

# A Linear Programming Approach For Optimal Relay Coordination Of Numerical Overcurrent Relay

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**Abstract:-** The transmission line fault results in a much larger postfault current than the prefault load current. So overcurrent relaying is widely used for transmission lines. The speed of relaying and selectivity can be sacrificed, to some extent, in LV or MV lines. The other line relays like distance protection relays are too expensive. The percentage cost of the protection, which can be justified is around 10% to 15% of the cost of the equipment or section to be protected. Hence directional overcurrent relays are commonly used as an economic alternative for the protection of interconnected sub transmission systems, distribution systems, or as a secondary protection of transmission systems. Nowadays, "Numerical Overcurrent relays" are widely used. So we have considered numerical relay in our case-study. The most important task when installing numerical overcurrent relay on the system is selecting their suitable settings such that their basic protection function is met under sensitivity, selectivity, reliability and speed. The settings should be designed for minimum relay time of operation. That is, the settings should be such that the overall time of operation is minimized while maintaining the selectivity and reliability. Thus, the numerical relay problem is an optimization problem, where the solution is the optimal settings of each relay. Various methods have been proposed to formulate and find the optimal solution for the numerical overcurrent relay problem. In, this paper the proposed techniques are discussed. In this paper the data taken for research is of the real field of The power system at TATA Motors Ltd, Pimpri. The problem is formulated as linear programming problem as a basic case study. The problem is solved using linprog function of the optimization toolbox of the MATLAB software.

**Keywords:-** Linear Programming, Numerical Overcurrent Relay, Power System Protection, Relay Coordination

## I. INTRODUCTION

The low voltage line is basically a radial line. The problem of transient stability does not exist for a radial line. However, the problems of power quality and voltage dip have become more significant nowadays for the LV and MV lines. So if the operation of the relay takes more time then the voltage dip caused by the fault persists for a longer time. This is critical for industrial and urban loads and especially for metros. This motivates to find out solutions to this problem in various ways.

## II. LINEAR PROBLEM FORMULATION USING A NUMERICAL RELAY

In the coordination problem, aim is to calculate the TDS and  $I_p$ , which would minimize the time of operation of the relays. The coordination problem of numerical relays can be stated as follows:

$$\text{Objective} = \min \sum W_i \times T_{ik} \quad (1)$$

Where  $T_{ik}$  indicates the operation time of the relay  $R_i$  for a fault in zone  $k$  and  $W_i$  is a coefficient and is usually set to 1. The objective is to minimize the time of operation of the relays under the following constraints.

### Coordination Criteria

$$T_{nk} - T_{ik} \geq \Delta T \quad (2)$$

where  $T_{nk}$  is the operation time of the first backup of  $R_i$  for a given fault in protection zone  $k$ .  $\Delta T$  is the coordination time interval and is taken to be 0.05 seconds. The coordination time interval is the minimum interval that permits the backup relay to clear the fault in its operating zone.

### B. Bound On Relay Settings And Operation Times

$$TDS_{imin} \leq TDS_i \leq TDS_{imax} \quad (3)$$

$$I_{pimin} \leq I_{pi} \leq I_{pimax} \quad (4)$$

$$T_{ikmin} \leq T_{ik} \leq T_{ikmax} \quad (5)$$

where  $TDS_i$  is the time dial setting and  $I_{pi}$  is the pickup current. The actual fault currents are generally much more higher than the normal load currents. So no need to keep pickup current between 1 to 2 times load current. Also sometimes for the double circuit lines, in case of outage of one line, the other line has to carry twice the load current to maintain the sufficient supply to the remote bus. The minimum pickup current is set equal to twice the normal load current. The maximum pickup current is chosen

such that it is less than the minimum fault current that passes through the relay. As the relay has to also protect for next zone fault as a backup relay, the minimum fault current will be for a fault at the end of the next section to be protected.

**C. Relay Characteristics**

All relays are assumed identical and with characteristic functions approximated as:

$$T_{ik} = \frac{[0.14 * TDS_i]}{[(I_{ik}/I_p)^{0.02} - 1]} \quad (6)$$

where  $I_{ik}$  is the fault current passing through the relay for a fault in zone  $k$ . As we can see from the equation (6) that the non-linearity comes from pickup current term. For LP formulation, the pickup current is assumed to be known and as a result eq. (6) now takes the form as:

$$T_{ik} = P_{ik} \times TDS \quad (7)$$

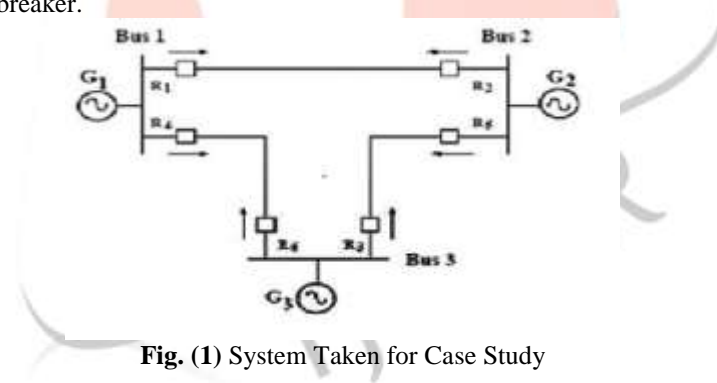
where  $P_{ik}$  is a constant depending on the values of  $I_p$  and fault current.

**III. A BASIC CASE STUDY USING LINEAR PROGRAMMING**

This example is solved to understand the relay coordination aspects for a network. The linear programming approach explained in section II is implemented here. The network is as shown in figure 2 below.

In this case, each protection zone corresponds with one of the transmission lines. As we know the fault immediately after the breaker will have the maximum fault current. So now as per the fault occurs in the network considered the achieved minimum time discrimination of 0.05 seconds. If TDS is based on this calculation, then it will be true for all other fault locations that a minimum time interval of 0.05 seconds is always provided. This will ensure proper coordination as well as selectivity. The system data and the pickup current values are provided in the Table I. The minimum allowed TDS is taken as 0.01 and upper bound is taken as 1.1 in steps of 0.01. The objective function weights  $W_{O1}$  shown in eq.1 were all set equal to one because the lines are short and their lengths are approximately equal.

The objective is to be minimized is the sum of time of operation of the relays for fault in their own primary zone and located immediately after the respective breaker.



**Fig. (1) System Taken for Case Study**

Now the diagram of stations in figure (1) at Bus-1, 2, 3 are shown as under :-

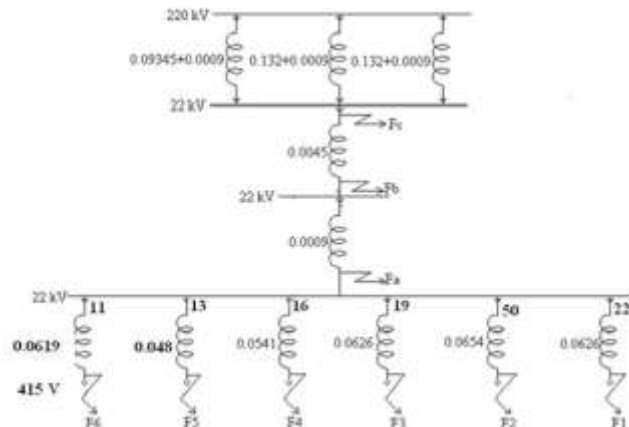


Figure (2) Station at bus-1

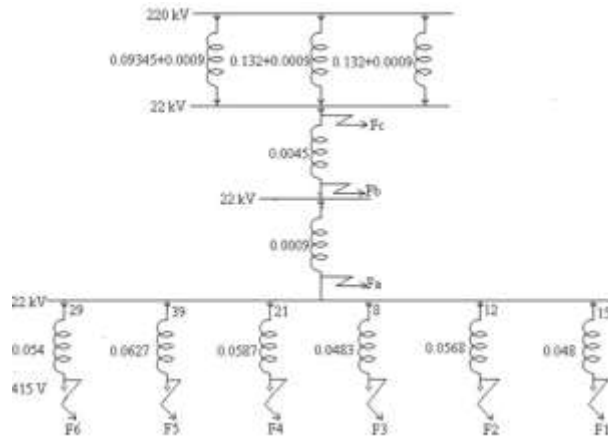


Figure (3) Station at bus-2

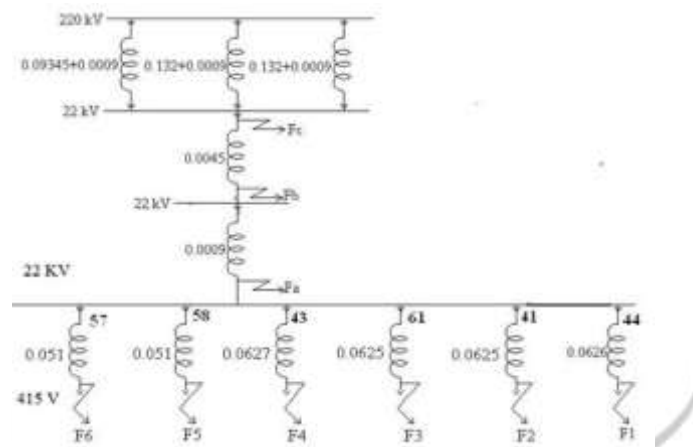


Figure (4) Station at bus-3

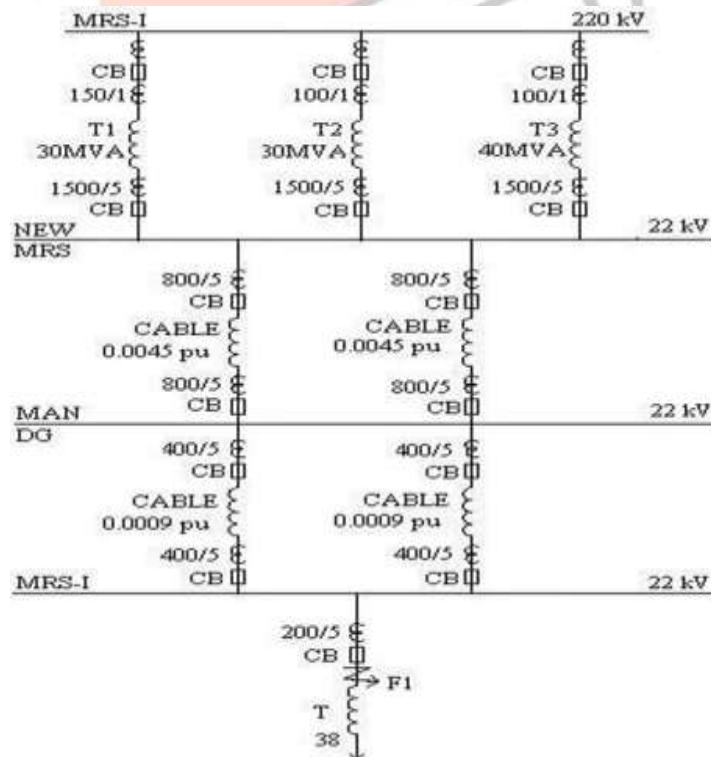


Figure (5) MRS-I

Here we have consider a three phase symmetrical fault. The short circuit analysis was done for 3-phase symmetrical fault in three buses. A proper calculation was done for the system. All the required matrices f, A, b, Ib, and ub were formed by writing the MATLAB program. The impedances of generators, transformers, cables and motors are contributing to the change in fault level at different locations.

The data and calculation is shown in the tables 1, table 2 and table 3.

**TABLE 1: - Data of Generators, Relays and Transformers**

Generator 1	30 MVA	22 KV	Z = 1.4285			
Generator 2	30 MVA	22 KV	Z = 1.4285			
Generator 3	40 MVA	22 KV	Z = 1.4285			
Transformers-(2 of 5)	2 MVA, 11K / 415 V					
Transformers-(3 of 5)	22 MVA, 11K / 22 KV					
Short Circuit Ratio	0.447					
Relay Number	R1	R2	R3	R4	R5	R6
Pickup Setting (Amps)	300	60	240	200	200	80

**TABLE 2: - Values of different constants for Relay's Different Characteristics**

Type of curve	$\alpha$	K	$\beta$
Normally inverse	0.02	0.14	2.97
Very Inverse	1.0	13.5	1.5
Extremely Inverse	2.0	80.0	0.808
Long-time Inverse	1.0	120.0	13.33

**TABLE 3: - Impedance values at various fault location**

Station A (MRS-I)		Station B (MRS-II)		Station C (MRS-III)	
Fault Location	Impedance Z (p.u)	Fault Location	Impedance Z (p.u)	Fault Location	Impedance Z (p.u)
F1	0.939	F1	0.96	F1	0.939
F2	0.981	F2	1.136	F2	0.9375
F3	0.939	F3	0.966	F3	0.9375
F4	1.082	F4	1.174	F4	0.9405
F5	0.96	F5	0.9405	F5	0.765
F6	0.9285	F6	1.028	F6	0.765

For the case study we have selected the values as:-

$k = 0.14$

$\alpha = 0.02$

$\beta = 2.97$

Plug Setting = 100% i.e. 1

Fault Current I = 19.70 kA

For first level, we assume TMS = 0.01.

The OLD-DG House and MAN-DG House contribute **1.68 kA** and **2.54 kA** respectively to the fault level at incomer of New MRS. So the changed fault level at the incomer of New MRS is **26.65 kA**.

Short circuit ratio of all eight generators is 0.447. From the short circuit ratio we calculate the transient reactance and sub-transient reactance of the generator.

The equivalent impedance of the transformers has to be taken into consideration for fault calculation. Here fault current contributed by all three generators is equal to the fault current contributed by a single generator multiplied by three. Hence

$$\text{Total Fault Current} = (\text{Fault current contributed} \\ \text{Contributed by single generator}) \times 3 \quad (8)$$

Here we do not consider the OLD-DG House and MAN-DG House then the values of impedance, Fault MVA and Fault current is calculated as under:-

$$Z_{(pu)T} = 0.039 + 0.0045 + 0.0009 - 0.1 \times (0.039 + 0.0045 + 0.0009) \\ = 0.0444 - 0.00444$$

We consider 10% negative tolerance as per IEC Standards

$$\text{So,} \\ Z_{(pu)T} = 0.03996 \text{ pu}$$

$$\text{Fault MVA} = 30 / (0.0996) \\ = 750.75$$

$$\text{Fault Current} = 750.75 / (\sqrt{3} \times 22000) \\ = 19.70 \text{ kA}$$

Now if we consider the OLD-DG House and MAN-DG House then the values of impedance, Fault MVA and Fault current is calculated as under:-

$$X_d = 1 / 0.447 \\ = 2.237 \text{ pu}$$

Also, sub-transient reactance is given by:-

$$X_d'' = 0.2 \times 2.237 \\ = 0.4474 \text{ pu}$$

MVA rating for all generators is 3.125. Hence fault MVA for all generators is given by:-

$$\text{Fault MVA} = 14.65 / 0.4474 \\ = 32.74$$

Now fault current contributed by each generator at 6.6 kV is given by:-

$$\text{Fault Current} = 32.74 / (\sqrt{3} \times 11) \\ = 1.72 \text{ kA}$$

Let us consider any one generator. It is connected to another 6.6 kV bus through a cable of impedance 0.0009 pu. So total impedance is given by:-

$$Z = X_d'' + 0.0009 \\ = 0.4474 + 0.0009 \\ = 0.4484 \text{ pu}$$

$$\text{Fault MVA} = 14.65 / 0.4484 \\ = 32.74$$

$$\text{Fault Current} = 32.74 / (\sqrt{3} \times 11) \\ = 1.72 \text{ kA}$$

Now this generator, with cable of 0.0009 pu in series, is connected to 22kV bus through a step up transformer of 11/22 kV. So total impedance is given by:

$$Z_T = (X_d'' + 0.0009 + Z_{\text{transformer}}) \\ = (0.4474 + 0.0009 + 0.00617) \\ = 0.4545 \text{ pu}$$

$$\text{Fault MVA} = 14.65 / 0.4545 \\ = 32.24$$

$$\text{Fault Current} = 32.24 / (\sqrt{3} \times 22) \\ = 0.85 \text{ kA}$$

Total fault current contributed

$$\begin{aligned} \text{by all generators} &= (\text{Fault current contributed by single generator}) \times 3 \\ &= 0.85 \times 3 \\ &= 2.54 \text{ kA} \end{aligned}$$

From incomer of New MRS to outgoing feeder the fault impedance is increasing while the voltage remains constant resulting in decrease in fault level. At low voltage side of distribution transformers the voltage level is significantly lower than high voltage side as the transformation ratio is high. The effect of lower voltage level is more than the effect of increase in fault impedance which causes the fault level to rise considerably as compared to the 22 kV level.

#### RESULTS: -

We co-ordinated the over current relays from the outgoing feeder of MRS-I to incomer of New MRS. The actual operating time for the relays at

Outgoing feeder of MRS-I is	<b>0.15 sec</b>
Incomer of MRS-I is	<b>0.30 sec</b>
Outgoing feeder of MAN-DG is	<b>0.40 sec</b>
Incomer of MAN-DG is	<b>0.60 sec</b>
Outgoing of New MRS is	<b>0.70 sec</b>
Incomer of New MRS is	<b>0.90 sec</b>

The fault currents obtained from short circuit analysis. After calculation enter all values in program and run the program. The *linprog* function was finally executed in the MATLAB program and the results obtained are reproduced here: -

```
x =
    0.1216
    0.0101
    0.0833
    0.0674
    0.1019
    0.0504

fval =
    1.3510

exitflag =
    1

output =

iterations: 7
cgiterations: 0
algorithm: 'lipsol'
```

New MRS incomer has the highest fault level amongst all the 22 kV buses. The selected operating time for this circuit breaker for a fault of **19.7 kA** at outgoing feeder of MRS-I is **0.9 sec**. Therefore, for the changed fault level of **26.65 kA** the sustainable time is

$$19.7^2 \times 0.9 = 26.65^2 \times t$$

$$t = \mathbf{0.5 \text{ sec.}}$$

For the system taken the fault clearing time obtained as per the real time settings is **t = 0.5 sec** for TDS as 0.05. According to the consideration and value taken in program the time of clearing fault (summation of all values of X) obtained is

$$t = 0.1216 + 0.0101 + 0.0833 + 0.0674 +$$

$$0.1019 + 0.0504$$

$$t = \mathbf{0.4347 \text{ sec.}}$$

#### IV. CONCLUSIONS

Thus we can conclude that the fault clearing time is less and better with Linear Programming than the actual settings. This is what is done while doing the case study here. The convergence is guaranteed in the LP method. Hence, we conclude that



results of Linear Programming based optimization are well in line with actual settings done. Though the fault level is maximum, fault will be cleared without any damage. The implementation can *further be extended* to bigger power system with more buses.

#### REFERENCES

- [1] Oza, Nair, Mehta, Makwana, Tata McGraw Hill , 2010.
- [2] D P Kothari, I J Nagrath, Modern Power System Analysis, Tata McGraw-Hill Publication, 2008.
- [3] V K Mehta, Rohit Mehta, Principles Of Power System, S Chand Prakashan, 4<sup>th</sup> revised edition, 2008.
- [4] Urdaneta A. J, Ramon N., Luis G.P.J “Optimal Coordination Of Directional Overcurrent Relays In Interconnected Power Systems”, IEEE Transactions on Power Delivery, Vol. 3, No. 3, July 1988.
- [5] Rashesh P Mehta, Optimal Relay Coordination, Journal Of Engineering And Technology, Sardar Patel University, Vol. 19, December 2006.
- [6] H.Zeineldin, E.F.El-Saadany, M.A.Salama, “Optimal Coordination Of Directional Overcurrent Relay Coordination”, Power Engineering Society General Meeting, IEEE CNF, June 12-16, 2005.
- [7] [www.google.com](http://www.google.com)
- [8] Wikipedia, the free encyclopedia
- [9] User’s manual and technical description “Combine overcurrent and earth-fault relay SPAJ 140c” of ABB, [www.abb.com](http://www.abb.com)

#### BIOGRAPHY: -



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