

Review Of Queuing And Reliability Model For Machine Interference And Repairing

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Abstract— In this paper, we consider a queueing model for machine repairing system with Bernoulli vacation schedule. The failure times, repair times and vacation times are all assumed to be exponentially distributed. In congestion, the server may increase or reduce the queue length. We assume that the server begins the working vacation when the repair rate with pressure coefficient system is empty. The server may go for a vacation of random length with probability p or may continue to repair the next (if available) failed machine with probability $q = 1 - p$. The whole system is modelled as a finite state Markov Chain and its steady state distribution is obtained by matrix recursive approach.

Keywords— Machine repair, Bernoulli vacation, pressure coefficient, Markov Chain, Matrix-recursive method.

I. INTRODUCTION

Every moment we spent waiting for some type of service, we are part of a queue. We wait in line in our cars in traffic jams or tool booths, we wait in line at supermarkets to check out, we wait in line at barber shop or beauty parlor, we wait in line at post offices etc. We, as customers, do not generally like these waits and the managers of the establishments at which we wait also do not like us to wait, since it may cost then business. Why then is there waiting? The answer is, there is facility for service available. In the present chapter, we introduce queueing models and related some definitions. We define machine interference problem which is finite source queueing model.

II. PRELIMINARIES OF THE QUEUEING SYSTEM

Modern age is the information technology and e-governance age, require innovations that are based on modeling, analyzing, designing, and finally implementing new systems. The whole developing process assumes a well-organized team work of experts including engineers, computer scientists, mathematicians, physicists etc. Modern info communication networks are one of the most complex systems where the reliability and efficiency of the components play a very important role. For the better understanding of the dynamic behavior of the involved processes one has to deal with constructions of mathematical models which describe the stochastic service of random arriving requests. Queueing theory is one of the most commonly used mathematical tool for the performance evaluation of such systems. A flow of customers from infinite / finite population towards the service facility forms a queue or waiting lines on accounts lack of capacity to serve them all at a time. "The queue is the group of customer's (people, tasks or objects) waiting to be served". Though we are most aware of our own waiting time, queueing is for most a problem for industry and government. The success of any organization depends on maximizing the utilization of its resources. Every minute that an employee spends waiting for another department and even minute that a job spends waiting to be processed is money wasted. Thus, success of any organization also depends on attracting and keeping customers. Every minute that a customary spends waiting to be served translates into lost business and lost revenue.

We explain some of the major characteristics of queueing systems which are given below:

- (i) Arrival pattern of customers.
- (ii) Service pattern of servers.
- (iii) Customers and customer behaviour.

Arrival Pattern of Customers

The arrival pattern or input to a queueing system is often measured in terms of the average number of arrivals per some unit of time (mean arrival rate) or by the average time between successive arrivals (mean inter-arrival time).

Service Pattern of Servers

Service pattern can be described by a rate (number of customers served per unit of time).

Customers and Customer Behaviour

- (a) Customers: By customers, we mean the arriving units that require some service to be performed. The customer may be of persons, vehicles, machines, etc., queues stands for a number of customers waiting to be serve
- (b) Customer Behaviour: Some of the customer behaviours are: reneging, balking and jockeying. Reneging is the act of leaving a queue before being served. Balking is the act of not joining a queue upon arrival. Conceptually, the two concepts are the same. The only difference is the timing of when the customer leaves (either immediately in the case of balking or later in the case of reneging). Jockeying is the act of switching from one queue to another.

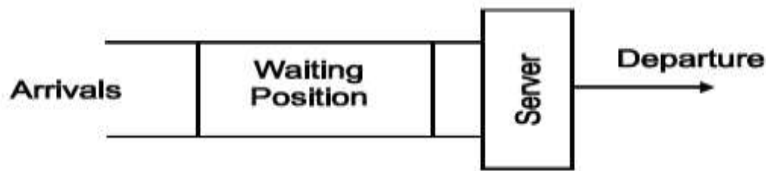


Figure 1.1: Standard graphical notation for queues.

III. PERFORMANCE MEASURES IN QUEUE

Let C be the number of identical servers and arriving unit requires service from any of the server.

Define $n P(t) = \text{Pr ob. \{exactly } n \text{ units in the system at time } t\}}$. Then

(i) $L_s(t)$ = Expected number of units in the system at time t is given as follows:

$$\sum_{n=1}^{\infty} n P_n(t)$$

(ii) $q L(t)$ = Expected Number of units in a queue at time t given by:

$$\sum_{n=c+1}^{\infty} (n-c) P_n(t)$$

(iii) $C(t)$ = Expected number of busy servers at time t is shown by:

$$c \sum_{n=1}^{\infty} n P_n(t) + \sum_{n=1}^{c-1} n P_n(t)$$

(iv) $C(t)$ = Expected number of servers not busy at time

$$c - c(t) = \sum_{n=1}^{\infty} (c-n) P_n(t)$$

IV. QUEUEING SYSTEM



Figure 1.2: Queueing system.

Steady-state (behaviour is independent of time t)

$$P_n(t) = \lim_{t \rightarrow \infty} P_n(t) = P_n$$

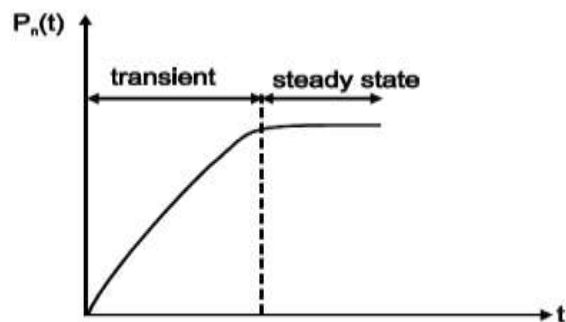


Figure 1.3: Transient phase.

Generally, in many situations only first three symbols A / B / X are used, because these are the three most important characteristics, the symbol Y is omitted if there is no restriction and the queueing system is FIFO. With a single queue, the most common performance measures are obtained as follows:

ρ : Is the server utilization which describes the fraction of time that the server is busy or the mean fraction of active serves, in the case of multiple servers.

Q : Is the queue length which is the number of jobs waiting in the queue at a given time.

w : Is waiting time which is the time that the jobs spend in the queue waiting to be served.

k : Is the number of jobs in the system at a given time.

P_n : Is the probability of a given number of jobs n in the system.

V. QUEUE DISCIPLINE

Queue discipline is a rule, according to which customers are selected for service when a queue has been formed. Most common queue discipline is the FCFS or "First Come First Served" or FIFO "First in First Out" rule under which the customers are serviced in the strict order of their arrivals. Other queue disciplines are LIFO "Last in First Out, SIRO "Selection for service In

Random Order” and a variety of priority schemes according to which a customer’s service is done in preference over some other customer’s service.

Cumulative Arrival and Departure Diagrams

The word cumulative means “resulting from accumulation”. A cumulative diagram indicates how many customers have accumulated up-to some point in time from an initial starting time. A cumulative arrival diagram indicates that how many customers have arrived up-to a point in time and a cumulative departure diagram indicates how many customers have departed up-to a point in time.

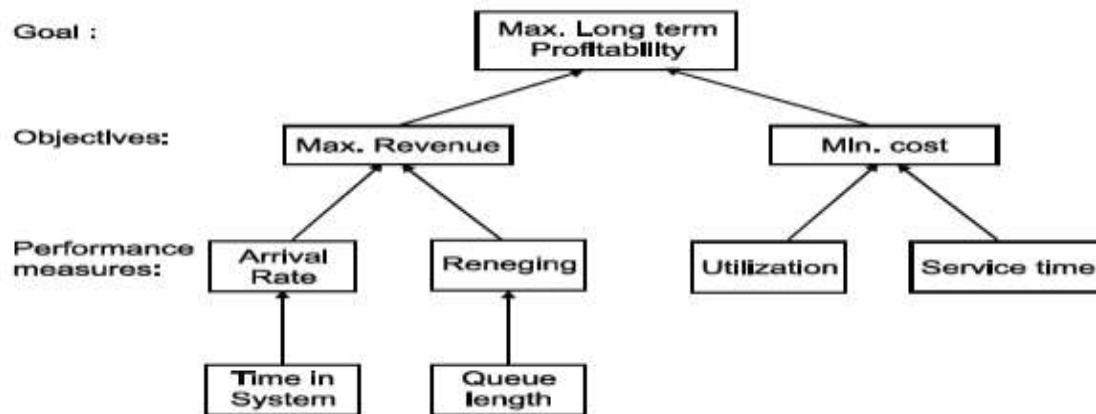


Figure 1.4: Performance measures indicate whether system objectives and goals are achieved.

VI. SIMULATION

Simulation is the technique that creates the data. According to Webster's dictionary, a simulation is the act of imitation. It is a word used in many contexts. NASA uses chambers to simulate zero gravity for astronauts. United airline uses computer programs to simulate Jet flight. The key characteristics of Simulation are dynamic and time varying behaviour. In the domain of Operations Research, Simulation is a powerful technique for evaluating the behaviour. Simulation is a very powerful and widely used management science technique for the analysis and study of complex system. A simulation model usually takes the form of a set of assumptions about the operation of the system, expressed as mathematical or logical relations between the objects of interest in the system has the process of designing a mathematical or logical model of a real-system and then conducting computer based experiments with the model to describe, explain and predict the behaviour of the real system.

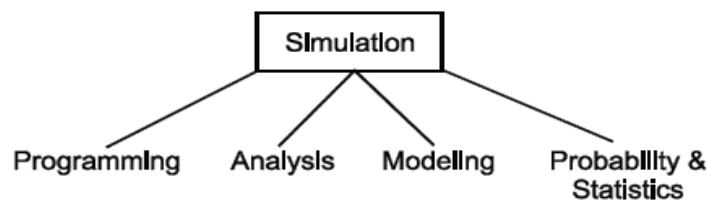


Figure 1.5: Simulation.

A system is a collection of entities that act and interact towards the accomplishment of some logical end. Systems generally tend to be dynamic, their status changes over time. To describe this status, we use the concept of the state of a system.

Examples (Simulation Model)

1. Emergency room (beds, doctors, nurses)
2. Power integration-Semiconductor capacity, and random machine downtime,

The simulation model provides a very flexible mechanism for evaluating alternative policies. Simulation runs usually result in variation due to its property and the random number used. Therefore, instead of depending on an average output the confidence interval of output is usually used to illustrate the numerical results. As mentioned by Banks [2004] “a confidence interval for the performance measures being estimated by the simulation model is a basic component of output analysis. Simulation is an alternative to solve the machine interference problem. Simulation is often used to solve complex problems that are too complicated to be derived by mathematical modeling or through the analytical approach. Simulation might not provide the exact solution, but it provided a good estimate for existing problems. Guild and Harnett [1982] has worked on the simulation study of the machine interference problem that involved N machines and R operators with two type of stoppage. In continuation to the application of the machine interference problem, Sztrik and Moller [2001] discussed characteristics of stochastic simulation of the Markov modulated finite source queueing systems also in [2002] Sztrik and Moller has studied on simulation of machine interference on randomly changing environments.

VII. QUEUEING PROBLEMS

Queueing problems can be classified into three types:

- (i) Perpetual queue.
- (ii) The predictable queue
- (iii) The Stochastic queue

The Perpetual Queue

It is a worst type queue. A perpetual queue is a queue for which all customers have to wait for service. All customers have to wait, because the servers have insufficient capacity to handle the demand for the service.

The Predictable Queue

This type queue occurs when the arrival rate is known to exceed the service capacity over finite intervals of time.

The Stochastic Queue

The Stochastic queues are not predictable. They occur by chance when customers happen to arrive at a faster rather than they are served. Stochastic queues can occur whether or not the service capacity exceeds the average arrival rate.

VIII. MARKOV CHAINS

The stochastic process $\{X_n, n = 0, 1, 2, \dots\}$ is called a Markov chain, if for $j, k, j_1, j_2, \dots, j_{n-1}$ (for any subset of I).

$$P_r(X_n=k | X_{n-1}=j, X_{n-2}=j_1, \dots, X_0=j_{n-1})$$

The outcomes are called the states of the Markov chain. The transition probabilities p_{jk} are basic to the study of the structure of the Markov-chain.

IX. M / M / 1: THE CLASSICAL QUEUEING SYSTEM

M/M/1 queue is the simplest non-trivial interesting queueing system and described by selecting the birth-death coefficients as follows:

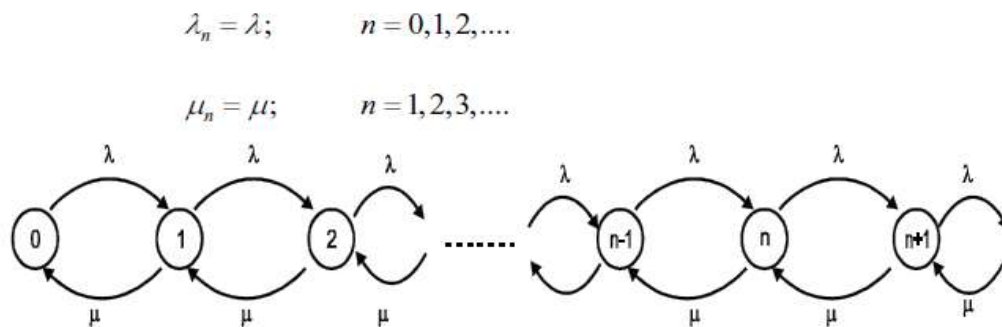


Figure 1.6: State-transition rate diagram for M/M/1 queue

X. MACHINE INTERFERENCE IN QUEUEING THEORY

With the advancement of latest technology, machining system has pervaded in every nook and corner of our lives. Thus, ensuring our almost dependence on machining system. As the time progresses a machine may fail due to wear and tear or due to some unpredicted fault and thus requires a corrective measure by providing the repair facility, after which it can be again start functioning properly. If at any time, failed machines need the repairman's attention, a queue of failed machines may develop due to unavailability of idle repairmen. This phenomenon needs attention of mathematicians as such a machine repair models have captivated the interest of many renowned researchers working in the area of queueing theory.

XI. SOME APPLICATIONS OF QUEUEING THEORY

Queueing theory or waiting lines is useful crowd safety management, entry and exist systems, concession planning and crowds flow assessment, venue ticket sales, transport loading (to and from avenue), traffic control and planning, finding the sequence of computer operations, telecommunication like waiting calls when server is busy, air port traffic, layout of manufacturing systems, capacity planning for buses and trains, health services (e.g. control of hospital bed assignments). Also queueing theory is useful in game e.g. when to remove a goalie in a hockey game. We have application of queueing theory in machine repairing system.

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