Effect of Geometry Factor I & J Factor Multipliers in the performance of Helical Gears

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Abstract— Gears are the prime device which is used to transfer the motion & power. Due to many positive properties of Gear drive, it placed at the top place in the stream of power transmission in mechanical engineering. In this paper first of all the introduction of Helical gear is mentioned for the reference of the reader & after that Kinematics of the Gear tooth formation profiles briefly stated. Then it will be shifted to the core topic of this paper & that is Geometry Factor & J Factor Multipliers.

Keywords—Helical Gear Kinematics, Geometry Factor I & J Factor Multipliers.

I. HELICAL GEARS – AN INTRODUCTION

The Gear is the prime device used to transmit the motion & power to the subsequent device from the parent device. The main property of the Gear drive is the transmission of motion & power with positive pattern, means with negligible slippage & variable velocity ratio.

In spur gears the axis of the teeth are parallel to the axis of wheel whereas in helical gears Fig.2 the teeth are inclined to the axis. Both the gears are transmitting power between two parallel shafts.

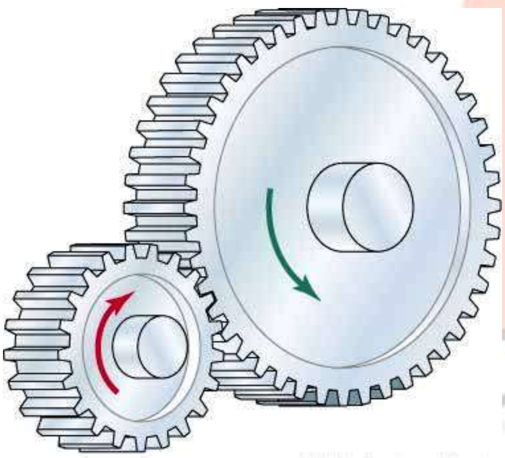


Fig.1 Spur gear

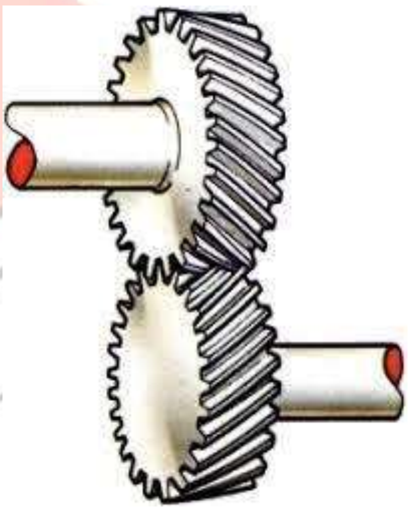


Fig.2 Helical gear

Helical gears are also one type of Spur Gear or it can be thought of as an ordinary spur gear which machined from a stack of thin shim stock. In Spur Gear all the teeth will be meshed with neighbor at an instance where as in Helical Gear the contact is uniformly applied on the surface but the limitation of which is rotated slightly with respect to its neighbors as in Fig.3. When power is transmitted both shafts are subjected to thrust load on the shaft.

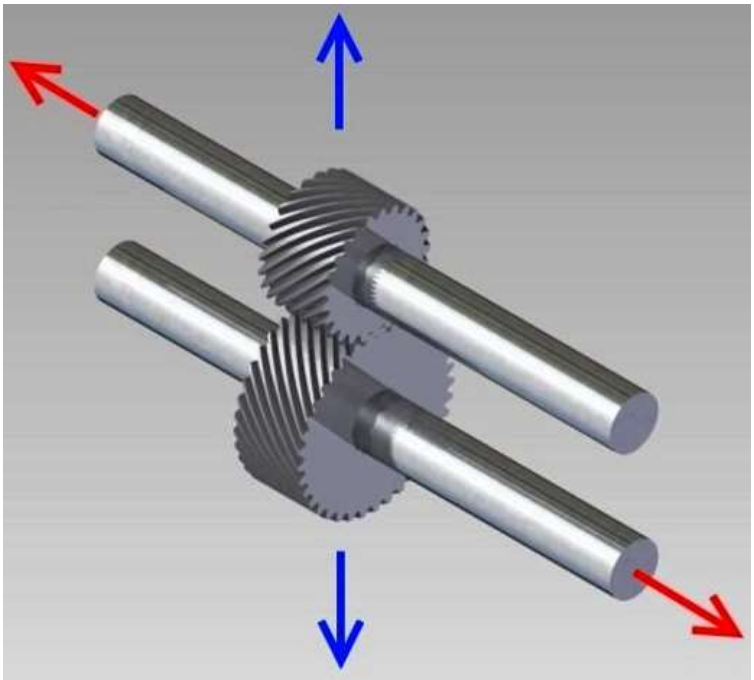


Fig.3 Illustration of concept of helical gear

Fig.4 Double helical gear or herringbone gear

Herringbone or double helical gear shown in Fig.4 is the solution of this Thrust Load generation. Here two helical gears with opposing helix angle stacked together. As a result, two opposing thrust loads generated which can cancel both the Thrust Loads and the shafts are not acted upon by any thrust load.

But the advantages of elimination of thrust load in Herringbone gears, is not much effective due to considerably higher mounting, manufacturing and machining costs. This limits their applications to very heavy power transmission.

Fig.5 Double helical gear of a cement mill rotary gear drive

Fig.6 Crossed helical gears.

Crossed helical gears As in Fig.6 are used for transmitting power between two non-parallel, non-intersecting shafts. Common application is distributor and pump drive from cam shafts in automotive engines.

II. HELICAL GEARS- KINEMATICS

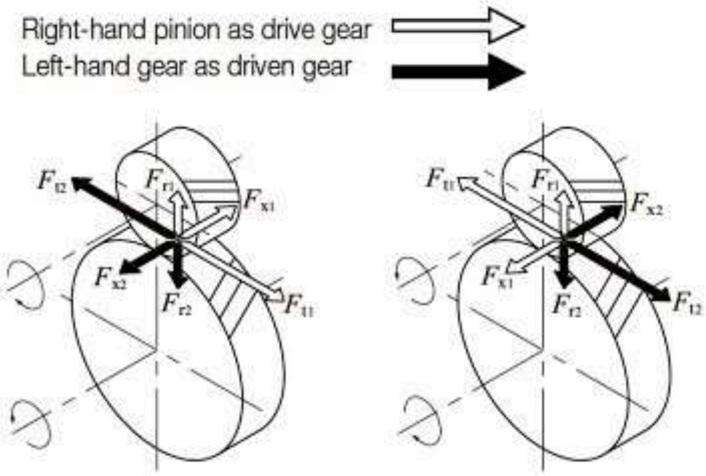


Fig.7 Helical gear

When two helical gears are engaged as in the Fig.7, the helix angle has to be the same on both gear, but one gear must have a right-hand helix and the other a left-hand helix.

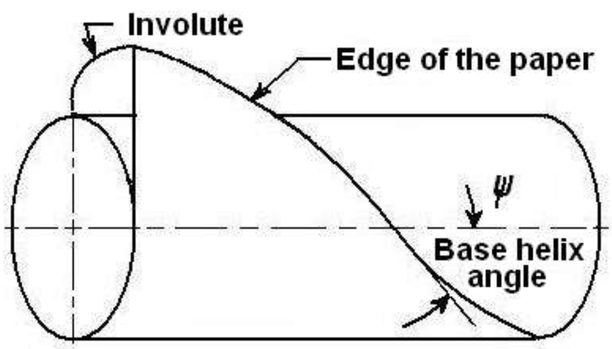
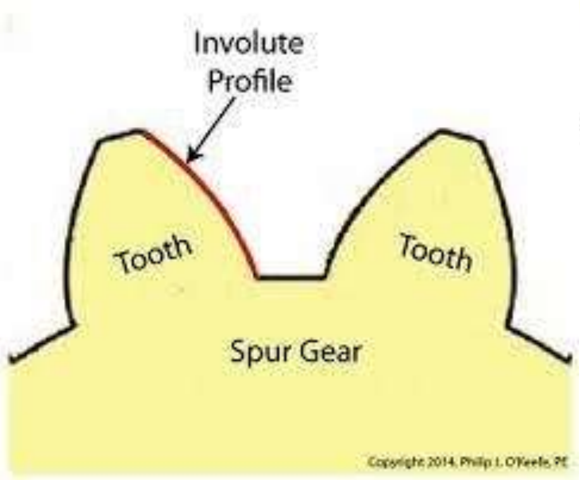
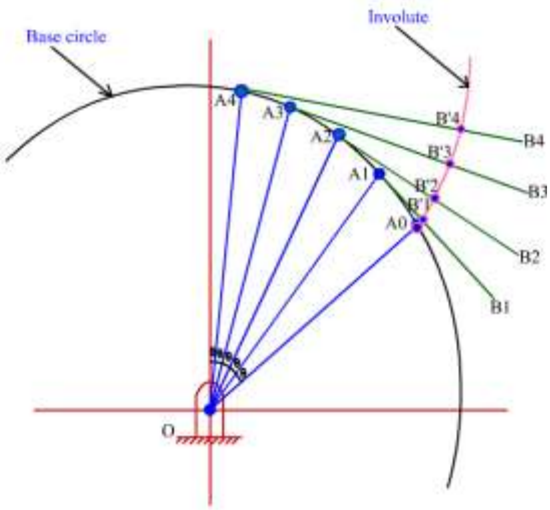


Fig.8 Illustration of helical gear tooth formation





The shape of the tooth is having a special profile. It is just same as involute helicoid as illustrated in the Fig.8. If a paper piece of the shape of a parallelogram of any angle is wrapped around a cylinder, the angular edge of the paper becomes the helix. If the paper is unwound, each point on the angular edge generates an Involute curve. The surface achieved when every point on the edge generates an Involute is called Involute helicoid.

In spur gear, the meshing depicts by initial contact line extends all the way across the tooth face. The initial contact of helical gear teeth is point which changes into a line as the teeth come into more engagement. In spur gears the line contact is parallel to the axis of rotation of wheel whereas in helical gear the line is diagonal across the face of the tooth. Due to this gradual engagement of the teeth, the load transfers smoothly from one tooth to another. This gradual engagement makes the gear operation smoother and quieter than with spur gears. The result is Lower Dynamic Factor K_v. Thus, it can transmit comparative heavy loads at high speeds. Typical usage is automotive transmission for compact and quiet drive.

III. HELICAL GEARS – GEOMETRY AND NOMENCLATURE

The Helix Angle ψ is the prime property to describe any Helical Gear. It always measured on the Cylindrical Pitch Surface Fig.8. In this paper the detailed properties of the Helical Gear is not discussed but some properties are taken as the reader is already going through the fundaments. The Helix Angle ψ value is not standardized. It ranges between 15° and 45°. Commonly used values are 15°, 16°, 23°, 30° or 45°. Above 45° is not recommended as a Helix Angle. The value of the Angle is having direct effects on the End Thrust Load generated. Lower values of the Helix Angle give less End Thrust but at a same time one must consider that Higher values result in Smoother Operation but more End Thrust.

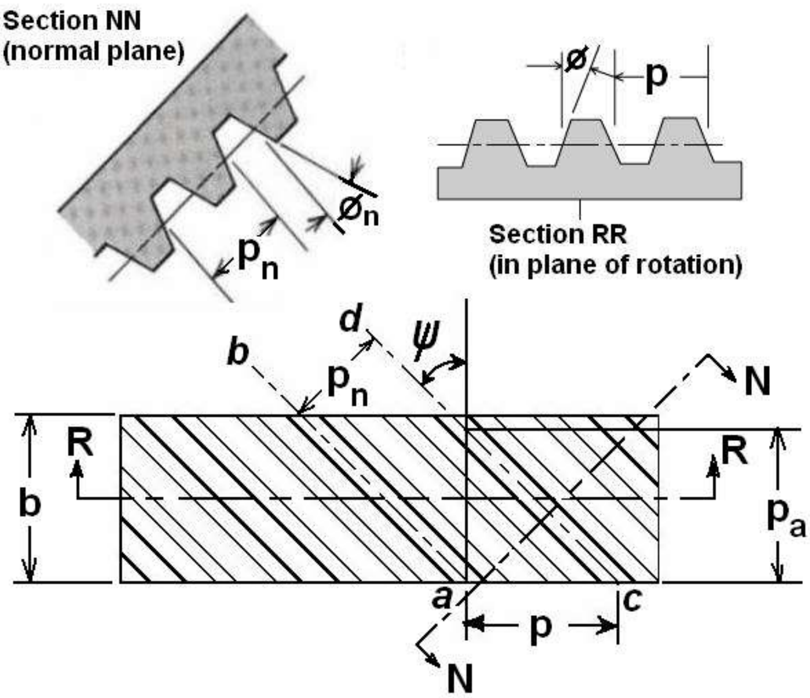


Fig.9 Portion of helical rack

The circular pitch (p) and pressure angle (ϕ) are measured in the plane of rotation, as in spur gears. These quantities in normal plane are denoted by suffix n (p_n, ϕ_n) as shown in Fig.9.

From geometry we have normal pitch as $\mathbf{p_n = p \cos \psi}$

Normal module m_n is $\mathbf{m_n = m \cos \psi}$

m_n is used for hob selection.

The pitch diameter (d) of the helical gear is:

$$\mathbf{d = Z\, m = Z\, \frac{m_n}{\cos \psi}}$$

The axial pitch (pa) is:

$$\mathbf{p_a = \frac{p}{\tan \psi}}$$

For axial overlap of adjacent teeth, $\mathbf{b \geq p_a}$

In practice b = (1.15 ~2) pa is used

The relation between normal and transverse pressure angles is

$$\mathbf{\tan \phi_n = \tan \phi \,.\cos \psi}$$

In the case of helical gear, the resultant load between mating teeth is always perpendicular to the tooth surface. Hence bending stresses are computed in the normal plane, and the strength of the tooth as a cantilever beam depends on its profile in the normal plane. Fig.10 shows the view of helical gear in normal and transverse plane.

The following figure shows the pitch cylinder and one tooth of a helical gear. The normal plane intersects the pitch cylinder in an ellipse.

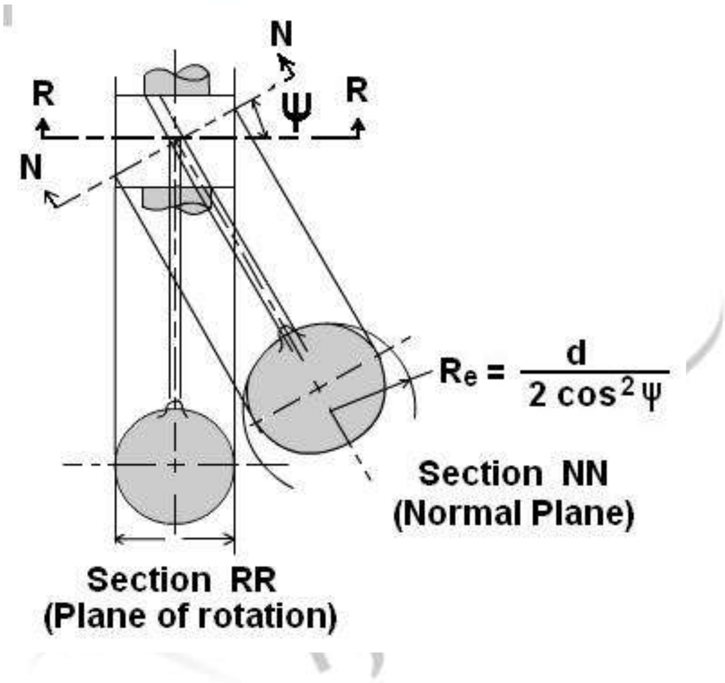


Fig.10 View of helical gear in normal and transverse sections

The shape of the tooth in the normal plane is nearly the same as the shape of a spur gear tooth having a pitch radius equal to radius R_e of the ellipse.

$$\mathbf{Re} = \frac{d}{(2cos^2\psi)}$$

The equivalent number of teeth (also called virtual number of teeth), Z_v, is defined as the number of teeth in a gear of radius R_e:

$$\mathbf{Z_v} = \frac{2R_e}{\mathbf{m_n}} = \frac{d}{(m_n cos^2\psi)}$$

Substituting $m_n = m \cos\psi$, and $d = Z\, m$

$$\mathbf{Z_v} = \frac{Z}{cos^3\psi}$$

When we compute the bending strength of helical teeth, values of the Lewis form factor Y are the same as for spur gears having the same number of teeth as the virtual number of teeth (Z_v) in the Helical gear and a pressure angle equal to ϕ_n.

IV. HELICAL GEARS – GEOMETRY FACTOR I AND J (Z_I AND Y_J)

The AGMA factors I and J are intended to accomplish the effect of tooth form into the stress equation in a more involved manner. The determination of I and J depends upon the face-contact ratio mF . This is defined as

$$m_F = \frac{F}{p_x}$$

where p_x is the axial pitch and F is the face width. For spur gears, m_F = 0.

Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ($m_F \leq 1$), and will not be considered here. Such gears have a noise level not too different from that for spur gears. Consequently we shall consider here only spur gears with $m_F = 0$ and conventional helical gears with $m_F > 1$.

V. BENDING-STRENGTH GEOMETRY FACTOR J (Y_J)

The AGMA factor J employs a modified value of the Lewis form factor, also denoted by Y; a fatigue stress-concentration factor K_f ; and a tooth load-sharing ratio m_N . The resulting equation for J for spur and helical gears is J

$$J = \frac{Y}{K_f m_N} \text{.................Equ.1}$$

It is important to note that the form factor Y in Equation 1 is not the Lewis factor at all. The value of Y here is obtained from calculations within AGMA 908-B89, and is often based on the highest point of single-tooth contact.

The factor K_f in Eq.1 is called a stress correction factor by AGMA. It is based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 50 years ago. The load-sharing ratio m_N is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio m_p, the face-contact ratio m_F , the effects of any profile modifications, and the tooth deflection. For spur gears, m_N = 1.0. For helical gears having a face-contact ratio $m_F > 2.0$, a conservative approximation is given by the equation

$$m_N = \frac{p_N}{0.95\, Z} \text{.................Equ.2}$$

where p_N is the normal base pitch and Z is the length of the line of action in the transverse plane.

Figs. 11 and 12 for helical gears having a 20◦ normal pressure angle and face-contact ratios of m_F = 2 or greater. For other gears, consult the AGMA standard.

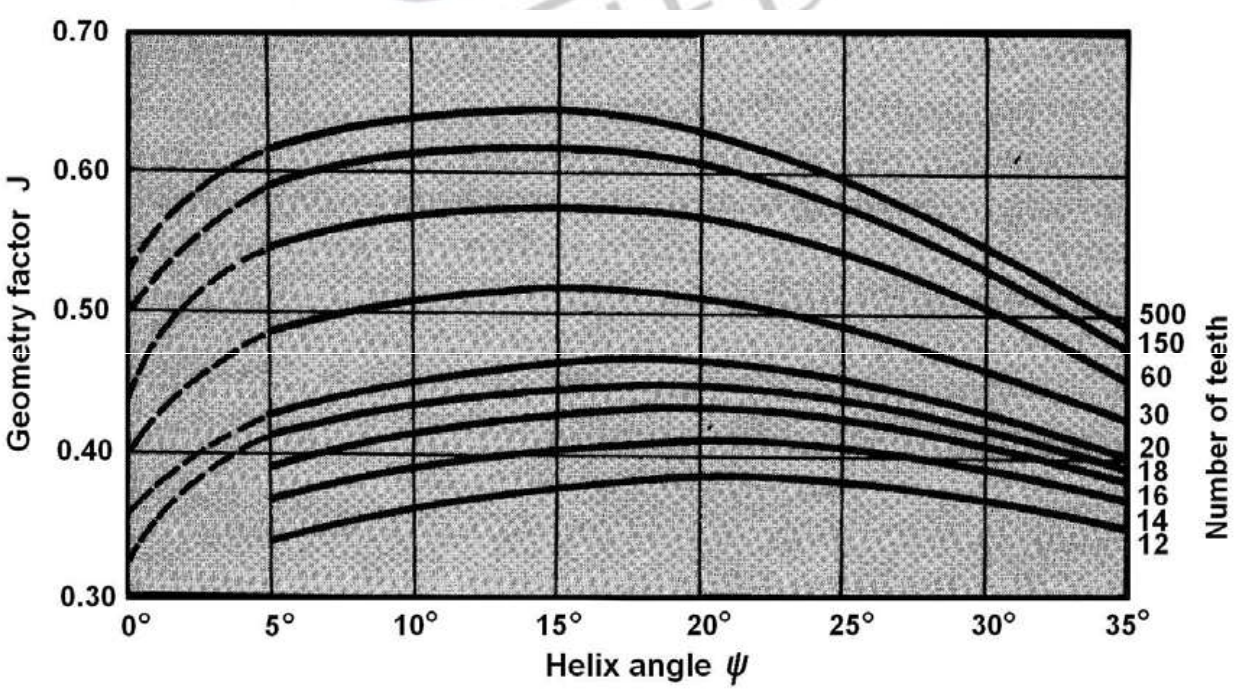


Fig.11 Helical-gear geometry factors J _. Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

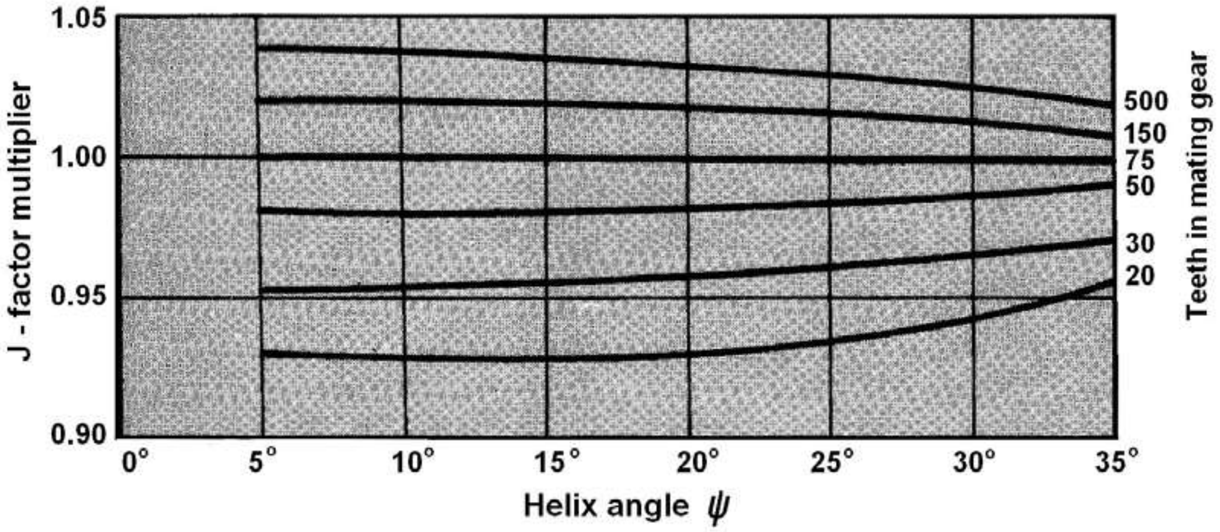


Fig.12 J factor multipliers for use with Fig.11 to find J. Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

VI. SURFACE-STRENGTH GEOMETRY FACTOR I (Z_I)

As it is noted that r_1 and r_2 are the instantaneous values of the radii of curvature on the pinion and gear tooth profiles respectively at the point of contact. By accounting for load sharing surface compressive stress can be solved for the Hertzian stress for any or all tooth contact points from the beginning to the end. Of course, pure rolling exists only at the pitch point. Elsewhere the motion is a mixture of rolling and sliding. surface compressive stress Equation does not account for any sliding action in the evaluation of stress. We note that AGMA uses μ for Poisson's ratio instead of ν as is used here. We have already noted that the first evidence of wear occurs near the pitch line. The radii of curvature of the tooth profiles at the pitch point are

$$r_1 = \frac{d_P \sin \emptyset}{2} \quad \& \quad r_2 = \frac{d_G \sin \emptyset}{2} \text{.................Equ.3}$$

According to AGMA, the factor I is also called the pitting-resistance geometry factor. The expression for I by noting that the sum of the reciprocals of Eq. 3 can be expressed as

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin \emptyset_t}\left(\frac{1}{d_P} + \frac{1}{d_G}\right) \text{.................Equ.4}$$

Here $\emptyset$ is replaced by $\emptyset_t$ the transverse pressure angle, so that the relation will apply to helical gears too. Now define speed ratio m_G as

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \text{.................Equ.5}$$

With the reference of Fig.12 the Equ.5 can now be written

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_P \sin \emptyset_t} \frac{m_G + 1}{m_G} \text{.................Equ.6}$$

Now substitute Equ.6 for the sum of the reciprocals in surface compressive stress. As it is compressive, the -ve sign is applied.

$$\sigma_c = -\sigma_c = C_p \left[\frac{K_V W^t}{d_P F} \frac{1}{\frac{\sin \emptyset_t \cos \emptyset_t}{2} \frac{m_G}{m_G + 1}} \right] \text{.................Equ.7}$$

The geometry factor I for external spur and helical gears is the denominator of the second term in the brackets in Equ.7. By adding the load-sharing ratio m_N, we obtain a factor valid for both spur and helical gears. The equation is then written as

$$I = \frac{\sin \emptyset_t \cos \emptyset_t}{2m_N} \frac{m_G}{m_G+1} \quad \text{for External Gear}$$

$$I = \frac{\sin \emptyset_t \cos \emptyset_t}{2m_N} \frac{m_G}{m_G-1} \quad \text{for Internal Gear}$$

By this the reader can understand the basic criteria of the factor I & J. To understand overall Gear geometry & also for the detailed analysis of the Gear with its meshing one must understand many more other factors also which can be detailed in the further papers.

VII. REFERENCES

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