{ij}}{k-1}$ This distance average is used to measure the centre of all the ranking for each quality characteristics.

Step3: Find a pair-wise comparison matrix P = $[P_{ij}]_{k*k}$ where $P_{ij} = \frac{\bar{d_j}}{\bar{d_i}}$

Step4: Evaluates weights by weight determination method of Saaty (1980) as

$w_j = \frac{1}{\sum_{i=1}^{k} P_{ij}}$; j= 1, 2,…… k where $\sum_{i=1}^{k} w_i = 1$

Step5: The importance of degree w_i represents the weight to be associated with l_i when estimating the mode of the fuzzy number. The fuzzy mode is given by $m = \sum_{i=1}^{k} p_i w_i$

Step6: Separate the sample quality characteristics l_i and find as fuzzy subset A and C, which is by obtained by m< l_i and m>l_i .The fuzzy subset A and C which is represented as follows A= $\{(f_1, w_1)\,(f_2, w_2)\dots\dots.\,(f_r, w_r)\}$ if m< l_i

C= $\{(m_1, w_1)\,(m_2, w_2)\,\dots\dots\,(m_t, w_t)\}$ if m> l_i and r + t = l_i

step7: Apply fuzzy multinomial distribution separately for the fuzzy subset A, M and C and find

$E[L_{IA}] = \sum_{i=1}^{r} p_i w_i$ $E[L_{iM}] = \sum_{i=1}^{k} p_i w_i$ $E\,[L_{iC}] = \sum_{i=1}^{s} p_i w_i$

Step8: Apply an α – cut to the fuzzy sets, the values are obtained as follows

$E[L_{iA}^{\propto}] = E[L_{iA}] + \alpha\,\{E[L_{iM}] - E[L_{iA}]\}$

$E[L_{iC}^{\propto}] = E[L_{iC}] - \alpha\,\{E[L_{iC}] - E[L_{iM}]\}$

Step9: Centre line of fuzzy based control chart = $\frac{(E[L_{iC}^{\propto}] + E[L_{iA}^{\propto}])}{2}$

Step10: The definition of α - level fuzzy midrange for Triangular FM control chart is defined

$S_{mr-j}^{\propto} = \frac{\{(E[L_{iA}]+E[L_{iC}])+\alpha\,[(E[L_{iM}]-E[L_{iA}])-(E[L_{iC}]-E[L_{iM}])]\}}{2}$

Step 11: Then the condition of process control for each sample can be defined by as

Process are {in control $LCL_{mr-p_i}^{\propto} \le S_{mr-j}^{\propto} \le UCL_{mr-pi}^{\propto}$

{out of control otherwise

Development of new chart for the high quality process

CL and *UCL*, *LCL* represented the centre line and control limits of fuzzy *P*-control charts respectively and they are triangular fuzzy sets; by applying α-cuts on fuzzy sets, the values of centre line are determined as follows

$\propto -cut$ Fuzzy based normal p chart

$$CL_{mr-p}^{\propto} = \frac{(E[L_{iC}^{\propto}] + E[L_{iA}^{\propto}])}{2}$$

And the two control lines will be,

$$LCL_{mr-p}^{\propto} = CL_{mr-p}^{\propto} - 3\sqrt{\frac{CL_{mr-p}^{\propto}(1 - CL_{mr-p}^{\propto})}{n}},$$

$$UCL_{mr-p}^{\propto} = CL_{mr-p}^{\propto} + 3\sqrt{\frac{CL_{mr-p}^{\propto}(1 - CL_{mr-p}^{\propto})}{n}}$$

$CL_{mr-p}^{\propto}$,$LCL_{mr-p}^{\propto}$, $UCL_{mr-p}^{\propto}$are the centre line, lower control limit and upper control limit of fuzzy based p chart

$\propto -cut$ Fuzzy Cornish-Fisher corrected p chart with one adjustment

The Fuzzy based Cornish Fisher P chart limits will be

$$UCL_{mr-p_1}^{\propto} = UCL_{mr-p}^{\propto} + \frac{4}{3n}(1 - 2CL_{mr-p}^{\propto})$$

$$LCL_{mr-p_1}^{\propto} = LCL_{mr-p}^{\propto} + \frac{4}{3n}(1 - 2CL_{mr-p}^{\propto})$$

$UCL_{mr-p_1}^{\propto}$ and $LCL_{mr-p_1}^{\propto}$ are upper and lower control limits of the proposed chart with one adjustment.

α-Cut fuzzy Cornish–Fisher corrected p-chart with two adjustment

$$UCL_{mr-p_2}^{\propto} = UCL_{mr-p_1}^{\propto} - \frac{[CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})+2]}{6n^2[\frac{CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})}{n}]^{1/2}}$$

$$LCL_{mr-p_2}^{\propto} = LCL_{mr-p_1}^{\propto} - \frac{[CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})+2]}{6n^2[\frac{CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})}{n}]^{1/2}}$$

$UCL_{mr-p_2}^{\propto}, LCL_{mr-p_2}^{\propto}$ are the upper and lower control limits of the proposed control chart with two adjustments

IV. NUMERICAL ILLUSTRATION

Data required for the numerical illustration was collected from connecting rod manufacturing process in SIFL, Athani Thrissur

No	sample size	rejected	poor quality	medium quality	good	excellent
1	30	0	6	4	20	0
2	25	1	2	1	20	1
3	30	0	1	3	24	2
4	30	0	1	2	22	5
5	25	0	1	2	21	1
6	25	3	1	1	20	0
7	25	0	1	2	19	3
8	30	0	4	4	20	2
9	25	1	2	1	20	1
10	30	0	1	3	24	2
11	25	3	1	1	20	0
12	30	0	1	2	22	5
13	30	0	1	2	24	3
14	25	0	2	0	21	2
15	25	1	2	1	20	1
16	25	4	0	1	19	1
17	30	0	4	4	20	2
18	25	1	0	4	20	0
19	30	0	4	1	21	4
20	25	0	1	2	19	3
21	25	2	1	1	20	1
22	25	0	1	2	21	1
23	30	0	4	6	20	0

Table 4.1: Grouping of items

Normal p chart

Centre line (p) = $\frac{16}{625}$ = .0256

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$= 0.0256 + 3\sqrt{\frac{0.0256(1-0.0256)}{27.17}}$$

= .11649

$$\text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

$$= 0.0256 - 3\sqrt{\frac{0.0256(1-0.02562)}{25}}$$

= -.06529

which is taken as zero

Cornish–Fisher p-chart with one adjustment

From equations (1) and (2),

$UCL_1 = UCL + \frac{4}{3n}(1-2p)$

= 0.11649 + $\frac{4}{3*27.17}(1-2*0.0256)$

= 0.16306

$LCL_1 = LCL + \frac{4}{3n}(1-2p)$

= -0.015+ $\frac{4}{3*27.17}(1-2*0.0256)$

= -0.0032

It is taken as zero.

Cornish–Fisher p-chart with two adjustment

From equations (3) and (4) the values for two adjustment was calculated as follows,

$UCL_2 = UCL_1 - \frac{[p(1-p)+2]}{6n^2[\frac{p(1-p)}{n}]^{1/2}}$

= 0.11795 – $\frac{[0.0256(1-0.0256)+2]}{6*27.17^2[\frac{0.0256(1-0.0256)}{27.17}]^{1/2}}$

= 0.14797

$LCL_2 = LCL_1 - \frac{[p(1-p)+2]}{6n^2[\frac{p(1-p)}{n}]^{1/2}}$

= 0.0032 – $\frac{[0.0256(1-0.0256)+2]}{6*27.17^2[\frac{0.0256(1-0.0256)}{27.17}]^{1/2}}$

= 0 which is taken as zero.

Fuzzy based p chart

The weights of the membership degree are assumed as 1, 0.75, 0.5, 0.25 and 0 for rejected, poor quality, medium quality, good and excellent quality respectively

$d_{ij} = |R_i - R_j|$

R_i= weightage × number of item

$d_{11} = |R_1 - R_1|$, $d_{12} = |R_1 - R_2|$, $d_{13} = |R_1 - R_3|$, $d_{14} = |R_1 - R_4|$, $d_{15} = |R_1 - R_5|$

For sample 1

$d_{11} = 0$, d_{12}= 4.5,$d_{13} = 2$,$d_{14} = 5$, $d_{15} = 0$

D = $[d_{ij}]_{k*k}$

D = $[d_{ij}]_{k*k}$

$$D = \begin{bmatrix} 0 & 4.5 & 2 & 5 & 0 \\ 4.5 & 0 & 2.5 & .5 & 4.5 \\ 2 & 2.5 & 0 & 3 & 2 \\ 5 & .5 & 3 & 0 & 5 \\ 0 & 4.5 & 2 & 5 & 0 \end{bmatrix}$$

P = $[P_{ij}]_{k*k}$ where $P_{ij} = \frac{d_j}{d_i}$

$$P = \begin{bmatrix} 1 & 1.043 & .8260 & 1.17 & 1 \\ .9583 & 1 & .7916 & 1.125 & .9583 \\ 1.210 & 1.263 & 1 & 1.4210 & 1.210. \\ .8514 & .888 & .785 & 1 & .8514 \\ 1 & 1.043 & .8260 & 1.17 & 1 \end{bmatrix}$$

$w_j = \frac{1}{\sum_{i=1}^{k} P_{ij}}$ $w_1 = \frac{1}{P_{11}+P_{21}+P_{31}+P_{41}+P_{51}}$,

w_1 = .19845 , w_2 = .2051, w_3 = .1638 , w_4 = .232644, w_5 = .19845

$E[L_{iA}] = \sum_{i=1}^{r} p_i w_i$ $E[L_{iM}] = \sum_{i=1}^{k} p_i w_i$ $E[L_{iC}] = \sum_{i=1}^{s} p_i w_i$

$E[L_{iA}^{\alpha}] = E[L_{iA}] + \alpha\{E[L_{iM}] - E[L_{iA}]\}$

$E[L_{iC}^{\alpha}] = E[L_{iC}] - \alpha\{E[L_{iC}] - E[L_{iM}]\}$

no	p1	p2	p3	p4	p5	w1	w2	w3	w4	w5	$E[L_{iA}^{\propto}]$	$E[L_{ic}^{\propto}]$
1	0	0.2	0.133	0.667	0	0.1985	0.2051	0.1638	0.23264	0.19845	0.02154857	0.02197219
2	0.04	0.08	0.04	0.8	0.04	0.1336	0.1475	0.1475	0.3863	0.1818	0.03347628	0.03439492
3	0	0.033	0.1	0.8	0.0667	0.1529	0.1388	0.1529	0.4027	0.1529	0.03476824	0.03568576
4	0	0.033	0.067	0.733	0.1667	0.151	0.1356	0.1405	0.4314	0.151	0.03510633	0.0359758
5	0	0.04	0.08	0.84	0.04	0.1522	0.1387	0.1434	0.4185	0.15219	0.03690912	0.03802
6	0.12	0.04	0.04	0.8	0	0.195	0.15	0.1554	0.315	0.185	0.02832925	0.02919395
7	0	0.04	0.08	0.76	0.12	0.1549	0.1369	0.143	0.41128	0.1549	0.03433117	0.03528443
8	0	0.133	0.133	0.667	0.0667	0.1786	0.2322	0.1428	0.262802	0.17587	0.02338615	0.02431214
9	0.04	0.08	0.04	0.8	0.04	0.1356	0.1475	0.1475	0.3863	0.1818	0.03355589	0.03433131
10	0	0.033	0.1	0.8	0.0667	0.1529	0.1388	0.1529	0.4027	0.1529	0.03464059	0.03640143
11	0.12	0.04	0.04	0.8	0	0.195	0.15	0.1554	0.315	0.185	0.02832565	0.02919755
12	0	0.033	0.067	0.733	0.1667	0.151	0.1356	0.1405	0.4314	0.151	0.03484203	0.0362401
13	0	0.033	0.067	0.8	0.1	0.1491	0.135	0.1395	0.428	0.1491	0.03664764	0.03757409
14	0	0.08	0	0.84	0.08	0.1522	0.1387	0.1435	0.4184	0.15219	0.03699495	0.03794969
15	0.04	0.08	0.04	0.8	0.04	0.1336	0.1475	0.1475	0.3863	0.1818	0.03343263	0.03443857
16	0.16	0	0.04	0.76	0.04	0.2266	0.1714	0.162	0.2688	0.1714	0.02505356	0.02572244
17	0	0.133	0.133	0.667	0.0667	0.1786	0.2322	0.1428	0.262802	0.17857	0.02338561	0.02403668
18	0.04	0	0.16	0.8	0	0.1461	0.1666	0.1666	0.35439	0.1666	0.03109656	0.03210584
19	0	0.133	0.033	0.7	0.1333	0.162	0.1994	0.1528	0.324	0.162	0.02758623	0.02842971
20	0	0.04	0.08	0.76	0.12	0.1549	0.1369	0.143	0.41128	0.1549	0.03440467	0.03521093
21	0.08	0.04	0.04	0.8	0.04	0.1452	0.1396	0.134	0.43612	0.14519	0.03721369	0.03823887
22	0	0.04	0.08	0.84	0.04	0.1522	0.1387	0.1434	0.4185	0.15219	0.03212035	0.03318802
23	0	0.133	0.2	0.667	0	0.1985	0.1638	0.2051	0.23264	0.19845	0.03196544	0.03247156

Table 4.2 : Fuzzy number construction

Fuzzy based p chart

$$CL_{mr-p}^{\propto} = \frac{(E[L_{iC}^{\propto}] + E[L_{iA}^{\propto}])}{2}$$

$$= \frac{(0.03170+0.03262)}{2}$$

= 0.03216

$$LCL_{mr-p}^{\propto} = CL_{mr-p}^{\propto} - 3\sqrt{\frac{CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})}{n}}$$

$$= 0.03216 - 3\sqrt{\frac{0.03216(1-0.03216)}{27.17}}$$

= -0.0694 which is taken as 0

$$UCL_{mr-p}^{\propto} = CL_{mr-p}^{\propto} + 3\sqrt{\frac{CL_{mr-p}^{\propto}(1-CL_{mr-p}^{\propto})}{n}}$$

$$= 0.03216 + 3\sqrt{\frac{0.03216(1-0.03216)}{27.17}}$$

= 0.13369

Fuzzy based Cornish–Fisher corrected p-chart with one adjustment

For fuzzy based corner fisher p chart, the values are substituted as shown**.** The control limits are:

$$UCL_{mr-p_1}^{\propto} = UCL_{mr-p}^{\propto} + \frac{4}{3n}(1 - 2CL_{mr-p}^{\propto})$$

$$= 0.13369 + \frac{4}{3*27.17}(1 - 2*0.03216)$$

= 0.1796131

$$LCL_{mr-p_1}^{\propto} = LCL_{mr-p}^{\propto} + \frac{4}{3n}(1 - 2CL_{mr-p}^{\propto})$$

$$= -0.0694 + \frac{4}{3*27.17}(1 - 2*0.03216)$$

= -0.02347, which is taken as zero.

Fuzzy based Cornish–Fisher corrected p-chart with two adjustment

For fuzzy based corner fisher p chart for two adjustment we substitute the values as shown**.** The control limits are:

$$UCL^{\propto}_{mr-p_2} = UCL^{\propto}_{mr-p_1} - \frac{[CL^{\propto}_{mr-p}(1 - CL^{\propto}_{mr-p}) + 2]}{6n^2[\frac{CL^{\propto}_{mr-p}(1 - CL^{\propto}_{mr-p})}{n}]^{1/2}}$$

$$= 0.1796131 - \frac{[0.03216(1-0.03216)+2}{6\times 27.17^2[\frac{0.03216(1-0..03216)}{25}]^{0.5}}$$

$$= 0.166064$$

$$LCL^{\propto}_{mr-p_2} = LCL^{\propto}_{mr-p_1} - \frac{[CL^{\propto}_{mr-p}(1 - CL^{\propto}_{mr-p}) + 2]}{6n^2[\frac{CL^{\propto}_{mr-p}(1 - CL^{\propto}_{mr-p})}{n}]^{1/2}}$$

$$= -0.02347 - \frac{[0.03216(1-0.03216)+2}{6\times 27.17^2[\frac{0..03216(1-0.03216)}{27.17}]^{0.5}}$$

$$= 0$$

The process condition was set to evaluate the process with an α-cut based on the α-level fuzzy midrange transformation technique for the $S^{\alpha}_{mr,j}$

$$S^{\propto}_{mr-j} = \frac{\{(E[L_{iA}]+E[L_{iC}])+\alpha\ [(E[L_{iM}]-E[L_{iA}])-(E[L_{iC}]-E[L_{iM}])]\}}{2}$$

sample number	$S^{\propto}_{mr-j}$	with one adjustment	with two adjustment
1	0.021725429	in control	in control
2	0.0339356	in control	in control
3	0.035227	in control	in control
4	0.035541067	in control	in control
5	0.03746456	in control	in control
6	0.0287616	in control	in control
7	0.0348078	in control	in control
8	0.018805144	in control	in control
9	0.0339436	in control	in control
10	0.026014765	in control	in control
11	0.0287616	in control	in control
12	0.035541067	in control	in control
13	0.037110867	in control	in control
14	0.03747232	in control	in control
15	0.0339356	in control	in control
16	0.025388	in control	in control
17	0.023711147	in control	in control
18	0.0316012	in control	in control
19	0.028007967	in control	in control
20	0.0348078	in control	in control
21	0.03772628	in control	in control
22	0.027843813	in control	in control
23	0.04264166	in control	in control

Table 4.3: $Smr{-}j \propto$ value for one and two adjustment

V. RESULTS AND DISCUSSION

Normal p chart with one and two adjustment

The values for each P chart, Cornish fisher graph is found out for the data collected from SIFL and the graph is drawn accordingly. The graph contains the p value for each sample size, centre value, UCL for each p chart, Cornish fisher graph. The LCL for the graph is not shown as all the LCL corresponds to zero.

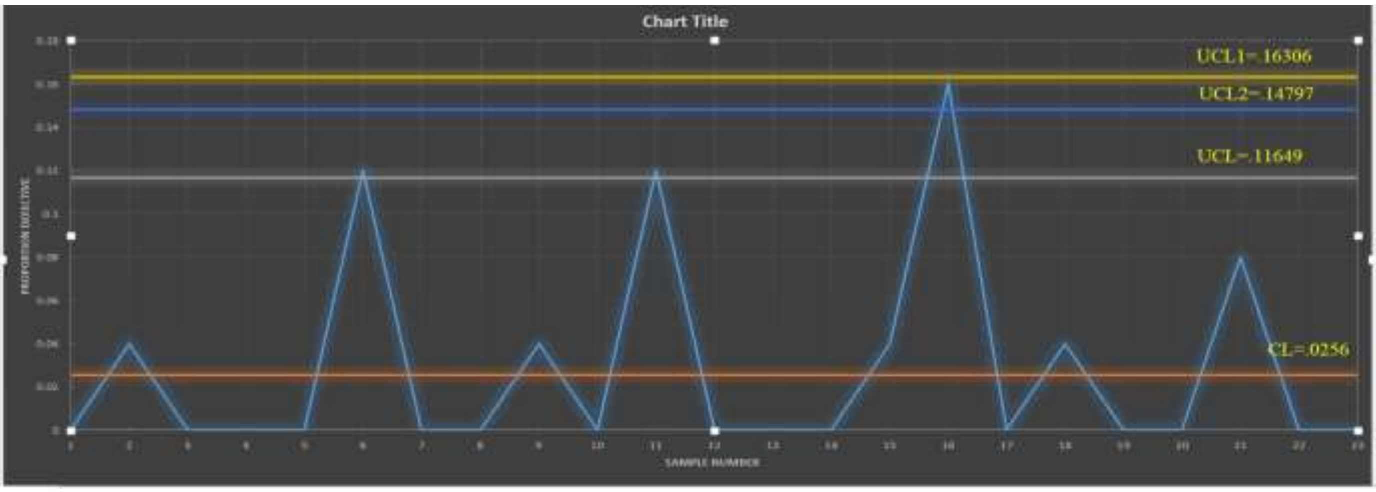

Fig 5.1: normal p chart with one and two adjustment

From the graph 5.1 it is seen that in a normal P chart 3 values are outside the control limits. The sample number that are outside the control limits are 6, 11 and 16. These values are out of control as shown in the normal P chart. But in the Cornish Fisher chart with one adjustment p values of all the samples are inside the control limits as these samples are in control in a Cornish fisher chart. Thus the Cornish Fisher is better than normal chart. The point of rejection becomes less in a Cornish fisher graph so these graphs can decrease rejection level in a processes.

Table 5.1 shows the α level risk values of the 3 charts. The α risk denotes the type 1 error of a control chart. As small the type 1 error less deviation of control level. Average Running Length (ARL) is the reciprocal of the α level denotes the possibility of rejecting a good lot. The α level risk thus denotes the efficiency of a control chart.

Type of Chart	UCL	nUCL	α- level risk
Normal P Chart	0.11649	3.165	0.03714
Cornish Fisher One Adjustment Chart	0.16303	4.4295	0.0018
Cornish Fisher Two Adjustment Chart	0.14797	4.067	0.00873

Table 5.1 : α level risk of normal p chart with Cornish – fisher adjustment

The table 5.1 shows that α risk is less for one adjustment Cornish fisher chart indicates that the error existing tendency of the chart is less than a normal P chart.The one adjustment Cornish fisher P chart has α risk of 0.0018 so less probability of false alarm so more controlled so it can be applied in high quality processes.

Fuzzy based p chart with Cornish-Fisher adjustment

The values for each fuzzy based P chart, fuzzy Cornish fisher graph is found out. And the graph is drawn accordingly. The graph 5.2 contains the p value for each sample size, centre value, UCL for each P chart, Cornish fisher graph. The LCL for the graph is not shown as all the LCL corresponds to zero.

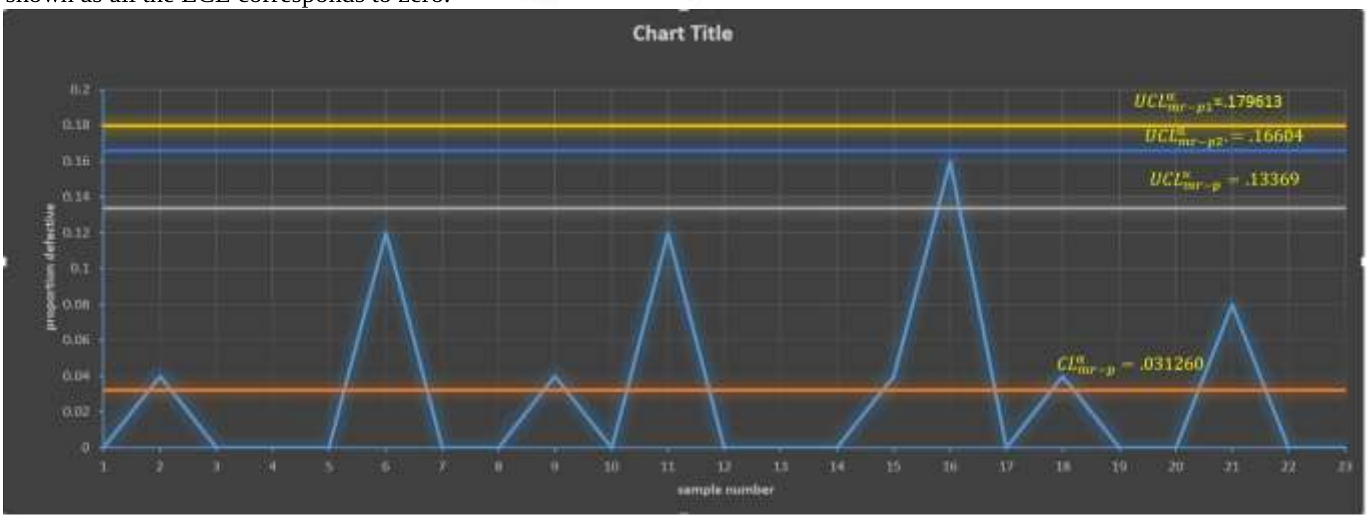

Figure 5.2 Fuzzy p chart and Cornish Fisher adjustment

From the graph 5.2 it is seen that in fuzzy based p chart sample number 16 is outside the control limit. Cornish fisher chart with one and two adjustment all points are inside the control limit. The control limit for UCL for Cornish fisher chart with one adjustment is 0.179613.The control limit for UCL for Cornish fisher with two adjustment is 0.1664.

Type of Chart	UCL	nUCL	α- level risk
Fuzzy p Chart	0.13369	3.08895	0.00747
Fuzzy Cornish Fisher One Adjustment Chart	0.17613	4.78545	0.000472
Fuzzy Cornish Fisher Two Adjustment Chart	0..16604	4.51130	0.001256

Table 5.2: α level risk of Fuzzy p chart and Cornish Fisher p chart

The table 5.2 shows that α risk is less for fuzzy one adjustment Cornish fisher chart indicates that the error existing tendency of the chart is less than a fuzzy p chart. The α risk of Cornish fisher with two adjustment is in between normal p chart and chart with two adjustment. The one adjustment Cornish fisher p chart has α risk of 0.000472 so less probability of false alarm so more controlled so can be applied to high quality processes.

When all the fuzzy and normal charts are compared the fuzzy charts has a less α risk compared to normal charts. The fuzzy charts are thus have less false alarm producing probability than normal charts. Thus a fuzzy charts can be used for high quality processes and can be more error free.

VI. CONCLUSION

When production processes reach high quality standards they are known as high quality processes. In high quality processes the values of fraction defectives (p) are usually very small and the sample sizes are not large enough. This situation conventional Shewhart p charts have serious drawbacks in detecting non- conforming products (excess of false alarm risk).

In this study, a fuzzy based p – chart based on the Cornish–Fisher quantile correction formula was presented. This modified p chart has some advantages especially in the sense that it allows monitoring lower values of p, as is the case of high quality processes and also used in the case of linguistics, uncertain data

From the comparative studies with traditional charts, it is clear that the fuzzy charts have less false alarm producing probability than normal charts. Thus a fuzzy charts can be used to monitor high quality processes and can be more error free. Also fuzzy based control chart have more efficiency than conventional p chart

However, the results are for a p value more than 0.01. It may differ when the value of p is less than 0.01. So there is a future scope for the work in the cases where p value is less (less than 0.01).

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