Assessment of Voltage Stability using Cumulant-based Probabilistic Load Flow including Wind and Photovoltaic Power Generation

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Abstract **- This paper applies a cumulant-based probabilistic power flow (PPF) algorithm to estimate voltage stability index in the presence of uncertainty due to wind and photovoltaic power generation. These renewable energy sources have introduced additional uncertainty to power system along with uncertainty due to load demand. To show the effect of these uncertainties, probabilistic power flow methods need to be applied. Voltage stability index (L-index) detects the vulnerable system states and predicts the voltage collapse. Deterministic methods cannot represent the effect of uncertainties on voltage stability index(L-index). So the Probabilistic load flow using the method of combined Cumulants and Gram-Charlier expansion is used to obtain the Probability distribution of voltage stability index. This method is computationally efficient with high degree of accuracy. The proposed approach has been tested on modified IEEE 14 bus and IEEE 30 bus systems. Results have also been compared with those from Monte-Carlo simulation.**

Keywords **- Probabilistic load flow; Cumulant method; Gram-Charlier expansion; Monte-Carlo simulation; Photovoltaic generation; Wind generation; Voltage stability index.**

I.INTRODUCTION

Deterministic load flow (DLF) is considered as a fundamental tool for analysis of power systems. This approach takes fixed values of power generation, load and a well defined network configuration. Accurate estimation of data for load flow studies is constrained due to its random nature. There are many uncertainties present in power systems such as load demand variation, change in network configuration and forced outage rate of generators. The large scale integration of renewable energy sources has increased the level of uncertainty in the power system. In PV generation, uncertainty arises from inaccurate prediction of solar irradiance, where as in wind power generation it arises from inaccurate prediction of wind speed. Probabilistic load flow considers effect of all these uncertainties by taking each input as a random variable. So Probabilistic load flow (PLF) studies give a better feel of future system conditions.

Application of PPF method to power system was first proposed in 1974 [1]. Several techniques have been introduced to carry out PLF studies. These techniques can be classified as (i) Monte-Carlo simulation (MCS), (ii) Analytical methods (AMs) and (iii) Approximate methods (APMs). MCS is a typical numerical technique which gives many solutions from deterministic LF based on samples of random variables (r.v.s) generated [16]. These samples are generated according to the distributions of the input random variables. The accuracy is the advantage of this method as it gives non-linear load flow equations directly; however, it usually takes more computational time. So this technique is generally used as reference method to compare the results obtained from proposed method. In Analytical methods, input random variables are represented by probability mass functions (PMFs) or probability density functions (PDFs). The output variables obtained are either in terms of probability density functions or cumulative distribution functions (CDFs). Fast Fourier Transforms (FFT) and cumulant method come under this category. Approximate methods include point estimate method [2], the first order second moment method [3] and unscented transforms method [4].

For large power systems CM (Cumulant method) is suitable since it has very less computational time without losing accuracy. Another advantage of finding cumulants is that, several type of expansions such as Gram-Charlier method (GCM), Cornish-Fisher method (CFM) can be used to shape the distribution of output variables. PLF computation using the method of combined cumulants and Gram-Charlier Expansion was proposed in [5]. In this, author uses DC model. The same method was applied to linearized AC model in [6]. Effect of wind uncertainty in PLF was considered by author in [13]. The author in [18] considered the photovoltaic generation uncertainty in probabilistic load flow using method of cumulants. Effect of branch outages in probabilistic load flow was taken as uncertainty in [21]. This paper proposes an approach to compute PDF and CDF of the voltage stability index and extends the cumulant method to find distributions of active and reactive power losses. Prediction of voltage collapse by using the L-index was proposed in [7]. It gives fixed values for a specified input data. But the proposed method gives PDF and CDF of the L-index which are used to effectively assess the reliability of the Power system.

In section II, mathematical model for Probabilistic load flow is given. Section III gives the computational procedure of proposed method. Next in Section IV, performance of the proposed method is tested on IEEE 14 and IEEE 30 bus systems and results are analyzed. Finally, conclusions are given in Section V.

II. PROBABILISTIC LOAD FLOW FORMULATION

(A)The Proposed PPF Model

The non linear load flow equations used for linearization are represented as

$$W = f(x) \quad (1)$$

$$Z = g(x) \quad (2)$$

Where W is vector of bus power injections, Z is vector of line power flows and X is state vector of nodal voltage magnitudes and their angles. f and g are functions of power injections and line flows respectively .

Assuming a small variation ΔW leads to a change ΔX and ΔZ in vectors of state variable and branch flows respectively. Expanding (1) and (2) around operating point X using Taylor series gives

$$\Delta X = J_1^{-1}\Delta W = K_1\Delta W \quad (3)$$

Where J_1 is the jacobian matrix and K_1 is sensitivity matrix of size *(N*-1+*L)x(N*-1+*L).*

Here *'N'* represents number of buses and *'L'* represents number of load buses.

Line flows represented by vector Z are given by

$$P_{ij} = V_iV_j(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) - t_{ij}G_{ij}V_i^2 \quad (4)$$

$$Q_{ij} = V_iV_j(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) + (t_{ij}B_{ij} - b_{ij})V_i^2 \quad (5)$$

Where P_{ij} and Q_{ij} are active and reactive power line flows from bus i and bus j, V_i and V_j are the r.m.s values of voltages at bus i and bus j, δ_{ij} are the angular difference between buses i and j, G_{ij} and B_{ij} are the conductance and susceptance between buses i and j, b_{ij} is the half line shunt susceptance of branch ij, and t_{ij} is the transformation ratio of branch ij.

Now, $\Delta Z = J_2\Delta X = J_2K_1\Delta W = K_2\Delta W$ (6)

Where J_2 and K_2 are Jacobian and sensitivity matrices respectively.

Size of K_2 is *(2b)*x*(N*-1+*L).* Where *'b'* represents number of branches in the network.

The similar line flow equations which represent active and reactive line flows from bus j to bus i are given by

$$P_{ji} = V_jV_i(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ji}) - \left(\frac{G_{ij}V_j^2}{t_{ij}}\right) \quad (7)$$

$$Q_{ji} = V_jV_i(G_{ij}\sin\delta_{ji} - B_{ij}\cos\delta_{ji}) + V_j^2\left(\frac{B_{ij}}{t_{ij}} - b_{ij}\right) \quad (8)$$

The line losses can be obtained by adding line flow equations (4) and (5) with (7) and (8) respectively.

$$P_l = 2V_jV_iG_{ij}\cos\delta_{ij} - G_{ij}\left(V_i^2t_{ij} + \frac{V_j^2}{t_{ij}}\right) \quad (9)$$

$$Q_l = -V_iV_jB_{ij}\cos\delta_{ij} + B_{ij}\left(V_i^2t_{ij} + \frac{V_j^2}{t_{ij}}\right) - b_{ij}(V_i^2 + V_j^2) \quad (10)$$

Where P_l and Q_l are active and reactive power line losses respectively

$$\Delta L = J_3\Delta X = J_3K_1\Delta W = K_3\Delta W \quad (11)$$

Where ΔL represents variation in loss vector which comprises of active and reactive power losses and J_3 is Jacobian matrix. Size of K_3 is same as that of K_4.

The voltage stability index (L-index) defined in is reproduced as

$$L_j = \left|1 - \sum_{i=1}^{n_g} F_{ji}\frac{V_i}{V_j}\right|, j \in N_{PQ} \quad (12)$$

To apply cumulant method, the above equation has to be linearized about mean value and the procedure used to calculate line flows can also be used for calculation of cumulants of L-index.

$$\Delta L_j = J_4\Delta X = J_4K_1\Delta W = K_4\Delta W \quad (13)$$

ΔL_j is the variation in L-index due to variation in state vector. Size of K_4 is *(L)x(N*-1+*L).*

(B)Gram-Charlier expansion

To obtain the P.D.F and C.D.F of selected random variables, Gram-Charlier series expansion is used. The coefficients of the series expansion are calculated from cumulants of random variables.

Considering a random variable 'y' with mean 'm' and standard deviation' σ' represented as standard normal variable x

Where $x = (y - m)/\sigma$ (14)

its C.D.F and P.D.F are denoted as F(x) and f(x) respectively. According to Gram-Charlier expansion, these can be formulated as

$$F(x) = \boldsymbol{\Phi}(x) + \frac{C_1}{1!}\boldsymbol{\Phi}'(x) + \frac{C_2}{2!}\boldsymbol{\Phi}''(x) + \frac{C_3}{3!}\boldsymbol{\Phi}^{(3)}(x) + \cdots \quad (15)$$

$$f(x) = \Phi(x) + \frac{C_1}{1!}\Phi'(x) + \frac{C_2}{2!}\Phi''(x) + \frac{C_3}{3!}\Phi^{(3)}(x) + \cdots \quad (16)$$

Where $\boldsymbol{\Phi}(x)$ and $\Phi(x)$ are C.D.F and P.D.F of normal distribution with m=0 and σ=1 and C_v are constant coefficients of series expansion.

(C) Uncertainty Modeling

(i) The real power generated by conventional generator is described by Bernoulli distribution where distribution parameter is related with forced outage rate of generating unit.

The p.m.f. of a binomial distribution is:

$$f_{\tilde{x}}(x; n, p) = \binom{n}{x}p^x(1-p)^{n-x} \quad (17)$$

Where '*n*' represents number of generating units and '*(1-p)*' represents Forced Outage Rate of generating unit.
(ii) As the load demand is time dependent, it has both random and deterministic components. The load demand at any bus is modelled in one of the three cases described in Table I.
The p.d.f. of a normal r.v. $\tilde{X}$.is:

$$f_{\tilde{X}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{X}}} \exp\left(-\frac{(x-m_{\tilde{X}})^2}{2\sigma_{\tilde{X}}^2}\right) \quad (18)$$

Table 1: MODELING OF LOAD DEMAND UNCERTAINTY

Name of distribution	Type of uncertainty	Specified parameters
Gaussian	Small Variance (forecasting error)	Mean and Standard deviation
Discrete	Large Variance	Discrete values and corresponding probability of occurrence
One point	Fixed forecasted value	Deterministic value

Where, $m_{\tilde{X}}$ is the mean, $\sigma_{\tilde{X}}^2$ is the variance and $\sigma_{\tilde{X}}$ is the standard deviation.

(iii) Distribution of the wind speed can be approximated by weibull one which has scale and shape parameters. Wind turbine characteristics curve is used to calculate the power corresponding to wind speed.
The p.d.f.. of a Weibull r.v. $\tilde{X}$ is:

$$f_{\tilde{X}}(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{(\alpha-1)} exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \quad (19)$$

where α > 0 and β > 0 are the shape and the scale parameters of the distribution, respectively.
(iv) The power output of PV generation is assumed to follow beta distribution whose parameters depend on expected value (mean) and standard deviation of output power.
The standard *Beta* distribution gives the probability density of a value *x* on the interval (0,1):

$$Beta(\alpha,\beta): prob(x!\,\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \quad (20)$$

Where '*B*' is the Beta function.

III. COMPUTATIONAL PROCEDURE

The procedure for implementing cumulant method for PLF is given below
1. Identify all input random variables including wind and solar power to obtain their Probability density function.
2. Obtain the expectation (mean) for each random variable
3. Run DLF using the mean values of random variables to get mean of voltages and line flows.
4. Calculate the sensitivity matrices K_1, K_2, K_3 and K_4 at solution obtained in above step.
5. Evaluate cumulants of nodal power injections upto order eight.
6. Calculate cumulants of all output random variables using equations (3), (6), (11) and (13).
7. Obtain the PDF and CDF of output random variables using Gram-Charlier expansion.

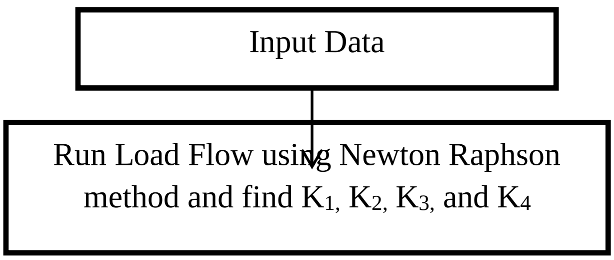
Flow chart to implement the proposed method is shown in Fig

IV. CASE STUDIES AND DISCUSSION

The proposed method to find the variation of voltage stability index in terms of PDF and CDF is tested on standard IEEE 14 and IEEE 30 bus systems. Later these are modified to include wind and photovoltaic power generations.

A) RESULTS OF IEEE 14 BUS SYSTEM

The probabilistic data of loads and generators are taken from [24]. The plots obtained using the cumulant method are compared with those of MCS method. For the results shown in case of monte carlo simulation method take the 10,000 samples of input probability distribution functions. So it runs DLF for 10,000 times and stores all the results. From these output samples PDF and CDF of variables are plotted. Cumulative distribution functions for voltage stability index at bus 13 using MCS and CM are shown in Fig 2 and Fig 3 respectively. From the graphs it is observed that both the methods give approximately same results which conclude that cumulant method is computationally efficient without losing accuracy.



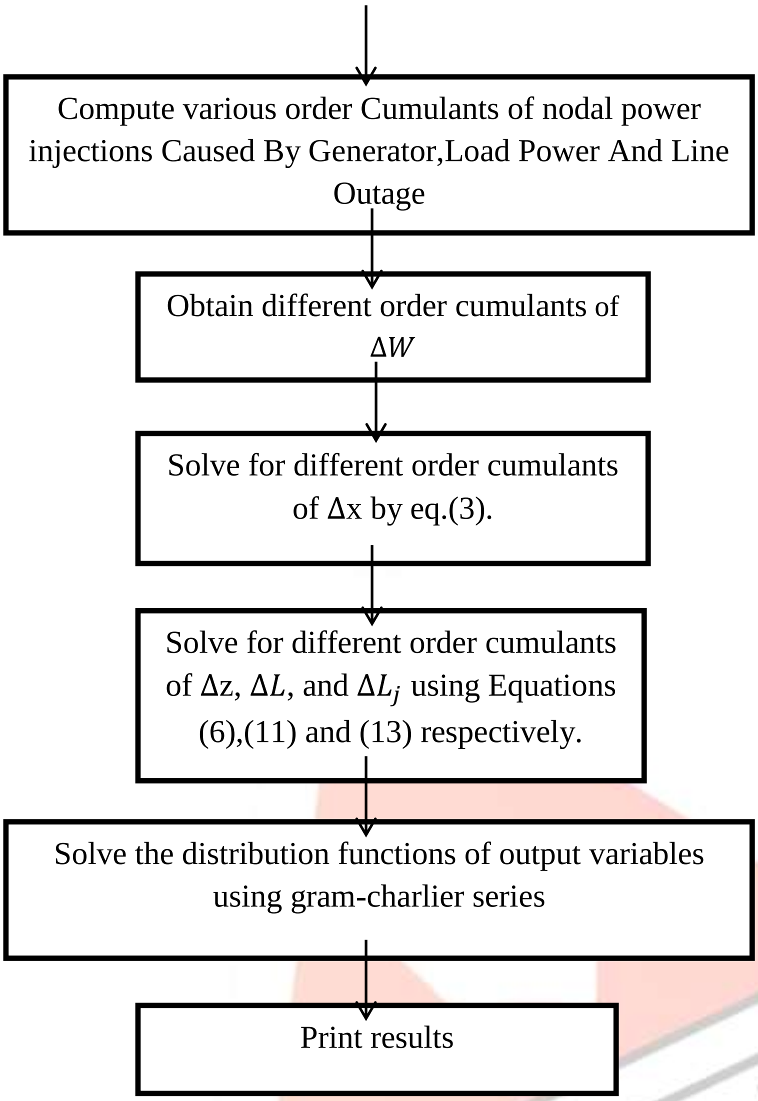


Fig 1: Flow chart for proposed Cumulant based PPF

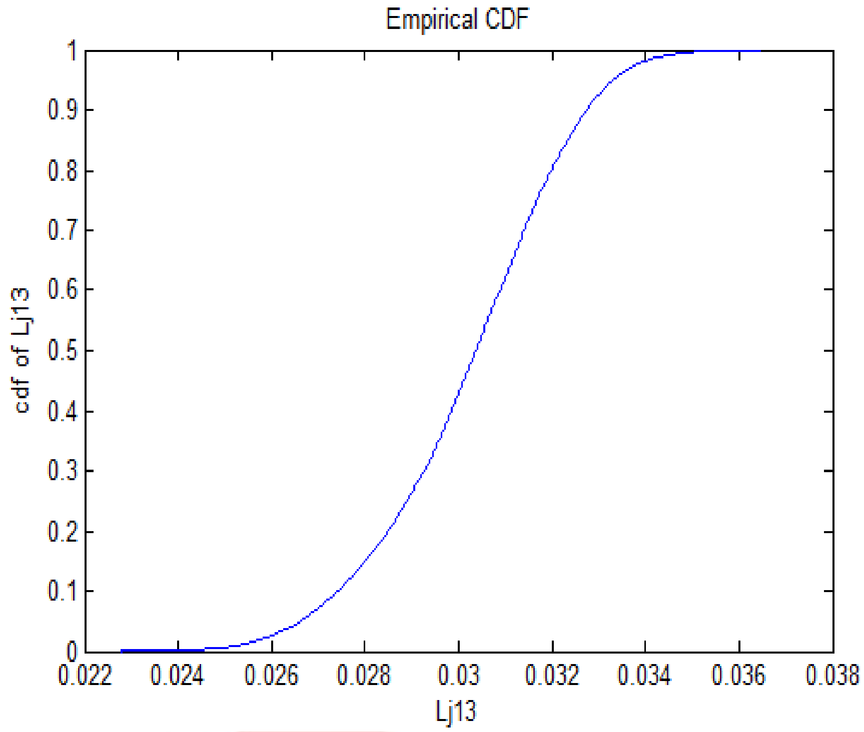


Fig 2 : CDF plot for voltage stability index at bus 13 using MCS

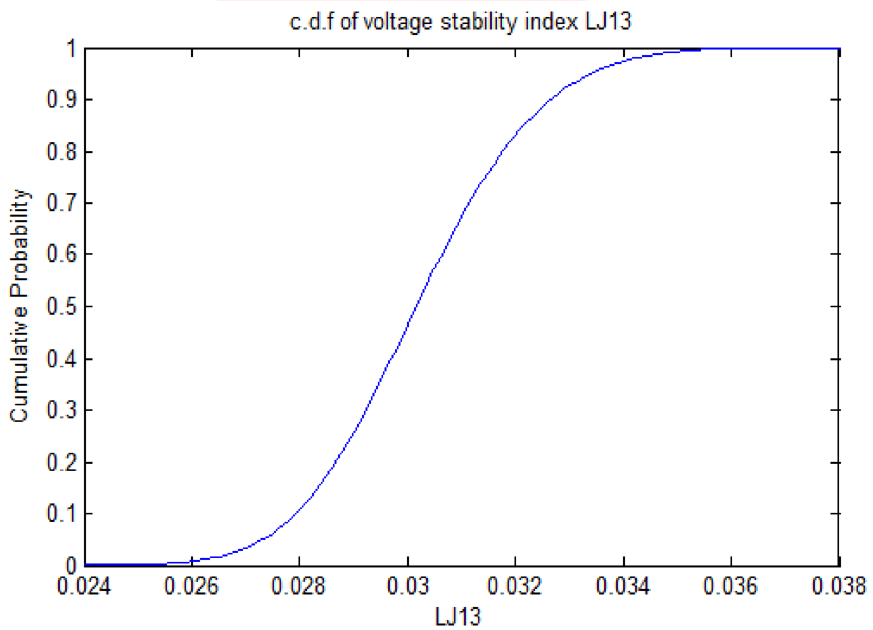


Fig 3: CDF plot for voltage stability index at bus 13 using CM

For a given value of voltage stability index, how much is the probability of variable less than or greater than the specified value, is given by the corresponding value of CDF. So this kind of information is very much useful to assess the reliability of power system. Cumulative distribution functions for total active power loss using MCS and CM are shown in Fig 4 and Fig 5 respectively.

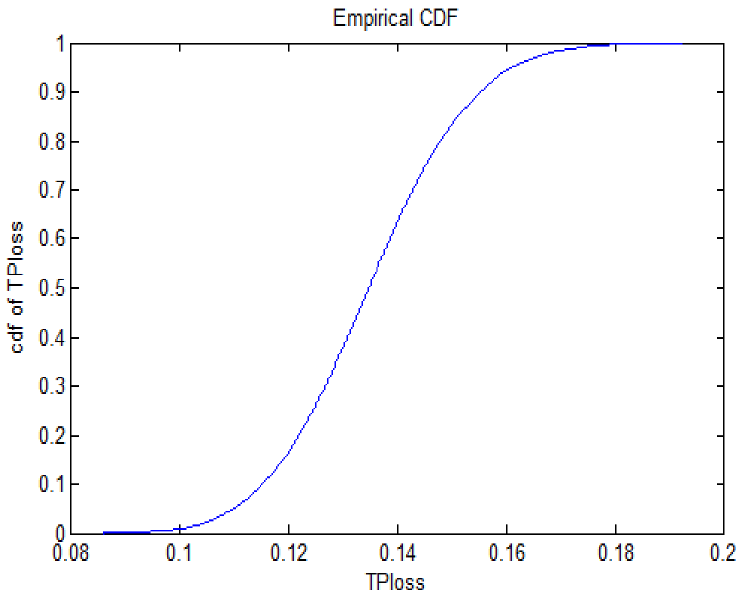


Fig 4: CDF plot for active power loss using MCS

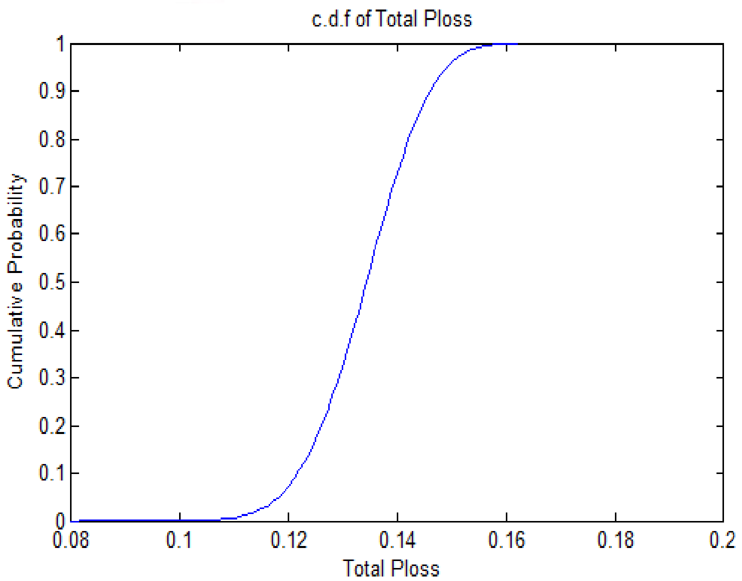


Fig 5: CDF plot for active power loss using CM

Now the IEEE 14 bus test system is modified to include solar generation at 13 bus and wind generation at 14 bus. Shape and scale factors of Weibull distribution are taken as 3.97 and 10.7 respectively. The cut-in, cut-out and rated speed of wind turbine is taken as 4m/s, 25m/s and 15m/s respectively. The parameters of beta distribution are taken as 2.9366 and 1.4139 and the rated power of solar generation is 0.04 p.u. Then the results obtained from the base case and the case with wind and solar energy systems are shown in Table 2.

Table 2: Mean and deviation of voltage stability index

Bus No	Voltage Stability Index(Base case)		Voltage Stability Index(including wind and PV system)	
	Expectation	Deviation	Expectation	Deviation

4	0.0299	0.0026	0.0288	0.0026
5	0.0203	0.0016	0.0197	0.0016
7	0.0348	0.0041	0.0318	0.0041
9	0.0598	0.0081	0.0540	0.0080
10	0.0569	0.0068	0.0521	0.0067
11	0.0325	0.0034	0.0301	0.0034
12	0.0230	0.0011	0.0189	0.0011
13	0.0302	0.0018	0.0216	0.0018
14	0.0708	0.0059	0.0512	0.0056

From the tables it is observed that percentage standard deviations of voltage stability index are increased due to the uncertainty injected by renewable energy sources. The percentage index is calculated by dividing standard deviation with corresponding mean value of the output variable.

B) RESULTS OF IEEE 30 BUS SYSTEM

Probabilistic data is same as that of IEEE-14 bus system except that standard deviation of loads are considered to be 10%. Then the IEEE 30 bus test system is modified to include solar generation at 29 bus and wind generation at 30 bus. Shape and scale factors of Weibull distribution are taken as 3.97 and 10.7 respectively. The cut-in, cut-out and rated speed of wind turbine is taken as 4m/s, 25m/s. Results obtained from cumulant based PPF with and without including wind and solar are given in Table 3. The mean values obtained in all the tables are obtained from deterministic load flow methods and standard deviations are obtained by the cumulant based probabilistic method.

Cumulative distribution functions for voltage stability index at bus 30 and total reactive power loss using the proposed method are shown in Fig 6 and Fig 7 respectively. Table 3 gives the voltage stability index of all the load buses in the bus system along with their standard deviation. In this case also the percentage standard deviation of variables was increased due to the uncertainty injected by wind and photovoltaic generation systems. This effect is more predominant at the buses connected to the bus where renewable energy sources are connected.

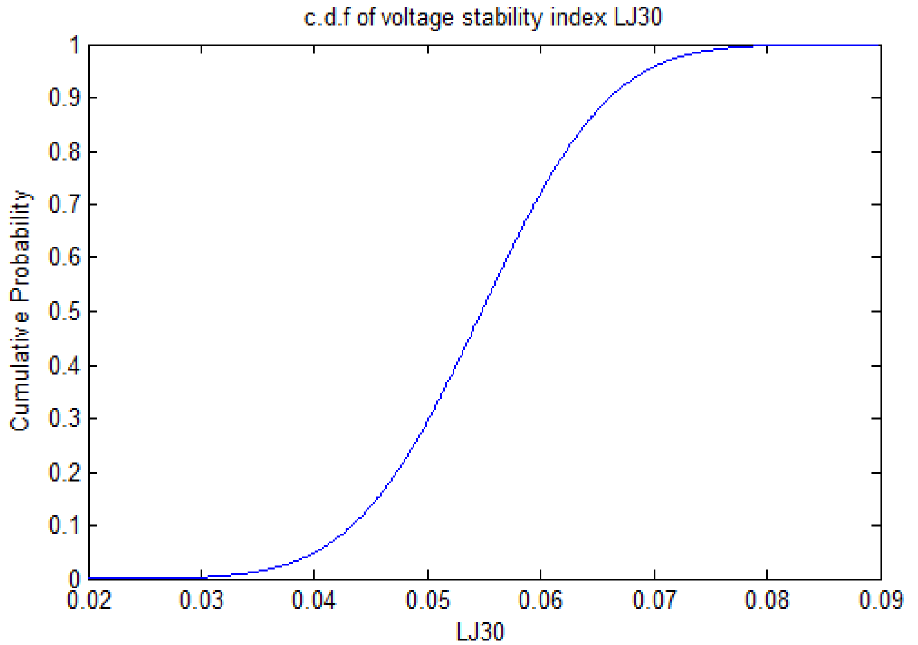


Fig 6: CDF plot for voltage stability index at bus 30 using CM with wind and solar

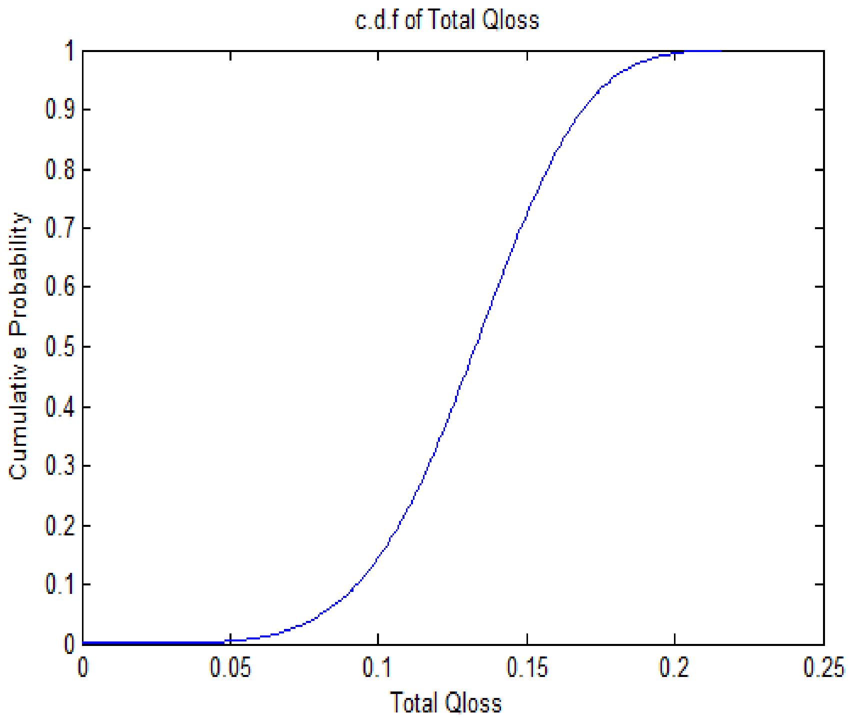


Fig 7: CDF plot for reactive power loss using CM with wind and solar .

From the graph shown in fig 6, it is observed that voltage stability index at bus 30 approximately varies from 0.03 to 0.08. Whereas, total reactive power loss varies from 0.05 p.u to 0.2 p.u. is shown in Fig 7

Table 3: Mean and deviation of voltage stability index

Bus No	Voltage Stability Index(Base case)		Voltage Stability Index(including wind and PV system)	
	Expectation	Deviation	Expectation	Deviation
3	0.0147	0.0004	0.0136	0.0004
4	0.0172	0.0005	0.0158	0.0005
6	0.0152	0.0004	0.0133	0.0005
7	0.0218	0.0013	0.0207	0.0013
9	0.0375	0.0012	0.0349	0.0012
10	0.0700	0.0024	0.0657	0.0024
12	0.0462	0.0013	0.0441	0.0013
14	0.0674	0.0021	0.0645	0.0021
15	0.0723	0.0021	0.0685	0.0021
16	0.0638	0.0019	0.0609	0.0019
17	0.0733	0.0024	0.0694	0.0024
18	0.0880	0.0028	0.0841	0.0028
19	0.0928	0.0032	0.0888	0.0032
20	0.0881	0.0029	0.0840	0.0029
21	0.0822	0.0031	0.0766	0.0031
22	0.0818	0.0030	0.0757	0.0030
23	0.0857	0.0025	0.0784	0.0026
24	0.0940	0.0031	0.0820	0.0032
25	0.0947	0.0035	0.0661	0.0039
26	0.1114	0.0044	0.0834	0.0046
27	0.0867	0.0038	0.0478	0.0047
28	0.0202	0.0007	0.0145	0.0008
29	0.1162	0.0061	0.0446	0.0083
30	0.1365	0.0085	0.0550	0.0089

V. CONCLUSION

In this paper the detailed analysis of cumulant based PPF is given with the help of CDF and PDF of output variables. The method proposed to find the probability distribution of voltage stability index and power losses has been successfully applied to IEEE 14 bus and IEEE 30 bus systems. Results obtained from this approach have been matched with those from MCS method. All the results of voltage stability index obtained from cumulant method have been tabulated. Then the renewable energy sources such as wind and photovoltaic generations are included in both the test systems. Results showed that the standard deviation of output variables in presence of renewable sources was increased due to the uncertainty of power injected by the wind and solar energy systems.

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