Cubic Harmonious Graphs

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***Abstract* - A graph *G* with *m* edges is said to be harmonious, if there is an injection *f* from the vertices of G to the group of integers modulo *q* such that when each edge *uv* is assigned the label *f(u)+f(v)(mod m)*,the resulting edge labels are distinct. In this paper we introduced a new harmonious labeling called Cubic Harmonious Labeling (CHL). A graph which admits cubic harmonious labeling is called Cubic Harmonious Graphs (CHG)).Here we prove that path graph, Bistar graph are cubic harmonious.**

***Index Terms*- Bistar, Cubic harmonious graph, Cubic harmonious labeling, Harmonious graph, Path graph.**

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1. Introduction

Throughout this paper we consider simple, finite, connected and undirected graph $G=(V(G),E(G))$ with n vertices and m edges. G is also called a (n,m) graph. For standard and terminology and notation we follow Graham and Sloane [4]. Graham and Sloane[4] defined a (n,m)- graph G of order n and size m to be harmonious, if there is an injective function $f: V(G) \rightarrow Z_m$,where Z_m is the group of integers modulo m, such that the induced function $f^*: E(G) \rightarrow Z_q$, defined by $f^*(uv) = f(u) + f(v)$ for each edge $uv \in E(G)$ is a bijection. Square harmonious graphs were introduced in [1].

Definition 1.1

The path on n vertices is denoted by P_n.

Definition1.2

A complete bipartite graph $K_{1,n}$ is called a star and it has (n+1) vertices and n edges

Definition1.3

The bistar graph $B_{m,n}$ is the graph obtained from a copy of a star $K_{1,m}$ and a copy of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.

II. MAIN RESULTS

CUBIC HARMONIOUS GRAPHS.

Definition 2.1

A (n,m) graph $G=(V,E)$ is said to be **Cubic Harmonious Graph(CHG)** if there exists an injective function $f: V(G) \rightarrow \{1,2,3,\dots\dots,m^3+1\}$ such that the induced mapping $f^*_{chg}: E(G) \rightarrow \{1^3,2^3,3^3,\dots\dots,m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. The vertex labels are distinct and edge labels are also distinct as well as cubic. The function f is called a **Cubic Harmonious Labeling (CHL)** of G.

Theorem 2.1.

Every path P_n is a cubic harmonious graph for all $n \geq 3$.

Proof:

Let P_n be a path graph with n vertices and m edges.

Let $V(P_n) = v_r$; $1 \leq r \leq n$

and

$$E(P_n) = \{ v_r v_{r+1} ; \quad 1 \leq r \leq n-1\}$$

Define an injection $f: V(P_n) \rightarrow \{1, 2, 3, \dots\dots, m^3+1\}$ by

$f(v_1) = m^3$

$f(v_2) = m^3 + 1$

$f(v_3) = (m-1)^3$

$f(v_r) = (m+2-r)^3 + (m^3+1) - f(v_{r-1})$; $4 \leq r \leq n$.

The induced edge mapping are

$f^*(v_r v_{r+1}) = (m+1-r)^3$; $1 \leq r \leq n$

The vertex labels are in the set $\{1,2,\dots\dots,m^3+1\}$.Then the edge label arranged in the set $\{1^3,2^3,\dots\dots,m^3\}$.So the vertex labels are distinct and edge labels are also distinct and cubic. So every path graph is cubic harmonious for all n ≥ 3.

Example 1

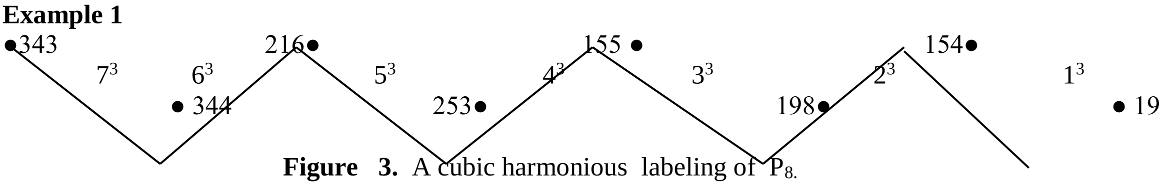

Figure 3. A cubic harmonious labeling of P_8.

Theorem 2. 2
Every bistar $B_{m,n}$ is a cubic harmonious graph.
Proof:
Let G be the bistar graph $B_{m,n}$ with $(m+n+2)$ vertices and $(m+n+1)$ edges.
Let $V(B_{m,n}) = \{u_r, v_s ;$ $1 \leq r \leq m+1, 1 \leq s \leq n+1 \}$
And $E(B_{m,n}) = \{u_r u_{m+1}, v_s v_{n+1}, u_{m+1} v_{n+1};$ $1 \leq r \leq m, 1 \leq s \leq n \}$
Define an injection $f : V(B_{m,n}) \rightarrow \{1, 2, 3, ..., [(m+n+1)^3 +1]\}$ by
$f(u_r) = (m+n+1-r)^3;$ $1 \leq r \leq m$
$f(u_{m+1}) = (m+n+1)^3 + 1;$
$f(v_{n+1}) = (m+n+1)^3;$
$f(v_s) = (n+1-s)^3 + 1;$ $1 \leq s \leq m$
The induced edge labels are
$f^*(u_r u_{m+1}) = (m+n+1-r)^3 ;$
$f^*(u_{m+1} v_{n+1}) = (m+n+1)^3 ;$
$f^*(v_s v_{n+1}) = (n+1-s)^3 ;$
In all the three cases, ie $m > n, m < n, m = n$, f induces a bijection $f^*: E(G) \rightarrow \{1^3, 2^3, 3^3, ..., (m+n+2)^3\}$.
The vertex labels are in the set $\{1,2, \dots \dots \dots, ((m+n+1)^3 + 1)\}$. Then the edge labels are arranged in the set $\{1^3, 2^3, 3^3, \dots., (m+n+1)^3\}$ So the vertex labels are distinct and the edge labels are also cubic and distinct . So the graph bistar $B_{m,n}$ is a cubic harmonious.

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