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Square Difference Prime Labeling for Some Tree Graphs

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***Abstract*— Square difference prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with absolute difference of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (*gcin*) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is one, then the graph admits square difference prime labeling. Here we identify some trees for square difference prime labeling.**

IndexTerms— **Graph labeling, square difference, greatest common incidence number, prime labeling, trees.**

I. INTRODUCTION

All graphs in this paper are trees. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In [5] , we introduced the concept, square difference prime labeling and proved that some snake graphs admit this kind of labeling. In [6], [7] and [8] we extended our study and proved that the result is true for some path related graphs, some planar graphs, fan graph, helm graph, umbrella graph, gear graph, friendship and wheel graph. In this paper we investigated square difference prime labeling of some tree graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,------------------, p-1} by $f(v_i) = i - 1$, for every i from 1 to p and define a 1-1 mapping f_{sdp}^* : E(G) → set of natural numbers N by $f_{sdp}^*(uv)$ = | f(u)^2 - f(v)^2 |. The induced function f_{sdp}^* is said to be a square difference prime labeling, if for each vertex of degree at least 2, the gcin of the labels of the incident edges is 1.

Definition 2.2 A graph which admits square difference prime labeling is called a square difference prime graph.

Definition 2.3 A graph G(V,E) obtained by a path by attaching exactly two pendent edges to each vertices of the path is called a centipede graph.

Definition 2.4 A graph G(V,E) obtained by a path by attaching exactly two pendent edges to each internal vertices of the path is called a Twig graph.

Definition 2.5 A coconut tree CT(m,n) is the graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n.

Definition 2.6 The Bistar graph B(m,n) is the graph obtained from path P_2 by joining *m* pendant edges to one end and *n* pendant edges to other end.

Definition 2.7 Star graph $K_{1,n}$ is a tree on n+1 vertices with one vertex having degree n and other n vertices having degree 1.

Definition 2.8 Let G be the graph obtained by adding pendant edges alternately to the vertices of a path P_n. G is denoted by P_n Θ A(K_1).

Theorem 2.1 Centipede graph C(2,n) admits square difference prime labeling.

Proof: Let G = C(2,n) and let v_1,v_2,----------------,v_{3n} are the vertices of G

Here |V(G)| = 3n and |E(G)| = 3n-1

Define a function f : V → {0,1,2,3,-----------------,3n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n+1

$f^*_{sdp}(v_{i+2}\ v_{n+i+2})$ = (n+2i+2)n, i = 1,2,------------,n-2
$f^*_{sdp}(v_{i+1}\ v_{2n+i})$ = (2n+2i-1)(2n-1), i = 1,2,------------,n
Clearly f^*_{sdp} is one to one.
gcin of (v_{i+1}) = gcd of $\{f^*_{sdp}(v_i\ v_{i+1}), f^*_{sdp}(v_{i+1}\ v_{i+2})\}$
= gcd of { 2i-1, 2i+1}
= gcd of {2, 2i-1} = 1. i = 1,2,------------,n
So gcin of each vertex of degree greater than one is one.
Hence C(2,n), admits square difference prime labeling.

Theorem 2.2 Twig graph $T_w(2,n)$ admits square difference prime labeling.
Proof: Let G = $T_w(2,n)$ and let v_1,v_2,----------------,v_{3n-4} are the vertices of G
Here |V(G)| = 3n-4 and |E(G)| = 3n-5
Define a function f : V → {0,1,2,3,----------------,3n-5 } by
$f(v_i)$ = i-1 , i = 1,2,------,3n-4
Clearly f^*_{sdp} is one to one.
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(v_i\ v_{i+1})$ = 2i-1, i = 1,2,------------,n-1
$f^*_{sdp}(v_{i+1}\ v_{n+i})$ = (n+2i-1)(n-1), i = 1,2,------------,n-2
$f^*_{sdp}(v_{i+1}\ v_{2n-2+i})$ = (2n+2i-3)(2n-3), i = 1,2,------------,n-2
Clearly f^*_{sdp} is one to one.
gcin of (v_{i+1}) = 1. i = 1,2,------------,n-2
So gcin of each vertex of degree greater than one is one.
Hence $T_w(2,n)$, admits square difference prime labeling.

Theorem 2.3 Coconut Tree graph CT(m,n) admits square difference prime labeling.
Proof: Let G = CT(m,n) and let v_1,v_2,----------------,v_{m+n} are the vertices of G
Here |V(G)| = m+n and |E(G)| = m+n-1
Define a function f : V → {0,1,2,3,----------------,m+n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,m+n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(v_i\ v_{i+1})$ = (2i-1), i = 1,2,------------,n-1
$f^*_{sdp}(v_n\ v_{n+i})$ = (2n+i-2)(i) i = 1,2,------------,m
Clearly f^*_{sdp} is one to one.
gcin of (v_{i+1}) = 1. i = 1,2,------------,n-1
So gcin of each vertex of degree greater than one is one.
Hence CT(m,n), admits square difference prime labeling.

Theorem 2.4 Comb graph admits square difference prime labeling.
Proof : Let G = C(n) and let v_1,v_2,----------------,v_{2n} are the vertices of G
Here |V(G)| = 2n and |E(G)| = 2n-1
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(v_i\ v_{i+1})$ = (2i-1), i = 1,2,------------,n+1
$f^*_{sdp}(v_{i+2}\ v_{n+i+2})$ = n(n+2i+2), i = 1,2,------------,n-2
Clearly f^*_{sdp} is one to one.
gcin of (v_{i+1}) = 1. i = 1,2,------------,n-1
So gcin of each vertex of degree greater than one is one.
Hence C(n) , admits square difference prime labeling.

Theorem 2.5 Star $K_{1,n}$ admits square difference prime labeling.
Proof: Let G = $K_{1,n}$ and let u,v_1,v_2,----------------,v_n are the vertices of G
Here |V(G)| = n+1 and |E(G)| = n
Define a function f : V → {0,1,2,3,----------------,n } by
$f(v_i)$ = i , i = 1,2,------,n
f(u) = 0
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows
$f^*_{sdp}(u\ v_i)$ = i^2, i = 1,2,------------,n
Clearly f^*_{sdp} is one to one.
gcin of (u) = 1.
So gcin of each vertex of degree greater than one is one.

Hence $K_{1,n}$, admits square difference prime labeling.

Theorem 2.6 Sub division graph of star $K_{1,n}$ admits square difference prime labeling.

Proof: Let G = $Sd(K_{1,n})$ and let u, v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 2n

Define a function f : V → {0,1,2,3,----------------,2n } by

$f(v_i) = i$, i = 1,2,------,2n

f(u) = 0

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(u\, v_{2i-1})$ = $(2i-1)^2$, i = 1,2,------------,n

$f_{sdp}^*(v_{2i-1}\, v_{2i})$ = 4i-1, i = 1,2,------------,n

Clearly f_{sdp}^* is one to one.

gcin of (v_{2i-1}) = gcd of $\{f_{sdp}^*(u\, v_{2i-1}), f_{sdp}^*(v_{2i-1}\, v_{2i})\}$

= gcd of { $(2i-1)^2$, 4i-1}

= gcd of {4i-1, 2i-1}

= gcd of {2i, 2i-1}

= 1. i = 1,2,------------,n

So gcin of each vertex of degree greater than one is one.

Hence $Sd(K_{1,n})$, admits square difference prime labeling.

Theorem 2.7 Bistar graph B(m,n) admits square difference prime labeling.

Proof: Let G = B(m,n) and let v_1, v_2,----------------,v_{m+n+2} are the vertices of G

Here |V(G)| = m+n+2 and |E(G)| = m+n+1

Define a function f : V → {0,1,2,3,----------------,m+n+1 } by

$f(v_i)$ = i-1 , i = 1,2,------,m+n+2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\, v_{i+1})$ = 2i-1, i = 1,2,3

$f_{sdp}^*(v_3\, v_{4+i})$ = (i+1)(i+5), i = 1,2,------------,n-1

$f_{sdp}^*(v_2\, v_{n+3+i})$ = (n+i+3)(n+i+1) i = 1,2,------------,m-1

Clearly f_{sdp}^* is one to one.

gcin of (v_{i+1}) = 1. i = 1,2.

So gcin of each vertex of degree greater than one is one.

Hence B(m,n), admits square difference prime labeling.

Theorem 2.8 $P_n \odot A(K_1)$ admits square difference prime labeling, if n is odd and pendant edge starts from the second vertex.

Proof : Let G = $P_n \odot A(K_1)$ and let v_1, v_2,----------------,$v_{\frac{3n-1}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = $\frac{3n-3}{2}$

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-3}{2}$} by

$f(v_i)$ = i , i = 1,2,------, $\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\, v_{i+1})$ = (2i-1), i = 1,2,------------,n-1

$f_{sdp}^*(v_{2i}\, v_{n+i})$ = (n-i)(n+3i-2), i = 1,2,------------,$\frac{n-1}{2}$

Clearly f_{sdp}^* is one to one.

gcin of (v_{i+1}) = 1. i = 1,2,------------,n-2

So gcin of each vertex of degree greater than one is one.

Hence $P_n \odot A(K_1)$, admits square difference prime labeling.

Theorem 2.9 $P_n \odot A(K_1)$ admits square difference prime labeling, if n is even and pendant edges starts from the first vertex.

Proof : Let G = $P_n \odot A(K_1)$ and let v_1, v_2,----------------,$v_{\frac{3n}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = $\frac{3n-2}{2}$

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-2}{2}$} by

$f(v_i)$ = i , i = 1,2,------, $\frac{3n}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{sdp}^* is defined as follows

$f_{sdp}^*(v_i\, v_{i+1})$ = (2i-1), i = 1,2,------------,n

$f_{sdp}^*(v_{2i+2}\, v_{\frac{3n+2-2i}{2}})$ = $(\frac{3n-2i}{2})^2 - (2i+1)^2$, i = 1,2,------------, $\frac{n}{2}$

Clearly f_{sdp}^* is one to one.

gcin of (v_{i+1}) = 1. i = 1,2,------------,n-1
So gcin of each vertex of degree greater than one is one.
Hence $P_n \Theta A(K_1)$, admits square difference prime labeling.

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