On Soft Gsr-Closed Sets In Soft Bitopological Spaces

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Abstract- In this paper, a new class of soft gsr-closed sets in soft bitopological spaces and some of its characteristics are investigated.

Keywords: $\widetilde{(1,2)}$soft gsr-closed set, $\widetilde{(1,2)}$soft gsr-open set, $\widetilde{(1,2)}$soft regular-open set, $\widetilde{(1,2)}$soft g-closed, $\widetilde{(1,2)}$soft semi-closed, $\widetilde{(1,2)}$soft α-closed, $\widetilde{(1,2)}$soft αg-closed, $\widetilde{(1,2)}$soft rg-closed, $\widetilde{(1,2)}$soft closed.

I. INTRODUCTION

Topology is a field of study which was developed from geometry and set theory. The term topology was introduced by Johann Benedict Listing in the 19th century, and middle of the 20th century Topology had become a major branch of Mathematics.The concept of generalized closed sets in the topological space was introduced by Levine [6] in 1970. Maki.et.al [9] and Bhattacharya and Lahiri [3] introduced gα-closed sets and sg-closed sets. The soft set theory was introduced by Molodstov [8] in the year 1999.Soft semi open set was introduced by Bin chen [2]. Muhammad shabir and Munazza naz [7] defined the soft topological spaces over the universe with fixed parameters in 2011. J.C. Kelly [5] introduced the concept of bitopological spaces in 1963. Basavaraj, M.Ittangi [1] initiated the concept in soft bitopological space in 2014. In this present study, we discuss soft-gsr-closed sets in soft bitopological spaces and their properties.

II. PRELIMINARIES

Definition: 2.1([2],[8],[10])

Let X be the initial universe and P(X) denote the power of X. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by F : A → P(X). In other words, a soft set over X is a parameterized family of subsets of the universe X. For Ɛ Є A, F(Ɛ) may be considered as the set Ɛ-approximate elements of the set (F,A).

Definition: 2.2 ([2],[8],[10])

For two soft sets (F,A) and (G,B) over a common universe X,we say that (F,A) is soft subset of (G,B) if

i) A $\widetilde{\subseteq}$ B and
ii) ∀ e ∈ A, F(e) $\widetilde{\subseteq}$ G(e)

we write (F,A) $\widetilde{\subseteq}$ (G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is soft subset of (F,A) and is denoted by (F,A) $\widetilde{\supseteq}$ (G,B).

Definition: 2.3 ([2],[8],[10])

The union of two soft sets of (F,A) and (G,B) over the common universe X is soft set (H,C), where C=A∪B and for all e∈ C, H(e) = F(e) if e ∈ A-B, H(e) = G(e) if e ∈ B-A and H(e) = F(e) ∪ G(e) if e ∈ A∩B. We write (F,A) ∪ (G,B) = (H,C).

Definition: 2.4 ([2],[8],[10])

The intersection (H,C) of two soft sets (F,A) and (G,B) over the common universe U denoted (F,A) ∩ (G,B) is denoted as C=A∩B and H(e) = F(e) ∩ G(e) for all e∈C.

Definition: 2.5 ([2],[8],[10])

For a soft set (F,A) over the universe X, the relative complement of (F,A) is denoted by $(F,A)^c$ = (F^c, A), where F^c: A → P(X) is a mapping defined by F^c(e) = X – F(e) for all e∈ A.

Definition: 2.6 ([4],[5])

Let $\tilde{X}$ ∈ S(X). Power soft set of $\tilde{X}$ is defined by

$$\tilde{P}(\tilde{X}) = \{\tilde{X}_i \,\widetilde{\subseteq}\, \tilde{X} : i \in I\}$$

and its cardinality is defined by

$| \tilde{P}\ (\tilde{X}\ ) | = 2^{\sum x \in E |F(x)|}$,where |F(x)| is cardinality of F(x).

Example: 2.7

Let X = $\{u_{1,}, u_{2,} u_3\}$, E = $\{x_{1,}\ x_2\}$ and $\tilde{X} = \{(x_1, \{u_{1,}, u_{2,} u_3\}, x_2\ , \{u_{1,}, u_{2,} u_3\})\}$. Then

$F_{A_1} = \emptyset$,
$F_{A_2} = \{(x_1, \{u_1\})\}$,
$F_{A_3} = \{(x_1, \{u_2\})\}$,
$F_{A_4} = \{(x_1, \{u_3\})\}$,
$F_{A_5} = \{(x_1, \{u_1, u_2\})\}$,
$F_{A_6} = \{(x_1, \{u_2, u_3\})\}$,
$F_{A_7} = \{(x_1, \{u_1, u_3\})\}$,
$F_{A_8} = \{(x_2, \{u_1\})\}$,
$F_{A_9} = \{(x_2, \{u_2\})\}$,
$F_{A_{10}} = \{(x_2, \{u_3\})\}$,
$F_{A_{11}} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_{12}} = \{(x_2, \{u_2, u_3\})\}$,
$F_{A_{13}} = \{(x_2, \{u_1, u_3\})\}$,
$F_{A_{14}} = \{(x_1, \{u_1\}, x_2, \{u_1\})\}$,
$F_{A_{15}} = \{(x_1, \{u_1\}, x_2, \{u_2\})\}$,
$F_{A_{16}} = \{(x_1, \{u_1\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{17}} = \{(x_1, \{u_2\}, x_2, \{u_1\})\}$,
$F_{A_{18}} = \{(x_1, \{u_2\}, x_2, \{u_2\})\}$,
$F_{A_{19}} = \{(x_1, \{u_2\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{20}} = \{(x_1, \{u_3\}, x_2, \{u_1\})\}$,
$F_{A_{21}} = \{(x_1, \{u_3\}, x_2, \{u_2\})\}$,
$F_{A_{22}} = \{(x_1, \{u_3\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{23}} = \{(x_1, \{u_3\}, x_2, \{u_3, u_1\})\}$,
$F_{A_{24}} = \{(x_1, \{u_1\}, x_2, \{u_3\})\}$,
$F_{A_{25}} = \{(x_1, \{u_1\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{26}} = \{(x_1, \{u_2\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{27}} = \{(x_1, \{u_2\}, x_2, \{u_3\})\}$,
$F_{A_{28}} = \{(x_1, \{u_2\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{29}} = \{(x_1, \{u_1\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{30}} = \{(x_1, \{u_3\}, x_2, \{u_3\})\}$,
$F_{A_{31}} = \{(x_1, \{u_3\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{32}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_1\})\}$,
$F_{A_{33}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_2\})\}$,
$F_{A_{34}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_1, u_2\})\}$
$F_{A_{35}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_1\})\}$,
$F_{A_{36}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_2\})\}$,
$F_{A_{37}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{38}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_1\})\}$,
$F_{A_{39}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_2\})\}$,
$F_{A_{40}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{41}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_3\})\}$,
$F_{A_{42}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{43}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_3\})\}$,
$F_{A_{44}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{45}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_3\})\}$,
$F_{A_{46}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{47}} = \{(x_1, \{u_1, u_2, u_3\})\}$,
$F_{A_{48}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_1\})\}$,
$F_{A_{49}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_2\})\}$,
$F_{A_{50}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_1, u_2\})\}$,
$F_{A_{51}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_3\})\}$,
$F_{A_{52}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_2, u_3\})\}$,
$F_{A_{53}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{54}} = \{(x_1, \{u_1\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{55}} = \{(x_1, \{u_2\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{56}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{57}} = \{(x_1, \{u_3\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{58}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{59}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{60}} = \{(x_1, \{u_1, u_3\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{61}} = \{(x_1, \{u_1, u_2\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{62}} = \{(x_1, \{u_2, u_3\}, x_2, \{u_1, u_3\})\}$,
$F_{A_{63}} = \{(x_2, \{u_1, u_2, u_3\})\}$,
$F_{A_{64}} = \{(x_1, \{u_1, u_2, u_3\}, x_2, \{u_1, u_2, u_3\})\} = \tilde{X}$.

$F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, \ldots F_{A_{64}}$ are all soft subsets of $\tilde{X}$. So $| \tilde{P}\ (\tilde{X}\ ) | = 2^6 = 64$.

Definition: 2.8 ([2],[10])

Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following axioms:

1. Φ , $\tilde{X}$ belongs to $\tilde{\tau}$.
2. The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
3. The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet (X, $\tilde{\tau}$, E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X, $\tilde{\tau}$, E) as X throughout work.

Example: 2.9

Let us consider the soft subsets of X that are given in example 2.7 then $\tilde{\tau_1}$ = { Φ, $\tilde{X}$ }, $\tilde{\tau_2}$ = { Φ, $F_{A_2}, F_{A_3}, F_{A_5}$, $\tilde{X}$ }, $\tilde{\tau_3}$ = { $\tilde{X}$ } are soft topologies on X.

Definition : 2.10 ([6],[8])

A set X together with two different topologies is called Bitopological Space. It is denoted by (X, τ_1, τ_2}.

Definition:2.11 ([6],[8])

A subset S of X is called $\widetilde{\tau_{1,2}}$-open if S = H∪ K such that H∈ $\tilde{\tau_1}$ and K∈ $\tilde{\tau_2}$ and the complement of $\widetilde{\tau_{1,2}}$-open set is $\widetilde{\tau_{1,2}}$-closed set.

Example: 2.12 ([6],[8])

Let X = {a,b,c}, $\tilde{\tau_1}$ = { Φ,{a}, X } and $\tilde{\tau_2}$ = { Φ, b, X}, $\widetilde{\tau_{1,2}}$= {Φ,{a},{b},{ab}} are called open set and $\widetilde{\tau_{1,2}}$= {Φ,{bc},{ac},{c}} are called closed set.

Definition: 2.13

Let S be subset of X. Then, (i) The $\widetilde{\tau_{1,2}}$-closure of S, denoted by $\widetilde{\tau_{1,2}}$-cl(S), is defined by $\tilde{\cap}${F :S ⊆F, F is a is $\widetilde{\tau_{1,2}}$-closed set}.

(ii) The $\widetilde{\tau_{1,2}}$-interior of S, denoted by $\widetilde{\tau_{1,2}}$-int(S), is defined by $\tilde{\cup}${A : A ⊆S, A is a $\widetilde{\tau_{1,2}}$-open set}.

Definition: 2.14

A soft subset (A,E) of X is called

I a $\widetilde{(1,2)}$soft generalized closed ($\widetilde{(1,2)}$ soft-g-closed) in a soft bitopological space (X, $\tilde{\tau}$, E) if $\widetilde{\tau_{1,2}}$cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is$\widetilde{(1,2)}$ soft open in X.

II a $\widetilde{(1,2)}$soft semi open if (A,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}(Int(cl(A,E)))$

III a $\widetilde{(1,2)}$regular open if (A,E) = $\widetilde{\tau_{1,2}}(Int(cl(A,E)))$

IV a $\widetilde{(1,2)}$soft α-open if (A,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}(Int(cl(Int(A,E))))$

V a $\widetilde{(1,2)}$soft b-open if (A,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}(cl(Int(A,E)))$ $\tilde{\cup}$ $\widetilde{\tau_{1,2}}(Int(cl(A,E)))$

VI a $\widetilde{(1,2)}$soft pre-open set if (A,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}(Int(cl(A,E)))$

VII a $\widetilde{(1,2)}$soft clopen is (A,E) is both $\widetilde{(1,2)}$soft open and $\widetilde{(1,2)}$soft closed.

The complement of the $\widetilde{(1,2)}$soft semi open, $\widetilde{(1,2)}$soft regular open, $\widetilde{(1,2)}$soft α-open, $\widetilde{(1,2)}$soft b-open, $\widetilde{(1,2)}$soft pre-open sets are their respective $\widetilde{(1,2)}$soft semi closed, $\widetilde{(1,2)}$soft regular closed, $\widetilde{(1,2)}$soft α-closed, $\widetilde{(1,2)}$soft b-closed and $\widetilde{(1,2)}$soft pre-closed sets.

The finite union of $\widetilde{(1,2)}$soft regular open sets is called $\widetilde{(1,2)}$soft open set and its complement is $\widetilde{(1,2)}$soft closed set. The $\widetilde{(1,2)}$soft regular open set of X is denoted by $\widetilde{\tau_{1,2}}$SRO(X) or $\widetilde{\tau_{1,2}}$SRO(X, $\tilde{\tau}$, E).

Definition: 2.15

The $\widetilde{(1,2)}$soft semi closure of (A,E) is the intersection of all $\widetilde{(1,2)}$soft semi closed sets containing (A,E). (i.e) The smallest $\widetilde{(1,2)}$soft semi closed set containing (A,E) and is denoted by $\widetilde{\tau_{1,2}}$sscl(A,E).

Definition: 2.16

The $\widetilde{(1,2)}$soft semi interior of (A,E) is the union of all $\widetilde{(1,2)}$soft semi open set contained in (A,E) and is denoted by $\widetilde{\tau_{1,2}}$ssint(A,E).

Similarly, we define $\widetilde{(1,2)}$soft regular-closure, $\widetilde{(1,2)}$soft α-closure, $\widetilde{(1,2)}$soft pre-closure, $\widetilde{(1,2)}$soft semi closure and $\widetilde{(1,2)}$soft b-closure of the soft set (A,E) of a bitopological space X and are denoted by $\widetilde{\tau_{1,2}}$srcl(A,E), $\widetilde{\tau_{1,2}}$sαcl(A,E), $\widetilde{\tau_{1,2}}$spcl(A,E), $\widetilde{\tau_{1,2}}$sscl(A,E) and $\widetilde{\tau_{1,2}}$sbcl(A,E) respectively.

Definition: 2.17

A subset (A,E) of a soft bitopological space X is called

I a $\widetilde{(1,2)}$soft rg-closed set if $\widetilde{\tau_{1,2}}$cl(A,E)$\tilde{\subseteq}$(U,E) whenever (A,E)$\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$ soft regular open.

II a $\widetilde{(1,2)}$soft αg-closed set if $\widetilde{\tau_{1,2}}$αcl(A,E)$\tilde{\subseteq}$(U,E) whenever (A,E)$\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft open.

Definition: 2.18

A set X together with two different soft topologies is called Soft Bitopological Space. It is denoted by (X, $\tilde{\tau_1}$, $\tilde{\tau_2}$).

Example: 2.19

Let X = $\{u_1, u_2, u_3\}$, E = $\{x_1, x_2\}$ and consider the example in 2.7. Let $\tilde{\tau_1}$={Φ, $F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}$} and

$\tilde{\tau_2}$ = {Φ, $F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}$} be a two different soft topologies.

$\tilde{\tau}$ = $\tilde{\tau_1}$ $\tilde{\cup}$ $\tilde{\tau_2}$= {Φ, $F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}$} are called $\widetilde{(1,2)}$soft open sets and

$\tilde{\tau}^c$ = {Φ, $F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}$} are called $\widetilde{(1,2)}$soft closed sets.

III. $\widetilde{(1,2)}$ SOFT-GSR-CLOSED SETS

Definition: 3.1

A soft subset (A,E) of a soft bitopological space X is called $\widetilde{(1,2)}$soft-gsr-closed set in X if $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever

(A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$ soft regular open in X.

Theorem: 3.2 Every $\widetilde{(1,2)}$soft closed set is $\widetilde{(1,2)}$soft-gsr-closed.

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft closed set in (U,E). Thus $\widetilde{\tau_{1,2}}$cl(A,E) = (A,E). Since $\widetilde{\tau_{1,2}}$ss(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Remark: 3.3 The converse of the above Theorem need not be true as seen from the following example.

Example:3.4

Let (X, $\tilde{\tau_1}$, $\tilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_{1,}, u_{2,} u_3\}$, E = $\{x_{1,} x_2\}$ and consider the soft sets over X in example 2.7, where $\tilde{\tau_1}$={$\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}$}, $\tilde{\tau_2}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}$}. $\widetilde{\tau_{1,2}}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}$} are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ ={$\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}$} are $\widetilde{(1,2)}$soft closed sets. Here, { $F_{A_2}, \dots, F_{A_{42}}, F_{A_{45}}, \dots, F_{A_{50}}, F_{A_{54}}, \dots, F_{A_{57}}, F_{A_{60}}, F_{A_{61}}, F_{A_{63}}$} is a $\widetilde{(1,2)}$ soft gsr-closed but not $\widetilde{(1,2)}$soft closed set.

Theorem: 3.5 Every $\widetilde{(1,2)}$soft g-closed set is $\widetilde{(1,2)}$soft-gsr-closed .

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft g -closed set in (U,E). Thus $\widetilde{\tau_{1,2}}$cl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft open in X. Since $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft g-closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Remark: 3.6 The converse of the above Theorem need not be true as seen from the following example.

Example: 3.7

Let (X, $\tilde{\tau_1}$, $\tilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_{1,}, u_{2,} u_3\}$, E = $\{x_{1,} x_2\}$ and consider the soft sets over X in example 2.7, where $\tilde{\tau_1}$={$\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}$}, $\tilde{\tau_2}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}$}. $\widetilde{\tau_{1,2}}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}$} are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ ={$\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}$} are $\widetilde{(1,2)}$soft closed sets. Here, { $F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}$} is a $\widetilde{(1,2)}$ soft gsr-closed but not $\widetilde{(1,2)}$soft g-closed set.

Theorem: 3.8 Every $\widetilde{(1,2)}$ soft α-closed set is $\widetilde{(1,2)}$ soft-gsr-closed but not conversely.

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft α-closed set in (U,E). Thus $\widetilde{\tau_{1,2}}$ (cl(int(cl(A,E)))) $\tilde{\subseteq}$ (A,E). Since $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft α-closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Example: 3.9

Let (X, $\tilde{\tau_1}$, $\tilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_{1,}, u_{2,} u_3\}$, E = $\{x_{1,} x_2\}$ and consider the soft sets over X in example 2.7, where $\tilde{\tau_1}$={$\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}$}, $\tilde{\tau_2}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}$}. $\widetilde{\tau_{1,2}}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}$} are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ ={$\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}$} are $\widetilde{(1,2)}$soft closed sets. Here, { $F_{A_2}, F_{A_5}, F_{A_7}, F_{A_8}, F_{A_9}, F_{A_{11}}, \dots, F_{A_{26}}, F_{A_{28}}, F_{A_{29}}, F_{A_{41}}, F_{A_{42}}, F_{A_{45}}, \dots, F_{A_{50}}, F_{A_{54}}, \dots, F_{A_{57}}, F_{A_{60}}, F_{A_{61}}, F_{A_{63}}$}is a $\widetilde{(1,2)}$soft gsr-closed but not $\widetilde{(1,2)}$ soft α-closed set.

Theorem: 3.10 Every $\widetilde{(1,2)}$soft semi-closed set is $\widetilde{(1,2)}$soft-gsr-closed but not conversely.

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft semi-closed set in (U,E). Thus $\widetilde{\tau_{1,2}}$ (int(cl(A,E))) $\tilde{\subseteq}$ (A,E). Since $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft semi-closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Example: 3.11

Let (X, $\tilde{\tau_1}$, $\tilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_{1,}, u_{2,} u_3\}$, E = $\{x_{1,} x_2\}$ and consider the soft sets over X in example 2.7, where $\tilde{\tau_1}$={$\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}$}, $\tilde{\tau_2}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}$}. $\widetilde{\tau_{1,2}}$ = {$\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}$} are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ ={$\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}$} are $\widetilde{(1,2)}$soft closed sets. Here, { $F_{A_{16}}, F_{A_{34}}, F_{A_{50}}, F_{A_{54}}, F_{A_{56}}, F_{A_{59}}, F_{A_{60}}$} is a $\widetilde{(1,2)}$soft gsr-closed but not $\widetilde{(1,2)}$ soft semi-closed set.

Theorem: 3.12 Every $\widetilde{(1,2)}$soft αg-closed set is $\widetilde{(1,2)}$soft-gsr-closed .

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft αg -closed set in (U,E). Thus $\alpha\,\widetilde{\tau_{1,2}}$(cl(A,E)) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft open in X. . Since $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft αg-closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Remark: 3.13 The converse of the above Theorem need not be true as seen from the following example.

Example: 3.14

Let (X, $\widetilde{\tau_1}$, $\widetilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_1, u_2, u_3\}$, E = $\{x_1, x_2\}$ and consider the soft sets over X in example 2.7, where $\widetilde{\tau_1}$=$\{\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}\}$, $\widetilde{\tau_2}$ = $\{\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}\}$. $\widetilde{\tau_{1,2}}$ = $\{\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}\}$ are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ =$\{\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}\}$ are $\widetilde{(1,2)}$soft closed sets. Here, $\{F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}\}$ is a $\widetilde{(1,2)}$soft gsr-closed but not $\widetilde{(1,2)}$ soft αg -closed set.

Theorem: 3.15 Every $\widetilde{(1,2)}$soft rg-closed set is $\widetilde{(1,2)}$soft-gsr-closed .

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft rg -closed set in (U,E). Thus $\widetilde{\tau_{1,2}}$ (cl(A,E)) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open. Since $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(U,E) whenever (A,E) $\tilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Then (A,E) is $\widetilde{(1,2)}$soft-gsr-closed. Hence Every $\widetilde{(1,2)}$soft rg-closed set is $\widetilde{(1,2)}$ soft-gsr-closed.

Remark: 3.16 The converse of the above Theorem need not be true as seen from the following example.

Example: 3.17

Let (X, $\widetilde{\tau_1}$, $\widetilde{\tau_2}$) be a soft bitopological space over X. Let X = $\{u_1, u_2, u_3\}$, E = $\{x_1, x_2\}$ and consider the soft sets over X in example 2.7, where $\widetilde{\tau_1}$=$\{\Phi, F_{A_8}, F_{A_9}, F_{A_{11}}, \tilde{X}\}$, $\widetilde{\tau_2}$ = $\{\Phi, F_{A_2}, F_{A_8}, F_{A_{14}}, \tilde{X}\}$. $\widetilde{\tau_{1,2}}$ = $\{\Phi, F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}, \tilde{X}\}$ are $\widetilde{(1,2)}$soft open sets and $\widetilde{\tau_{1,2}}$ =$\{\Phi, F_{A_{43}}, F_{A_{44}}, F_{A_{51}}, F_{A_{52}}, F_{A_{53}}, F_{A_{58}}, F_{A_{62}}, \tilde{X}\}$ are $\widetilde{(1,2)}$soft closed sets. Here, $\{F_{A_2}, F_{A_8}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_{A_{15}}\}$ is a $\widetilde{(1,2)}$soft gsr-closed but not $\widetilde{(1,2)}$ soft rg -closed set.

Remark: 3.18 From the above result the following implication is made:

$\widetilde{(1,2)}$Soft- α-closed ↘   ↙ **$\widetilde{(1,2)}$Soft-αg-closed**

$\widetilde{(1,2)}$Soft-closed → **$\widetilde{(1,2)}$Soft-gsr-closed** ← **$\widetilde{(1,2)}$Soft-g-closed**

$\widetilde{(1,2)}$Soft-semi-closed ↗   ↖ **$\widetilde{(1,2)}$Soft-rg-closed**

Figure.1

Theorem: 3.19 If (A,E) is $\widetilde{(1,2)}$soft regular open and $\widetilde{(1,2)}$soft-gsr-closed then (A,E) is $\widetilde{(1,2)}$soft semi-closed.

Proof: Assume (A,E) is $\widetilde{(1,2)}$soft regular open and $\widetilde{(1,2)}$soft-gsr-closed. Then by definition of $\widetilde{(1,2)}$soft gsr-closed sets, $\widetilde{\tau_{1,2}}$sscl(A,E) $\tilde{\subseteq}$(A,E). But (A,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sscl(A,E). Thus (A,E) = $\widetilde{\tau_{1,2}}$sscl(A,E). Hence (A,E) is $\widetilde{(1,2)}$soft semi closed.

Theorem: 3.20 If (A,E) is $\widetilde{(1,2)}$soft regular open and $\widetilde{(1,2)}$soft-gsr-open then (A,E) is $\widetilde{(1,2)}$soft semi-closed.

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft regular open and $\widetilde{(1,2)}$soft-gsr-open. Then by definition of $\widetilde{(1,2)}$soft gsr-open sets, $\widetilde{\tau_{1,2}}$ssint(A,E) $\tilde{\supseteq}$(A,E). But (A,E) $\tilde{\supseteq}$ $\widetilde{\tau_{1,2}}$ssint(A,E). Thus (A,E) = $\widetilde{\tau_{1,2}}$ssint(A,E). Hence (A,E) is $\widetilde{(1,2)}$soft semi closed.

Theorem: 3.21 If (A,E) is $\widetilde{(1,2)}$soft-gsr-closed in X and (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sscl(A,E). Then (B,E) is also $\widetilde{(1,2)}$soft-gsr-closed.

Proof: Assume (A,E) is $\widetilde{(1,2)}$soft-gsr-closed in X and (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sscl(A,E) and let (B,E) $\tilde{\subseteq}$ (U,E) and (U,E) be $\widetilde{(1,2)}$soft regular open in X. Since (A,E) $\tilde{\subseteq}$ (B,E) and (B,E) $\tilde{\subseteq}$ (U,E), we have (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is $\widetilde{(1,2)}$soft-gsr-closed,

$\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\subseteq}$(U,E). Since (B,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sscl(A,E), we have $\widetilde{\tau_{1,2}}$sscl(B,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\subseteq}$ (U,E). Hence (B,E) is $\widetilde{(1,2)}$soft-gsr-closed.

Theorem: 3.22 If (A,E) and (B,E) are $\widetilde{(1,2)}$ soft-gsr-closed sets then (A,E) $\widetilde{\cup}$ (B,E) is also a $\widetilde{(1,2)}$soft-gsr-closed.

Proof: Assume that (A,E) and (B,E) are $\widetilde{(1,2)}$soft-gsr-closed sets in X. Let (U,E) be a $\widetilde{(1,2)}$soft open set in X such that ((A,E) $\widetilde{\cup}$ (B,E)) $\widetilde{\subseteq}$ (U,E). Then (A,E) $\widetilde{\subseteq}$(U,E) and (B,E) $\widetilde{\subseteq}$(U,E). Since (A,E) and (B,E) are $\widetilde{(1,2)}$soft-gsr-closed sets, $\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\subseteq}$(U,E) and $\widetilde{\tau_{1,2}}$sscl(B,E) $\widetilde{\subseteq}$(U,E) whenever (A,E) $\widetilde{\subseteq}$(U,E) and (B,E) $\widetilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Hence $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cup}$ (B,E)) = ($\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cup}$ $\widetilde{\tau_{1,2}}$sscl((B,E)) $\widetilde{\subseteq}$(U,E). That $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cup}$ (B,E)) $\widetilde{\subseteq}$(U,E) whenever ((A,E) $\widetilde{\cup}$ (B,E)) $\widetilde{\subseteq}$ (U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Therefore (A,E) $\widetilde{\cup}$ (B,E) is also a $\widetilde{(1,2)}$soft-gsr-closed sets in X.

Theorem: 3.23 The intersection of a $\widetilde{(1,2)}$soft-gsr-closed set and a $\widetilde{(1,2)}$soft closed set is a $\widetilde{(1,2)}$soft-gsr-closed.

Proof: Let (A,E) be a $\widetilde{(1,2)}$soft-gsr-closed set and (B,E) be a $\widetilde{(1,2)}$soft closed set. Since (A,E) is a $\widetilde{(1,2)}$soft-gsr-closed set, $\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\subseteq}$(U,E) whenever (A,E) $\widetilde{\subseteq}$(U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. To show that (A,E) $\widetilde{\cap}$ (B,E) is $\widetilde{(1,2)}$soft-gsr-closed set, it is enough to show that $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$(U,E), whenever ((A,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$(U,E), where (U,E) is $\widetilde{(1,2)}$soft regular open set. Let (H,E) = X-(B,E) then (A,E) $\widetilde{\subset}$ ((U,E) $\widetilde{\cup}$ (H,E)). Since (H,E) is $\widetilde{(1,2)}$soft regular open set. (U,E) $\widetilde{\cup}$ (H,E) is a $\widetilde{(1,2)}$soft regular open and (A,E) is a $\widetilde{(1,2)}$ soft-gsr-closed set, $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\subseteq}$ ((U,E) $\widetilde{\cup}$ (H,E)). Now, $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$(sscl(A,E) $\widetilde{\cap}$ sscl(B,E)) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$ sscl((A,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$(((U,E) $\widetilde{\cup}$ (H,E)) $\widetilde{\cap}$(B,E)) $\widetilde{\subseteq}$ ((U,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\cup}$ ((H,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$ ((U,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\cup}$ Φ = (U,E). This implies that ((A,E) $\widetilde{\cap}$ (B,E)) is a $\widetilde{(1,2)}$ soft-gsr-closed set.

IV. $\widetilde{(1,2)}$SOFT GSR-OPEN SETS

Definition: 4.1 Let (X, E, $\tilde{\tau}$) be a soft bitopological space over X. A soft set (A,E) is called $\widetilde{(1,2)}$soft gsr-open set in X if the relative complement $(A,E)^c$ is $\widetilde{(1,2)}$soft-gsr-closed.

Theorem: 4.2 A soft set (A,E) is $\widetilde{(1,2)}$soft gsr-open set in a soft bitopological space X if and only if (H,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$ssint(A,E) whenever (H,E) is $\widetilde{(1,2)}$soft regular closed in X and (H,E) $\widetilde{\subseteq}$ (A,E).

Proof: Assume (H,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$ssint(A,E) whenever (H,E) is $\widetilde{(1,2)}$soft regular closed in X and (H,E) $\widetilde{\subseteq}$ (A,E). Let $(A,E)^c$ $\widetilde{\subseteq}$ (U,E),where (U,E) is $\widetilde{(1,2)}$soft regular open. Then $(U,E)^c$ $\widetilde{\subseteq}$ (A,E) where $(U,E)^c$ is $\widetilde{(1,2)}$soft regular closed. By assumption, $(U,E)^c$ $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$ssint(A,E). That is ($\widetilde{\tau_{1,2}}$ssint$(A,E)^c$) $\widetilde{\subseteq}$ (U,E). Similarly, ($\widetilde{\tau_{1,2}}$sscl$(A,E)^c$) $\widetilde{\subseteq}$ (U,E). Therefore $(A,E)^c$ is $\widetilde{(1,2)}$soft-gsr-closed. Hence (A,E) is $\widetilde{(1,2)}$soft-gsr-open.

Conversely, Assume (A,E) is $\widetilde{(1,2)}$soft gsr-open, (H,E) $\widetilde{\subseteq}$ (A,E) and (H,E) is $\widetilde{(1,2)}$soft regular closed. Then $(H,E)^c$ is $\widetilde{(1,2)}$soft regular open. Then $(A,E)^c$ $\widetilde{\subseteq}$ $(H,E)^c$. Since $(A,E)^c$ is $\widetilde{(1,2)}$soft-gsr-closed. Hence we have ($\widetilde{\tau_{1,2}}$sscl$(A,E)^c$) $\widetilde{\subseteq}$ $(H,E)^c$. Hence (H,E) $\widetilde{\subseteq}$ ($\widetilde{\tau_{1,2}}$sscl$(A,E)^c)^c$= $\widetilde{\tau_{1,2}}$ssint(A,E).

Theorem: 4.3 If (A,E) is $\widetilde{(1,2)}$ soft-gsr-open in X and $\widetilde{\tau_{1,2}}$sint(A,E) $\widetilde{\subseteq}$ (B,E) $\widetilde{\subseteq}$ (A,E) then (B,E) is $\widetilde{(1,2)}$soft-gsr-open set.

Proof: Assume (A,E) is $\widetilde{(1,2)}$soft-gsr-open in X and $\widetilde{\tau_{1,2}}$sint(A,E) $\widetilde{\subseteq}$ (B,E) $\widetilde{\subseteq}$ (A,E). Let (H,E) $\widetilde{\subseteq}$ (B,E) and (H,E) is $\widetilde{(1,2)}$soft regular closed in X. Since (B,E) $\widetilde{\subseteq}$ (A,E) and (H,E) $\widetilde{\subseteq}$ (B,E), we have (H,E) $\widetilde{\subseteq}$ (A,E). Hence (H,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sint(A,E). Since (A,E) is $\widetilde{(1,2)}$soft gsr-open and $\widetilde{\tau_{1,2}}$sint(A,E) $\widetilde{\subseteq}$ (B,E), we have (H,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sint(A,E) $\widetilde{\subseteq}$ $\widetilde{\tau_{1,2}}$sint(B,E). Hence (B,E) is $\widetilde{(1,2)}$soft-gsr-open.

Theorem: 4.4 The intersection of two $\widetilde{(1,2)}$soft-gsr-open sets is again a $\widetilde{(1,2)}$soft-gsr-open set.

Proof: Suppose (A,E) and (B,E) are $\widetilde{(1,2)}$soft-gsr-closed sets in (X, $\tilde{\tau}$, E). Then $\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\subseteq}$(U,E) and $\widetilde{\tau_{1,2}}$sscl(B,E) $\widetilde{\subseteq}$(U,E) whenever (A,E) $\widetilde{\subseteq}$ (U,E) and (B,E) $\widetilde{\subseteq}$ (U,E) and (U,E) is $\widetilde{(1,2)}$soft regular open in X. Hence $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cap}$ (B,E)) = ($\widetilde{\tau_{1,2}}$sscl(A,E) $\widetilde{\cap}$ $\widetilde{\tau_{1,2}}$sscl(B,E)) $\widetilde{\subseteq}$ (U,E). which implies $\widetilde{\tau_{1,2}}$sscl((A,E) $\widetilde{\cap}$ (B,E)) $\widetilde{\subseteq}$ (U,E). Thus (A,E) $\widetilde{\cap}$ (B,E) is $\widetilde{(1,2)}$soft gsr-closed set in (X, $\tilde{\tau}$, E).

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