Effect of hub radius on natural frequency of a pre-twisted composite beam

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Abstract: **In this paper, twisted, uniform, rotating cantilever beam was analysed numerically using finite element method with ANSYS^{TM} software. These kind of beam components are more prevalently seen in turbomachinery blades and helicopter rotor blades. Composites give an added advantage of being light weight structure and more beneficial mechanical and physical properties. Modal frequencies of isotropic and composite materials with the present model have been compared for model verification. The effect of hub radius on pre-twisted blade has been studied.**

Keywords: **Twisted composite beams, Finite element method.**

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1. Introduction:

In the recent period there has been tremendous advancement in material science and engineering of composite materials. The low density, high strength, stiffness to weight ratio, excellent durability and design flexibility of composites are the primary reasons for their choice to be used in many structural components especially in aircrafts, automotive, marine, turbomachinery are few to say.

Engineers expect that novel materials and their combinations should exhibit higher performance and efficiency, which could be better than available traditional materials. The advent and usage of developed composite materials appears to be an appropriate solutions for this problem. However, usage of composite materials in manufacturing of beam like structures is important and a practical problem; wherein, utilizing these materials for applications like turbomachinery blades, helicopter blades, construction of sensors/ actuator are possible and feasible to develop in beam like structures.

Cantilever beams involves to form an important class of problems. A system with one end is rigidly fixed to a support and the other end is set free to move is said to be a cantilever beam. Problems with turbomachinery blades, helicopter blades falls on to special cases of cantilever beams with complex nature are an interest of study for many researchers. The construction of turbomachinery blades is generally acceptable as cantilever beam model [5]. In aircraft applications, cantilever is set of fixed-wing design. Turbine blades, helicopter blades, yokes, satellite antennas and robot arms falls on to the vital applications of the rotating cantilever beams.

In cantilever bridges the cantilevers are usually built as pairs, with each cantilever used to support one end of a central section. Another use of the cantilever is in fixed-wing aircraft design; cantilevered beams are the most ubiquitous structures in the field of micro electro mechanical systems (MEMS).A cantilever rack is a type of warehouse storage system. Turbine blades, helicopter blades, yokes, satellite antennas and robot arms these are the applications of rotating cantilever beams. Rotating cantilever beam is an important physical problem with complex parameters.

Pre-twisted rotating beams are also treated as cantilever beams to design and analyse the effects of different characteristics of the beam in action [4]. Pre-twisted rotating beams are widely been used in engineering structural components such as turbine blades, helicopter blades, wind turbine blades, aircraft rotary wings, and so forth. Evaluation of their modal characteristics is a critical problem, to the design and analysis of such rotating structures using composite materials. Due to the influence of the centrifugal inertial force the modal characteristics of the rotating beam vary significantly when compared to non-rotating beams.

The modal characteristics of rotating beams often vary significantly from those of non-rotating beams due to the influence of centrifugal inertia force. Due to this significant variation of modal characteristics resulted from rotation, the modal characteristics of rotating beams have been widely investigated.

R.C.DiPrima and G.H. Handelman (1954), presented a clean and detailed study on vibrations of twisted beams and gave a definition to twisted beams in vector form. They assumed a uniform beam of constant twist, where the minimum principle characterizes the lowest Eigen value.

Mohamad.S.Qatu et.al. (1991), studied laminated composite twisted cantilever plates where they observed a 3-D solution to clamped edge conditions provided better results than experimental one using Ritz method applying strain energy & kinetic energy functions foe laminated twisted plates. Maximum functional frequencies had been observed when the fibres are in perpendicular to the clamped edge. Advantage of Ritz method reasonably eadds to accurate results with less DOF compared to FEM but its lengthy and costly which not always required.

S.S.Rao and R.S.Gupta (2001), studied the vibration analysis of rotating Timoshenko beams using finite element method. They suggested that effect of twist of beam is significant than rotation which is smooth to effect the natural frequency. They studied shear deformation which indeed reduces the values of higher natural frequencies of vibration problem of the beam. When taper is considered, breadth to depth ratio has a significant effect on the natural frequency. So for designing a vibration less beam, tapered considerations become crucial.

E.Ghafari & J.Rezaeepazhand (2016), proposed a new approach called dimensional reduction method for vibration of rotating composite beams, based on polynomials and Rayleigh-Ritz method. Simple polynomial series were used to derive the essential cross sectional warpings. Natural frequencies of rotating and non-rotating beams have been obtained for isotropic, laminated composite beams and thin-walled composite box beams.

Varadaraja.D.N et.al. (2009), used higher shear deformation theory (HSDT) to study the vibration characteristics of rotating pre-twisted thin-walled beams with embedded macro fibre composites applied to actuators and sensor applications. Taper in a beam enhances the first two natural frequencies of the system. Hamilton’s principle had been used to derive linear equation of motion in axial, bending & rotational motions including gyroscopic coupling and centrifugal stiffness.

L.W.Chen et.al. (1993), studied the general orthotropic characteristics of pre-twisted rotating beams. Vibration of coupled-bending-bending tortion of a pre-twisted rotating cantilever beam was modelled with fibre reinforced material for dynamic stability, free vibration, and rotary inertia with warping effects. Finite element method was used to study the effect of fibre orientation, twist angle and rotation of the beam and compared the results of both isometric and orthotropic materials.

J.R.Benerjee (2001), used dynamic stiffness method to study the free vibration of a twisted beam whose flexural displacements are coupled in two planes. Twisted beams in general are considered with cantilever condition, where boundary conditions for bending are imposed in both the planes. Beams has been considered as per Bernoulli-Euler beam theory for which results are found to show significant effect on mode shapes of twisted blade.

Metin Aydogdu et.al. [11] Investigated Flapwise vibration of rotating composite beams based on different beam theories such as Euler–Bernoulli, Timoshenko and Reddy beam theories. The Ritz method with the algebraic polynomials is used in the formulation of the problem. The effect of the displacement field has been investigated using different theories. Non dimensional frequency parameters are obtained for different orthotropic ratios, rotation speed, hub ratio, length to thickness ratio of the rotating composite beam and different boundary conditions.

B. P. Pate1et.al. [12] Investigated Free Vibrations Analysis of Laminated Composite Rotating Beam using C' Shear Flexible Element .The governing equations for the free vibration of rotating beam are derived using Lagrange's equation of motion. The element employed is based on shear flexible theory. It also includes in-plane and rotary inertia terms. The formulation takes care of continuity conditions for stresses and displacements at the interfaces between the layers of a laminated beam. The results obtained indicate the significance of hub radius, slenderness ratio and modular ratio on the values of natural frequencies.

Jun Li et.al. [13] Presented Comparison of various shear deformation theories for free vibration of laminated composite beams with general lay-ups. All the laminated composite beams are discretized using only one element. A unified type of assumed displacement field is considered and the influences of shear deformation, rotary inertia and Poisson effect are included in the formulation. Numerical results obtained using the ESDBT, HSDBT and TSDBT and their comparison with the solutions evaluated by the FSDBT and TOSDBT and the experimental results reveal that the higher-order shear deformation beam models and the spectral finite element formulation introduced in this study can provide accurate and reliable results.

Priyanka Dhurvey et.al. [14] Review on various studies of composite laminates with ply drop-off. Static and dynamics analysis, buckling analysis, vibration analysis, delamination and interlaminar stress analysis of laminated composite plates and beams, tapered laminated structure with ply drop-off. The ply-drop zones contain high interlaminar stress gradients and hence potential sites for the onset and the subsequent growth of delamination. The size of resin pocket is shown to be crucial parameter affecting the nature of stress distribution. Dropping of 0ff-axis piles leads to higher failure factors due to the presence of significant in-plane shear stresses, whereas the dropping of 900 piles has negligible effect on the stress distribution.

Jagadish Babu Gunda et.al. [15] A numerically efficient super element is proposed as a low degree of freedom model for dynamic analysis of rotating tapered beams. The element uses a combination of polynomials and trigonometric functions as shape functions in what is also called the Fourier- p approach. The super element also allows an easy incorporation of polynomial variations of mass and stiffness properties typically used to model helicopter and wind turbine blades. Comparable results are obtained using one super element with only 14 degrees of freedom compared to 50 conventional finite elements with cubic shape functions with a total of 100 degrees of freedom for a rotating cantilever beam. Excellent agreement is also shown with results from the published literature for uniform and tapered beams with cantilever and hinged boundary conditions. The element developed in this work can be used to model rotating beam substructures as a part of complete finite element model of helicopters and wind turbines.

The present work deals with the free vibration analysis of rotating twisted composite beam. The features of rotating beam models differ significantly from non-rotating beams. Due to rotational motion of a structural element, centrifugal inertia force exerts certain variation in the bending stiffness, which shows significance in natural frequencies and mode shapes. However, the stiffness properties of composite material differs with isotropic materials. The number of plies, fibre orientation, stacking sequence are the key parameters which can be used to modulate the stiffness properties of composite materials. Twist along the length of the beam can create a physical model of cantilever beam to be applicable in a field of interest where rotating beams are a subject of investigation. Comparison of both isotropic and orthotropic natural frequencies have been presented. Hence the objective of the present work is to conduct vibration analysis of rotating composite twisted beam. Analysis is carried out using finite element analysis based software ANSYS. In the present work the effect of hub radius ratio on natural frequency of rotating composite twisted beam has been investigated.

Finite Element modelling

A 3 layered cantilever twisted beam has been designed by using Solid Works later the beam so forth modelled has been imported to ANSYS. The element used here for analysis of the beam is a two node beam element with four degree of freedom on each node. This beam element is analysed in ANSYS and layup has been modelled by using its ACP module. The beam here is a composite beam modelled with IM7_5250-4RTM a carbon fibre composite material. The element used here is shell 281 to mesh the model.

Rotational speed could be applied to the beam in the vertical direction and frequencies are analysed at stationary condition (i.e., zero rps). The beam so forth modelled has been analysed and compared with the beam of isotropic in nature. Now for the same beam frequencies are analysed under different hub radius at rest for free vibration analysis. Here the ply angles with stacking sequence of (+45/-45/+45) have been used for the present study. The variation of frequency has more change in higher modes with change in hub radius. Graphs are plotted for frequency vs. hub radius.

Composite beam acting as a cantilever beam hanged over a hub was fixed at the hub side and set free on the other side. The first case was studied without any twist in the beam and validated the same with the beam of isotropic nature where the results are in good agreement. The model of this case was shown in figure-1 and the results of the same have been presented in the table-1.

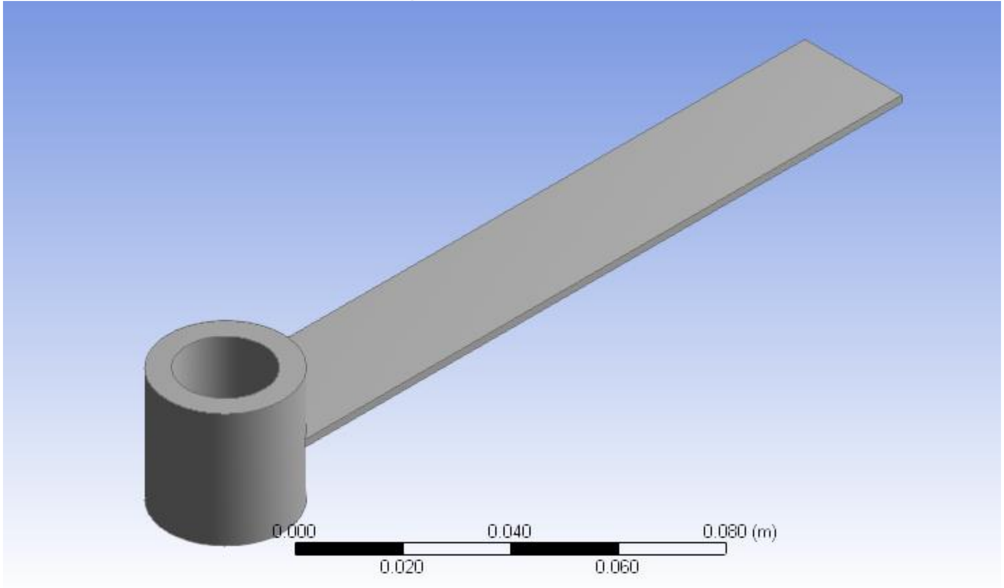


Figure-1: Composite beam without any twist

The second case of this problem was studied with the twist angle of 45^0 to the beam which forges from route of the hub to the end of the beam. The same was compared with the beam of isotropic in nature and results were tabulated in table-2 with model shown in figure-2.

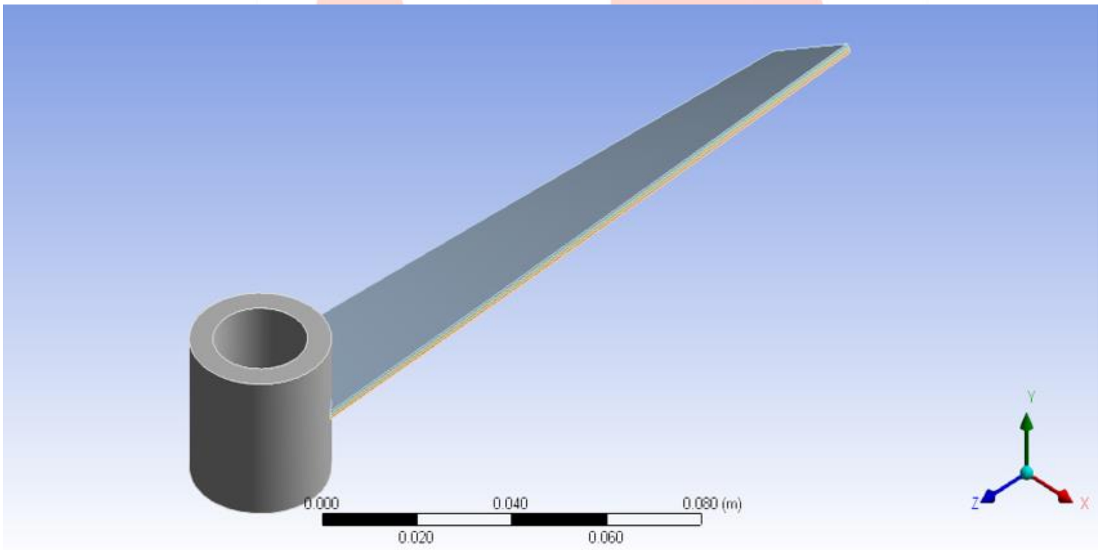


Figure-2: Composite beam with twist angle 45^0

The effect of hub radius was analysed in ANSYS which uses finite element technique, a free vibrational analysis (at 0rps) was studied. The natural frequencies are obtained for various hub radius from 25.4mm to 50mm at 0rps rotational speed. The variation of natural frequencies with respect to hub radius ratio was plotted in figure-3, and figure-4 for speed 0 rps respectively.

Table-1 Comparison of non-rotating beam at zero twist of isotropic and composite material.

Mode No.	Isotropic	Composite material	Error %
1	63.64	63.42	0.34569453
2	398.54	396.52	0.50685
3	744.15	735.65	1.14224283

4	860.88	859.36	0.17656352
5	1117.9	1109.1	0.78719027
6	2198.4	2171.7	1.21451965

Table-2 Comparison of non-rotating beam at 45^0 twist of isotropic and composite material.

Mode No.	Isotropic	Composite material	Error %
1	63.93	63.691	0.373846
2	323.35	321.67	0.519561
3	787.37	779.82	0.958888
4	954.21	949.13	0.532378
5	1170.1	1164.7	0.461499
6	2153.9	2127.3	1.234969

From these figures it is observed that the frequency increases with increasing hub radius. This increase is more prominent especially for higher hub radius i.e. the hub radius affects the higher frequencies at stationary condition. As the centrifugal force is very less in lower speeds irrespective of hub radius which affects higher frequencies.

From figures 3, 4; as hub radius increases its effect is prevalent and influential over higher frequencies. Significant effect of hub radius can be observed over natural frequencies. In figure 3, first three natural frequencies have been observed and found that as hub radius increases the frequency of the twisted cantilever beam increases incrementally. The same can be observed in figure 4, which shows that the variation of first six natural frequencies have an incremental increase with the increase of the hub radius.

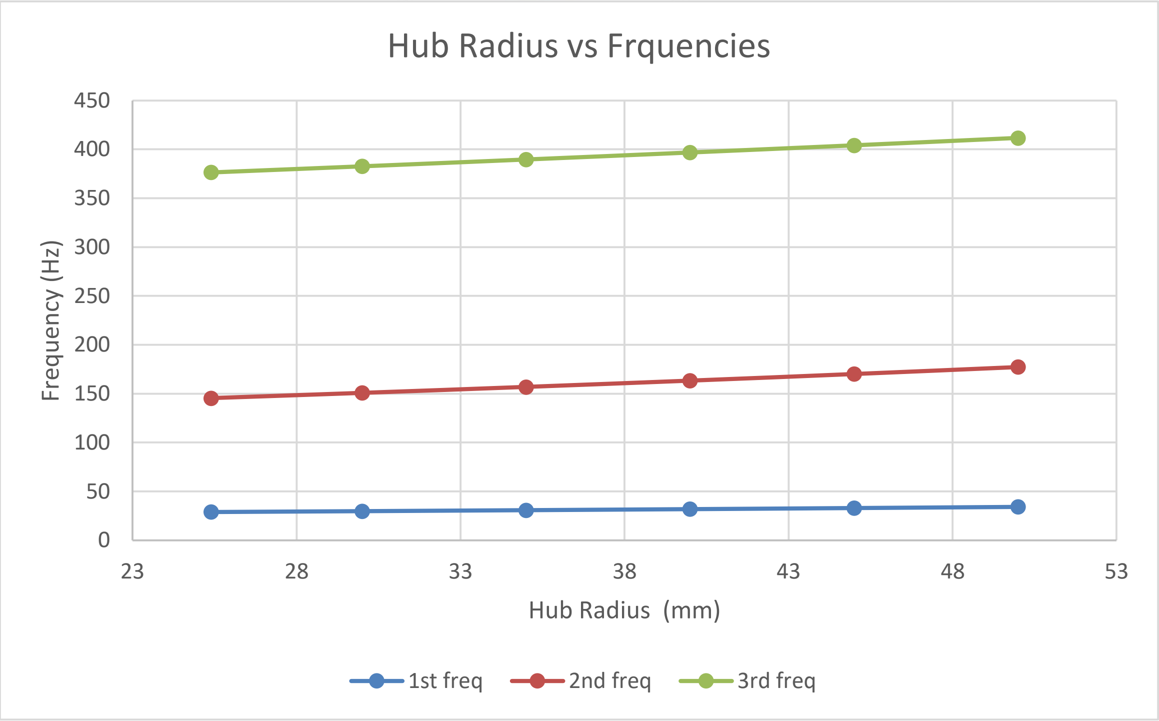
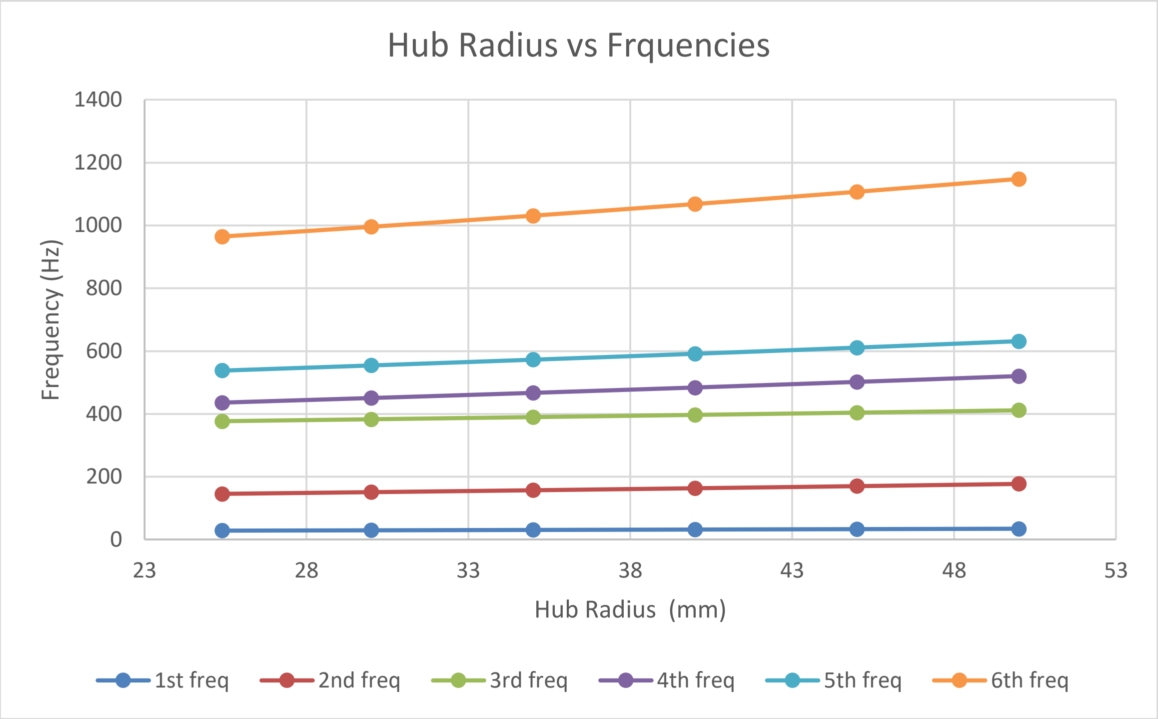


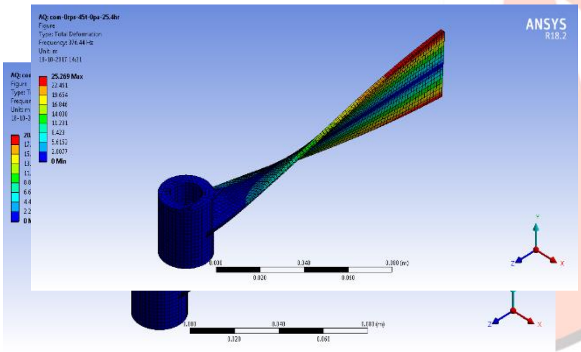
Figure-3: Variation of first three natural frequencies with hub radius

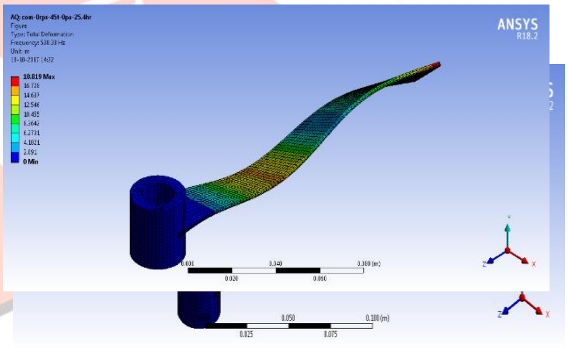
Figure-4: Variation of first six natural frequencies with hub radius

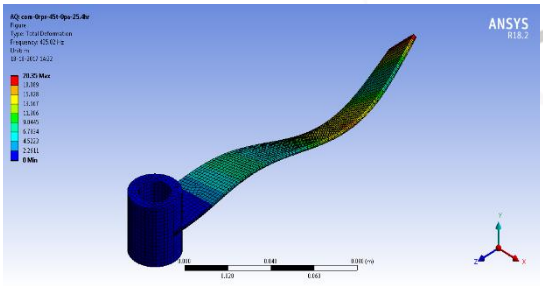


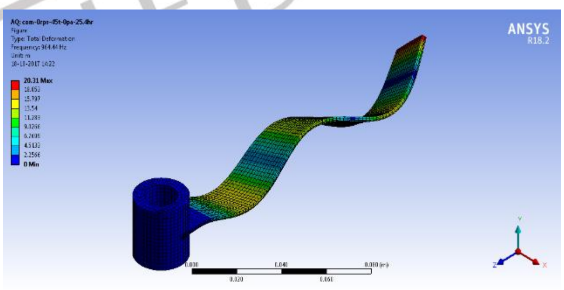
The mode shapes of these six cases with effect of change in hub radius have been presented case-wise from 25.4mm to 50mm.

I. Mode shapes for hub radius 25.4mm.

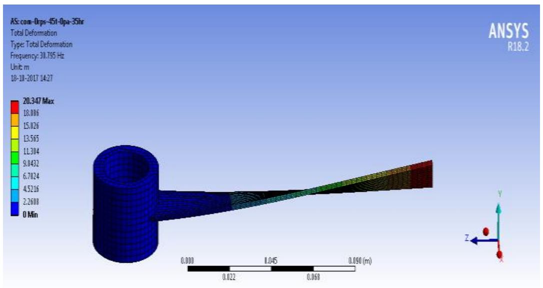


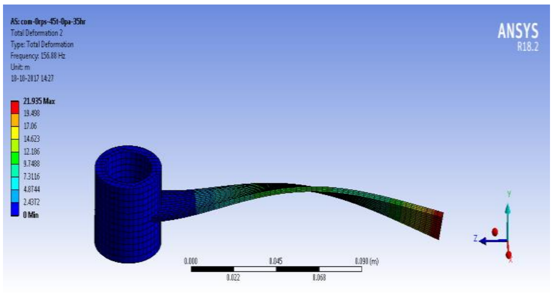


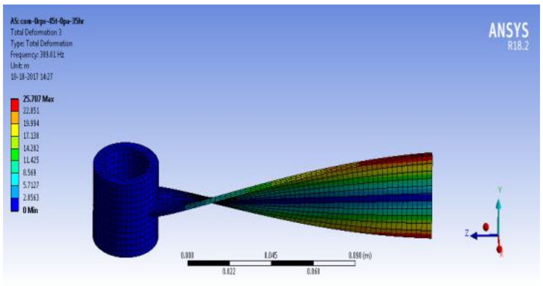


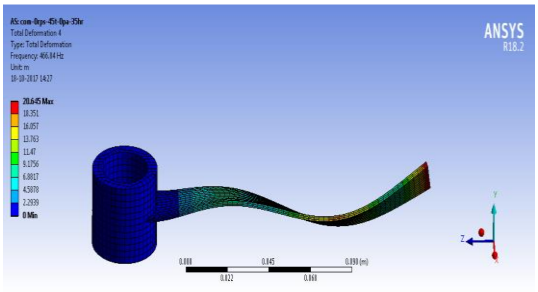


II. Mode shapes for hub radius 35mm.

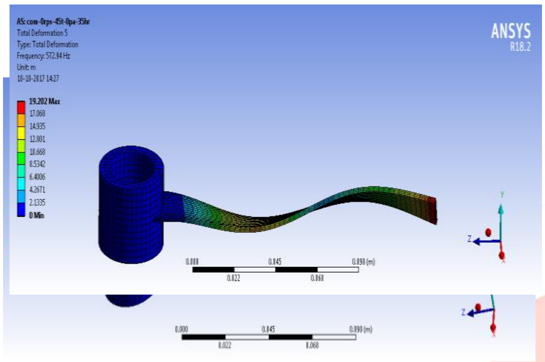


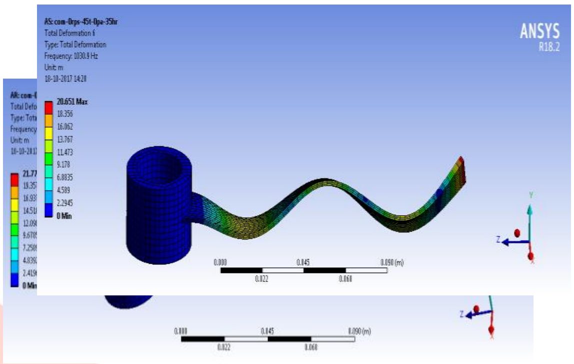




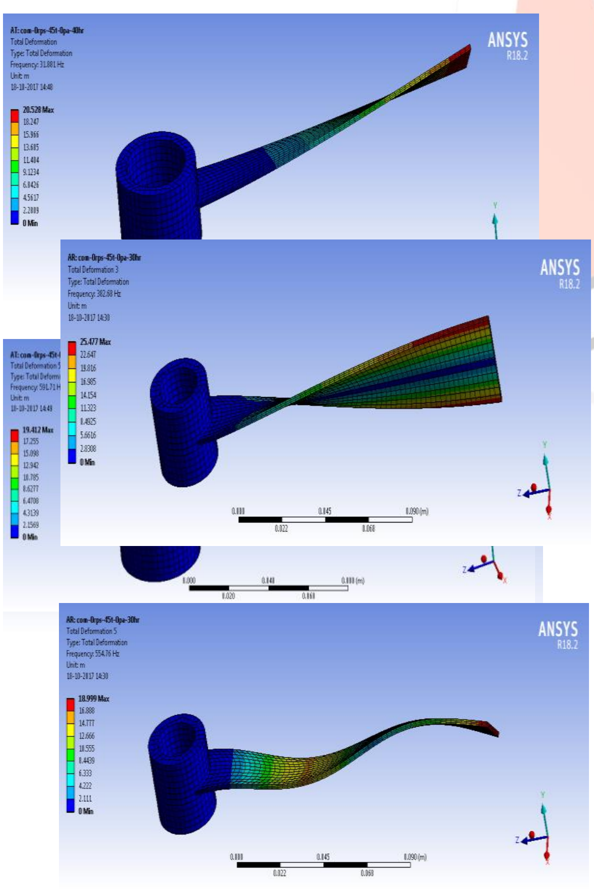


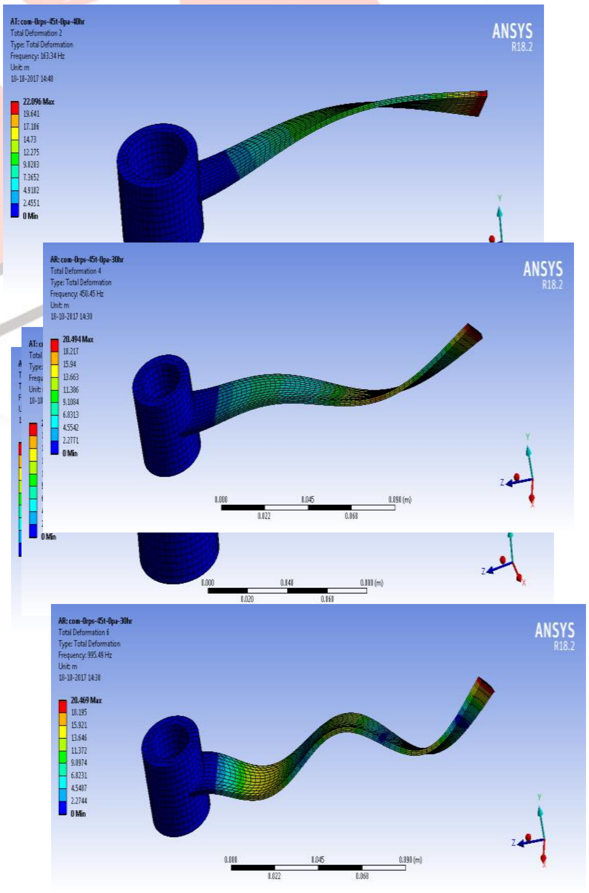
III. Mode shapes for hub radius 30mm.



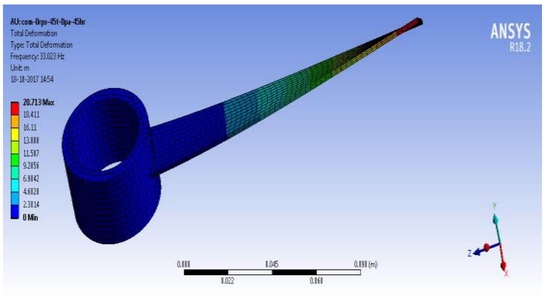


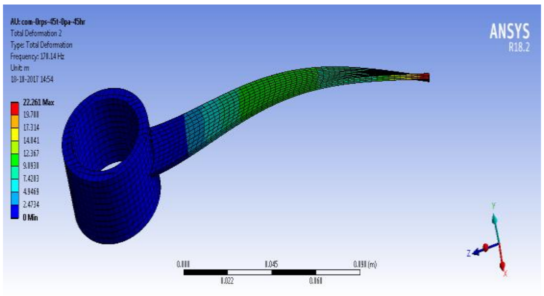
IV. Mode shapes for hub radius 40mm.

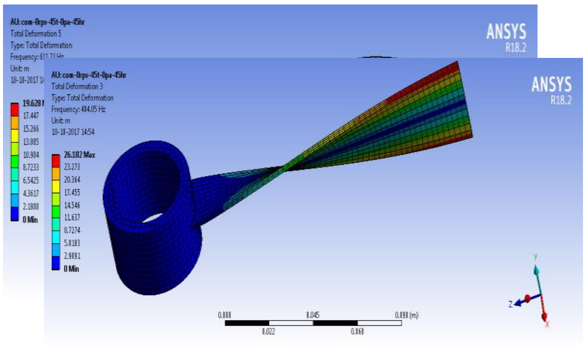


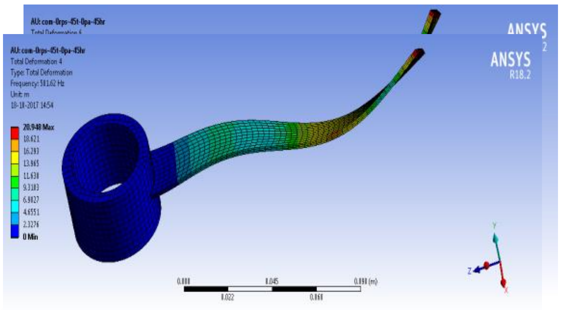


V. Mode shapes for hub radius 45mm.

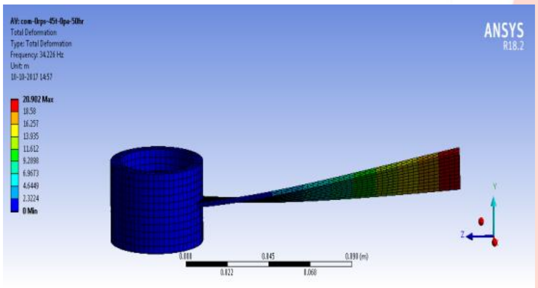


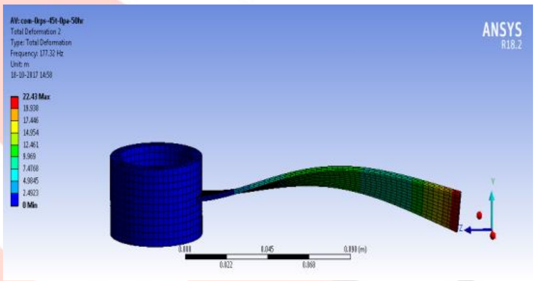


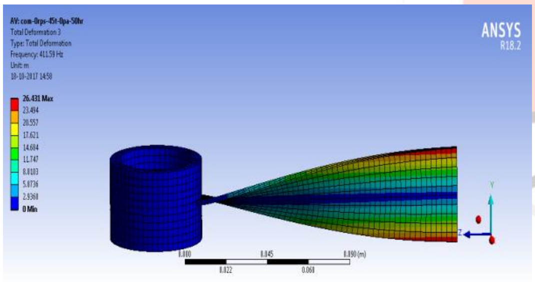


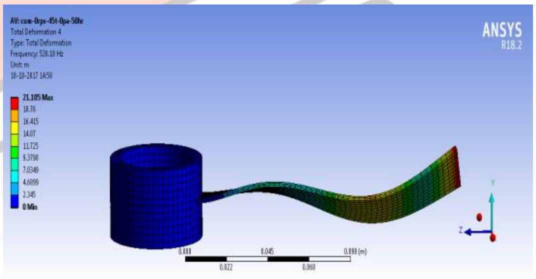


VI. Mode shapes for hub radius 50mm.



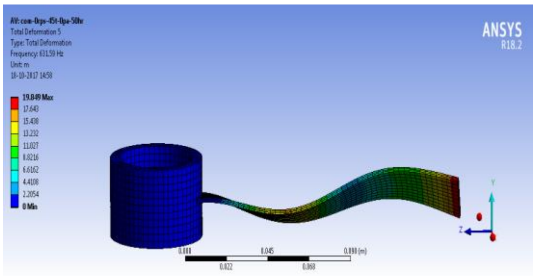


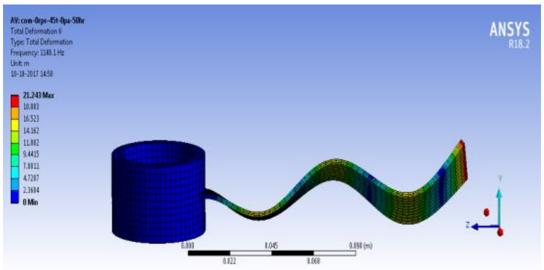




Conclusion

In the present work, free vibration analysis of rotating twisted composite beam is performed. A composite uniform beam model and composite twisted beam model has been compared with isotropic beam model for validation purpose. The results are in coherence for the validation purpose. Later, the effect of hub radius on the natural frequency of rotating composite beam has been studied and presented. With the change in hub radius the natural frequencies are being affected at higher level only. The mechanical behaviour of the twisted composite beam is to be different from that of the uniform beam. Henceforth, it is essential to consider the effect of the laminate stiffness of the composite beam initiated by the twist angle which could be an interest of future study.





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