Analysis of A 2DOF Nonlinear Damping System for Effectively Suppressing Vibrations Using Linear Vibration Absorber

[1]Visakh S Kumar, [2]Dr Y V K S Rao
[1]MTech Scholar, [2]Rtrd. Professor in Mechanical Engineering
[1]Department of Mechanical Engineering, Mar- Baselios College of Engineering, Trivandrum, India

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Abstract - **In this study the nonlinear analysis of 2 DOF vibration systems with weakly nonlinear damper is studied. A linear vibration absorber is used to suppress the nonlinear vibrations of forced nonlinear damping system. A simple perturbation method known as straight forward expansion is used to find the approximate analytical solution. On the basis of this solution, a time with displacement graph is obtained. Using the same system parameters, a computer generated graph is also formed. Both graphs are studied. As the graphs are found to be valid, further studies are being conducted with the help of frequency response curves, time plots and phase planes.**

IndexTerms **- Degrees of Freedom, Dynamic Vibration Absorber.**

I. INTRODUCTION

Many practical systems we are experiencing in our day to day life are nonlinear in nature. However; the presence of non-linearity introduces dangerous instabilities, which in some cases may result in amplification rather than reduction of the vibration amplitudes. Many researchers have used springs as nonlinear system. Here an attempt is made to analyse the nonlinearity of a vibrating system using nonlinear damper for the reduction of vibration. The mathematical models of the nonlinear systems are represented by nonlinear differential equations. Hence, there are no general methods for the analysis and synthesis of nonlinear control systems. Various methods of solving the nonlinear vibration problems are Lindstedt's perturbation method, the iterative method and the Ritz- Galerkin method. The nonlinear systems do not obey superposition principle. For this reason, the response of nonlinear systems to a particular test signal is no guide to their behaviour to other inputs. The nonlinear system response may be highly sensitive to input amplitude. Hence, in a nonlinear system, the stability is very much dependent on the input and also the initial state.

Our main aim in this paper is to analyze the nonlinearity of a vibration system with weakly nonlinear damper. The approximate analytical solution for nonlinear system with nonlinear damper is carried out by using straight forward expansion. The time plots with displacement are also plotted. A comparison of plots is made with the computer generated solution obtained from MATLAB to ensure the validation of the solution.

Abbreviations

DOF- Degrees of freedom
DVA- Dynamic Vibration Absorber

II. RESEARCH METHODOLOGY

2.1 Mathematical Modeling Using Nonlinear Damper

The mass, M is attached to a rigid boundary through a spring & viscous damper of linear plus nonlinear characteristic, as shown in **Figure 1**. The displacement of the nonlinear primary system & the linear absorber system are denoted by x & x_a respectively. By applying Newton's second law of motion, two equations of motion for the new system composed of the nonlinear primary system incorporated by a linear absorber system may be written as

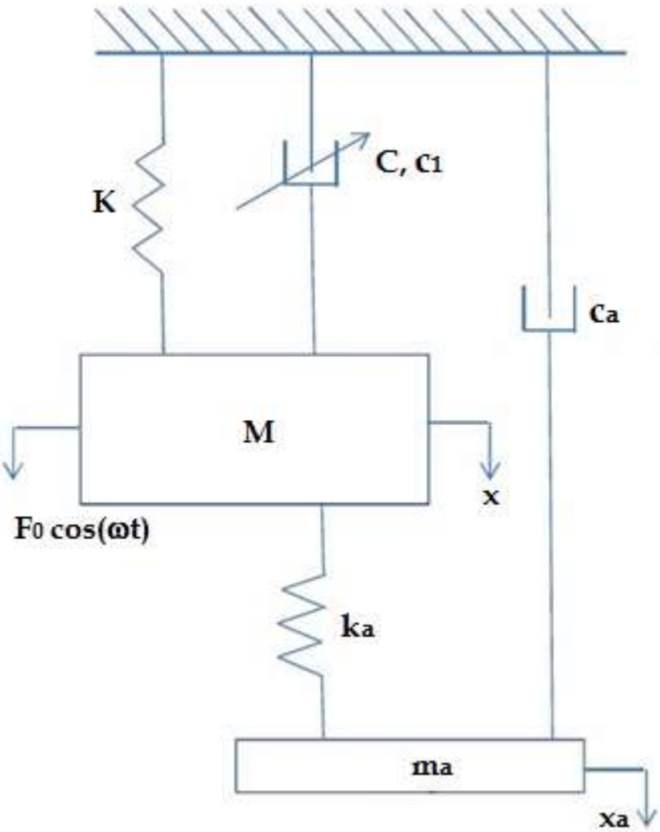

Fig.1 System with Nonlinear Damper

$$M\ddot{x} + Kx - k_a(x_a - x) + C\dot{x} + c_1x^2\dot{x} = F_0\cos\omega t \quad (1)$$
$$m_a\ddot{x}_a + k_a(x_a - x) + c_a\dot{x}_a = 0 \quad (2)$$

Where M, K, C, c_1 and m_a, k_a, c_a are the system parameters for primary nonlinear system and secondary absorber system respectively. Dividing M on both sides of Eq.1 & dividing m_a on both sides of Eq.2 & then rewriting the resultant equations yields the following equations.

$$\ddot{x} + \left(\frac{K+k_a}{M}\right)x - \left(\frac{k_a}{M}\right)x_a + \left(\frac{C}{M}\right)\dot{x} + \left(\frac{c_1}{M}\right)x^2\dot{x} = (F_0/M)\cos\omega t \quad (3)$$

$$\ddot{x}_a + \left(\frac{k_a}{m_a}\right)(x_a - x) + \left(\frac{c_a}{m_a}\right)\dot{x}_a = 0 \quad (4)$$

$$\ddot{x} + \omega_1^2 x - m\omega_a^2 x_a + \mu_1\dot{x} + \epsilon x^2\dot{x} = F\cos\omega t \quad (5)$$

$$\ddot{x}_a + \omega_a^2(x_a - x) + \mu_2\dot{x}_a = 0 \quad (6)$$

The solution x of our problem is a function of the independent variable t and the parameter, ϵ i.e. $x = x(t;\epsilon)$. One of the perturbation method known as the straight forward expansion is used to expand the above equations to determine the analytical solution. The straight forward expansion in the form of a power series in ϵ is given by

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) + \epsilon^2 x_2(t) + \epsilon^3 x_3(t) + \cdots \quad (7)$$

Here only the first term in the correction series is considered and neglecting the higher order terms, so that the approximate solution in the form

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) \quad (8)$$

Substituting Eq.8 into Eq.5 and Eq.6, and equating each of the coefficients of ϵ^0 & ϵ^1 to zero

$$\ddot{x}_0 + \omega_1^2 x_0 + \mu_1\dot{x}_0 = F\cos\omega t + m\omega_a^2 x_{a0} \quad (9)$$

$$\ddot{x}_1 + \omega_1^2 x_1 + {x_0}^2 \dot{x}_0 + \mu_1 \dot{x}_1 = m\omega_a^2 x_{a1} \quad (10)$$

$$\ddot{x}_{a0} + \omega_a^2 x_{a0} + \mu_2 \dot{x}_{a0} = \omega_a^2 x_0 \quad (11)$$

$$\ddot{x}_{a1} + \omega_a^2 x_{a1} + \mu_2 \dot{x}_{a1} = \omega_a^2 x_1 \quad (12)$$

Since Eq.9 is inhomogeneous, its general solution can be obtained as the sum of a homogeneous solution and any particular solution. Therefore the complete solution is given by

$$x_0(t) = x_h(t) + x_p(t) \quad (13)$$

$$x_0(t) = Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi) + X_0\cos(\omega t - \phi) \quad (14)$$

To find $x_1(t)$, solve for unknowns x_1 & x_{a1}. Eq.10 & Eq.12 on rearranging gives

$$\ddot{x}_1 + \omega_1^2 x_1 + \mu_1 \dot{x}_1 - m\omega_a^2 x_{a1} = -({x_0}^2 \dot{x}_0) \quad (15)$$

$$\ddot{x}_{a1} + \omega_a^2 x_{a1} + \mu_2 \dot{x}_{a1} - \omega_a^2 x_1 = 0 \quad (16)$$

$$(D^2 + \omega_1^2 + D\mu_1)x_1 - m\omega_a^2 x_{a1} = -({x_0}^2 \dot{x}_0) \quad (17)$$

$$(D^2 + \omega_a^2 + D\mu_2)x_{a1} - \omega_a^2 x_1 = 0 \quad (18)$$

In order to find the solution of x_1 & x_{a1} from the above Eq.17 & Eq.18, multiply Eq.17 by ω_a^2 & Eq.18 by $(D^2 + \omega_1^2 + D\mu_1)$ and then adding the resulting equations will give

$$(D^2 + \omega_1^2 + D\mu_1)(D^2 + \omega_a^2 + D\mu_2)x_{a1} - m\omega_a^4 x_{a1} = -(x_0^2 \dot{x}_0)\omega_a^2 \quad (19)$$

$$(D^4 + D^2(\omega_a^2 + \omega_1^2 + \mu_1\mu_2) + D^3(\mu_1 + \mu_2) + D(\mu_1\omega_a^2 + \mu_2\omega_1^2) + \omega_1^2\omega_a^2 - m\omega_a^4)x_{a1} = -(x_0^2 \dot{x}_0)\omega_a^2$$

$$(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)x_{a1} = (\omega X_0^3 \cos^2(\omega t - \phi))\, \omega_a^2 \sin(\omega t - \phi)$$

$$(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)x_{a1} = C_5 \cos^2(\omega t - \phi)\sin(\omega t - \phi) \quad (20)$$

Using trigonometric relations,

$$\cos^2(\omega t - \phi) = 1 - \sin^2(\omega t - \phi) \quad (21)$$

$$\sin^3(\omega t - \phi) = \frac{3}{4}\sin(\omega t - \phi) - \frac{1}{4}\sin 3(\omega t - \phi) \quad (22)$$

Substituting Eq.21 & using the trigonometric relation in Eq.22 into Eq. 20, will give

$$x_{a1} = \frac{1}{(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)}\left(C_5 \sin(\omega t - \phi) + \frac{C_5}{4}\sin 3(\omega t - \phi) - \frac{3C_5}{4}\sin(\omega t - \phi)\right.$$

$$x_{a1p} = \frac{1}{(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)}(C_5 \sin(\omega t - \phi) + C_7 \sin 3(\omega t - \phi) + C_6 \sin(\omega t - \phi))$$

$$x_{a1p} = \frac{1}{(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)}(C_8 \sin(\omega t - \phi) + C_7 \sin 3(\omega t - \phi)) \quad (23)$$

$$x_{a1p} = PI_1 + PI_2 \quad (24)$$

$$PI_1 = \frac{1}{(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)}(C_8 \sin(\omega t - \phi))$$

Put $D^2 = -\omega^2$

$$PI_1 = \frac{1}{(\omega^4 - C_2\omega^2 + C_4 + (C_3 - C_1\omega^2)D)}(C_8 \sin(\omega t - \phi)$$

$$PI_1 = \frac{1}{(C_9 + C_{10}D)}(C_8 \sin(\omega t - \phi)$$

Multiply both numerator and denominator by the conjugate of $(C_9 + C_{10}D)$ & then on solving gives

$$PI_1 = \frac{1}{(C_9^2 + C_{10}^2\omega^2)}(C_8C_9\sin(\omega t - \phi) - \omega C_8C_{10}\cos(\omega t - \phi)) \quad (25)$$

$$PI_2 = \frac{1}{(D^4 + C_1D^3 + C_2D^2 + C_3D + C_4)}(C_7 \sin 3(\omega t - \phi))$$

Put $D^2 = -9\omega^2$

$$PI_2 = \frac{1}{(81\omega^4 - 9C_2\omega^2 + C_4 + (C_3 - 9C_1\omega^2)D)}(C_7 \sin 3(\omega t - \phi))$$

$$PI_2 = \frac{1}{(C_{11} + C_{12}D)}(C_7 \sin 3(\omega t - \phi))$$

Multiply both numerator and denominator by the conjugate of $(C_{11} + C_{12}D)$ & on solving gives

$$PI_2 = \frac{1}{(C_{11}^2 + 9C_{12}^2\omega^2)}(C_7C_{11} \sin 3(\omega t - \phi) - 3\omega C_7C_{12} \cos 3(\omega t - \phi)) \quad (26)$$

$$x_{a1p} = \frac{1}{(C_9^2 + C_{10}^2\omega^2)}\left(C_8C_9\sin(\omega t - \phi) - \omega C_{10}C_8\cos(\omega t - \phi)\right) + \frac{1}{(C_{11}^2 + 9C_{12}^2\omega^2)}(C_7C_{11} \sin 3(\omega t - \phi) - 3\omega C_7C_{12} \cos 3(\omega t - \phi))$$

$$x_{a1p} = C_{13}\sin(\omega t - \phi) - C_{14}\cos(\omega t - \phi) + C_{15}\sin3(\omega t - \phi) - C_{16}\cos3(\omega t - \phi) \quad (27)$$

Substitute Eq.27 into Eq.18 & then on solving gives

$$x_{1p}(t) = \frac{\omega^2}{\omega_a^2}(-C_{13} \sin(\omega t - \phi) + C_{14}\cos(\omega t - \phi) - 9C_{15}\sin3(\omega t - \phi) + 9C_{16}\cos3(\omega t - \phi) + \left(C_{13}\sin(\omega t - \phi) - C_{14}\cos(\omega t - \phi) + C_{15}\sin3(\omega t - \phi) - C_{16}\cos3(\omega t - \phi)\right) + \frac{\mu_2\omega}{\omega_a^2}(C_{13}\cos(\omega t - \phi) + C_{14}\sin(\omega t - \phi) + 3C_{15}\cos3(\omega t - \phi) + 3C_{16}\sin3(\omega t - \phi)) \quad (28)$$

Where C_1, C_2, C_3 ….C_{16} are constants. The complete solution is given by $x_1(t) = x_{1h}(t) + x_{1p}(t)$. For an under damped system $x_{1h}(t)$ is given by

$$x_{1h}(t) = Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi) \quad (29)$$

Thus the approximate solution can be obtained by substituting Eq.14, Eq.28 & Eq.29 into Eq.8

$$x(t;\epsilon) = x_0(t) + \epsilon x_1(t) + \cdots$$

$$= Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi) + \frac{F}{\left[\left\{(\omega_1^2 - \omega^2) - \frac{1}{\omega_a^2 - \omega^2}m\omega_a^4\right\}^2 + \mu_1^2\omega^2\right]^{\frac{1}{2}}}\cos(\omega t - \phi) +$$

$$\epsilon\left[\frac{\omega^2}{\omega_a^2}\left(-C_{13} \sin(\omega t - \phi) + C_{14}\cos(\omega t - \phi) - 9C_{15}\sin3(\omega t - \phi) + 9C_{16}\cos3(\omega t - \phi) + \left(C_{13}\sin(\omega t - \phi) - C_{14}\cos(\omega t - \phi) + C_{15}\sin3(\omega t - \phi) - C_{16}\cos3(\omega t - \phi)\right) + \frac{\mu_2\omega}{\omega_a^2}(C_{13}\cos(\omega t - \phi) + C_{14}\sin(\omega t - \phi) + 3C_{15}\cos3(\omega t - \phi) + 3C_{16}\sin3(\omega t - \phi))\right) + Ae^{(-\xi\omega_1 t)}\cos(\omega_d t - \psi)\right] \quad (30)$$

III. VALIDITY OF THE SOLUTION

This section deals with the comparison of the graphs obtained from analytical solution with that of a computer generated mat lab solution, so that the validity of the solution can be assessed.

3.1 Analytical Solution

On the basis of the above solutions, a time with displacement graph is generated. The time plots generated from the analytical solutions for the system parameters M=10kg, m_a=0.8kg, K=15N/m, k_a=10N/m, C=0.15Ns/m, c_a=0.03Ns/m, c_1=0.1Ns/m^3, F_0=5.5N are shown in **Figure 2**.

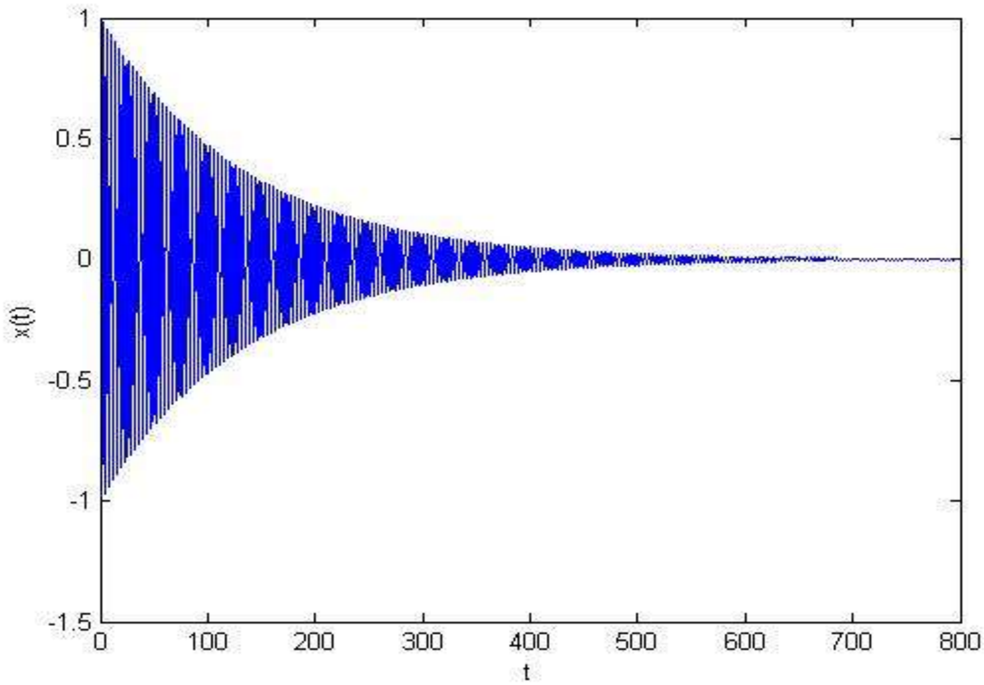

Fig.2 Analytical Solutions

3.2. MAT-LAB Solution

In equations of motion, take $x = y(1)$, $\dot{x} = y(2)$, $x_a = y(3)$ & $\dot{x}_a = y(4)$, so that the derivatives of y(1), y(2), y(3) & y(4) are y(2), $\ddot{x}$, y(4) & $\ddot{x}_a$. For obtaining the time plots, y(2), $\ddot{x}$, y(4) & $\ddot{x}_a$ are taken as functions f(1), f(2), f(3) & f(4) respectively. By using these functions different time plots with displacement, velocity, acceleration of the system can be plotted.

For the system with nonlinear damper, say

$$f(1) = y(2),$$

$$f(2) = \frac{F_0}{M}\cos\omega t - \frac{K + k_a}{M}y(1) + \frac{k_a}{M}y(3) - \frac{C}{M}y(2) - \frac{c_1}{M}y(1)^2 y(2)$$

$$f(3) = y(4),$$

$$f(4) = -\frac{k_a}{m_a}\big(y(3) - y(1)\big) - \frac{c_a}{m_a}y(4)$$

The time plot generated from these functions using the same system parameters used above are shown in **Figure 3**.

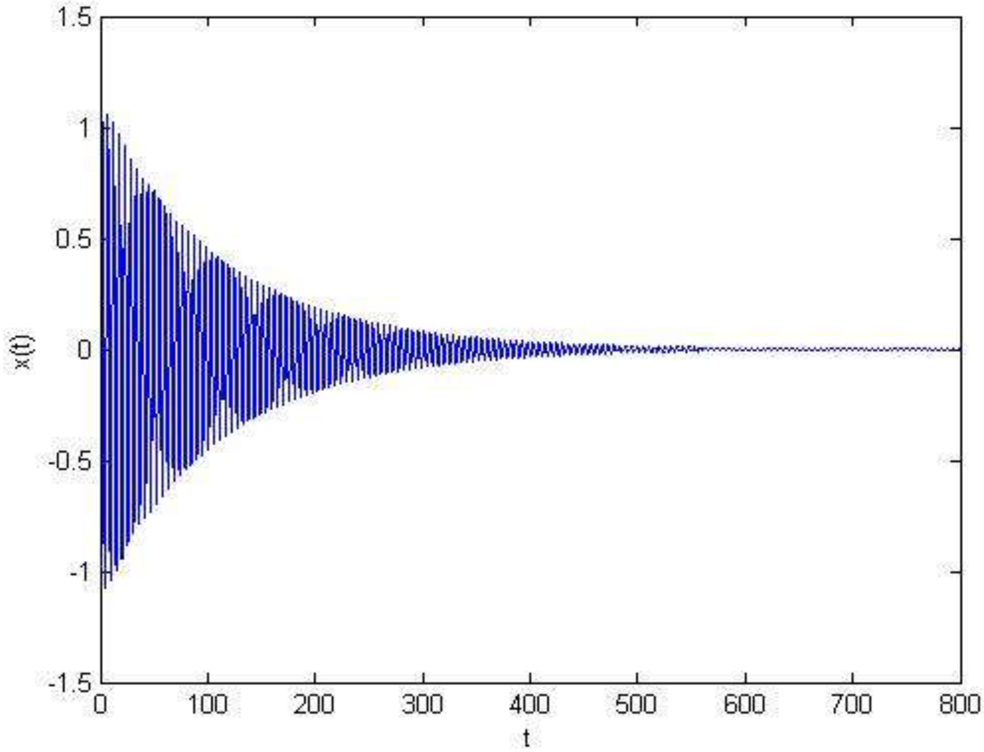

Fig.3 Mat lab solutions

It is observed that the graphs are almost similar in various situations which ensures the validity of the approximate analytical solutions. The slight discrepancy seen in the graphs are due to the approximations that already taken earlier in Eq.8.

IV. RESULTS AND DISCUSSION

4.1. Numerical Simulations

This section presents illustrative examples to show the effectiveness of the nonlinear damping system for suppressing the nonlinear vibrations under primary resonance conditions. As the graphs are found to be valid, further studies are being conducted with the help of frequency response curves, time plots and phase planes. Numerical simulations have been performed under the following values of the system parameters shown in **Table 1**. The linearized natural frequencies of the nonlinear primary system before and after being attached by the vibration absorber are found to be approximately, ω_{10}=2.098 rad/sec, ω_1=2.280 rad/sec and natural frequency of the absorber be ω_a=3.651 rad/sec.

Table 1 System parameter values

Primary mass, M (kg)	Absorber mass, m_a (kg)	Linear stiffness, K (N/m)	Absorber stiffness, k_a (N/m)	Primary damping, C (Ns/m)	Absorber damping, c_a (Ns/m)	Nonlinear damper, c_1 (Ns/m^3)	External Excitation, F_0 (N)
10	0.6	44	8	0.1	0.08	0.01	4.5

Using the system parameters given above, the displacements of the primary system for different time period have been plotted as shown in **Figure 4**. It is observed that for a small value of damping the amplitude goes on decreasing with time. For time t=450s, the amplitude almost reaches zero.

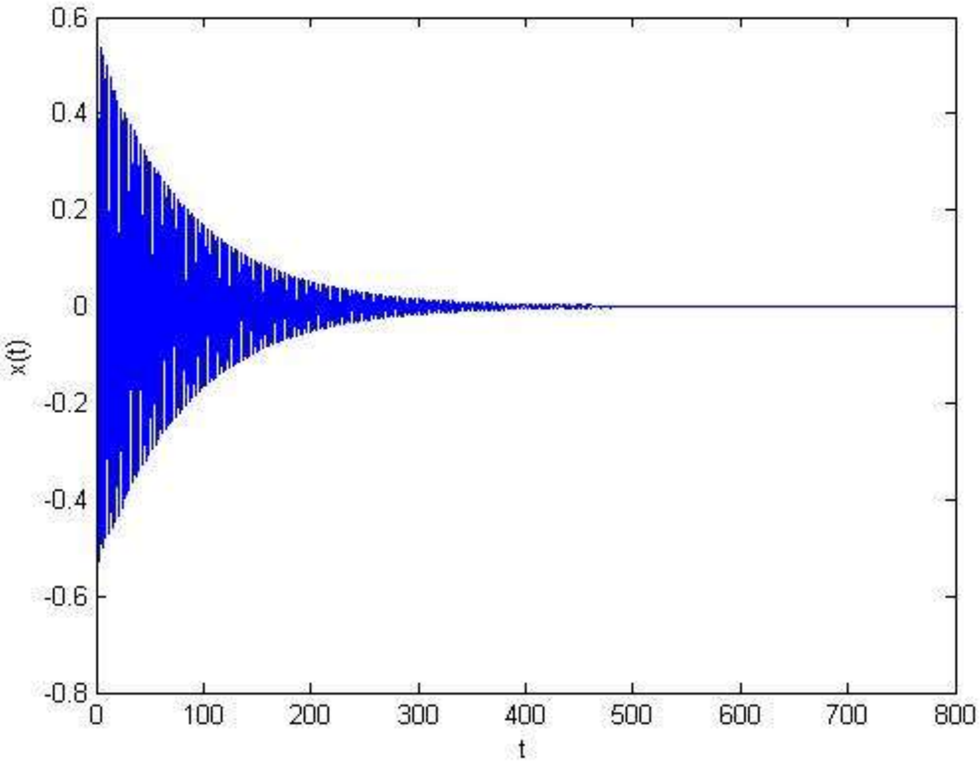

Fig.4 Response of the nonlinear system at c_1=0.01

The response of the system for different values of c_1 =0, 0.01, 0.1 & 0.5 are shown in **Figure 5.** It is seen that the nature of the graph may not be changed for smaller nonlinear damping values.

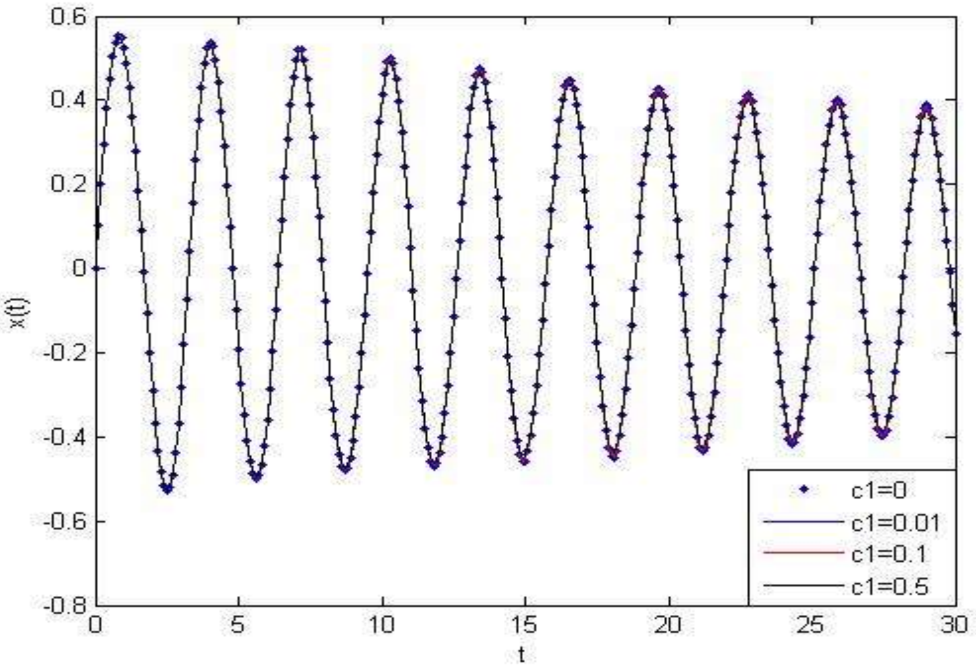

Fig.5 Response of the nonlinear system at c_1= 0, 0.01, 0.1, 0.5

4.2. Stability of the System

The trajectory in the phase plane for the system with nonlinear damping as shown in **Figure 6.** It is seen that the system is stable as the trajectory approaches to zero. It is also observed that smaller nonlinear damping values will have no effect as the phase plane remains in the same fashion.

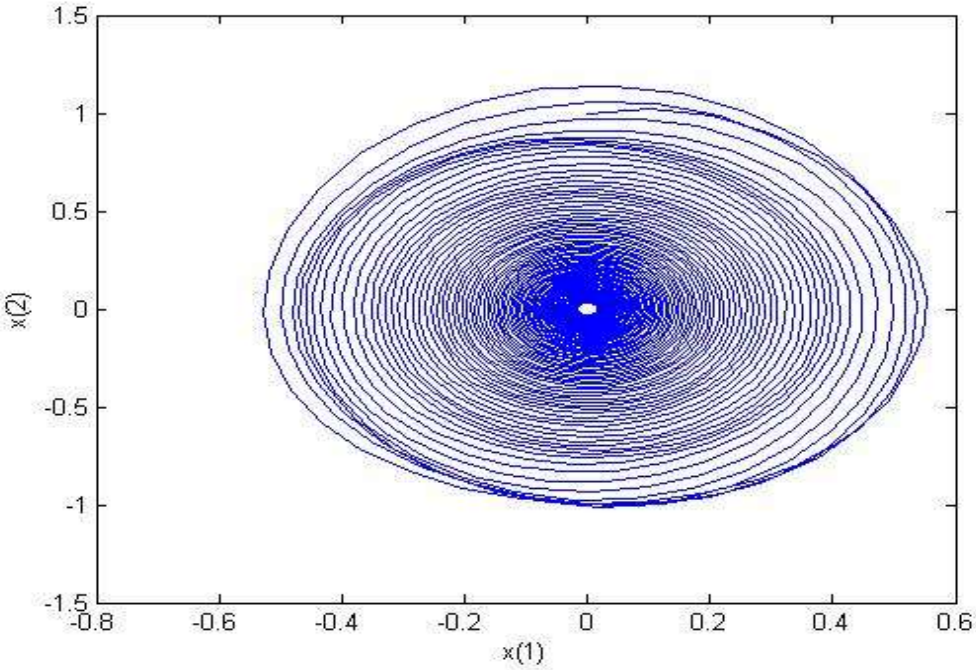

Fig. 6 Phase plane for the nonlinear system with c_1=0.01

4.3. Frequency Response Curves

The amplitude spectrum for the system with nonlinear damper with the function of frequency as shown in **Figure 7.** It is found that increase of absorber damping as well as nonlinear damping leads to reduction of peak amplitude at resonant frequencies. It is seen that for a nonlinear system, a small amplitude peak is observed. This is because the proposed nonlinear structure is a 2DOF system so that the system has two linearized natural frequencies.

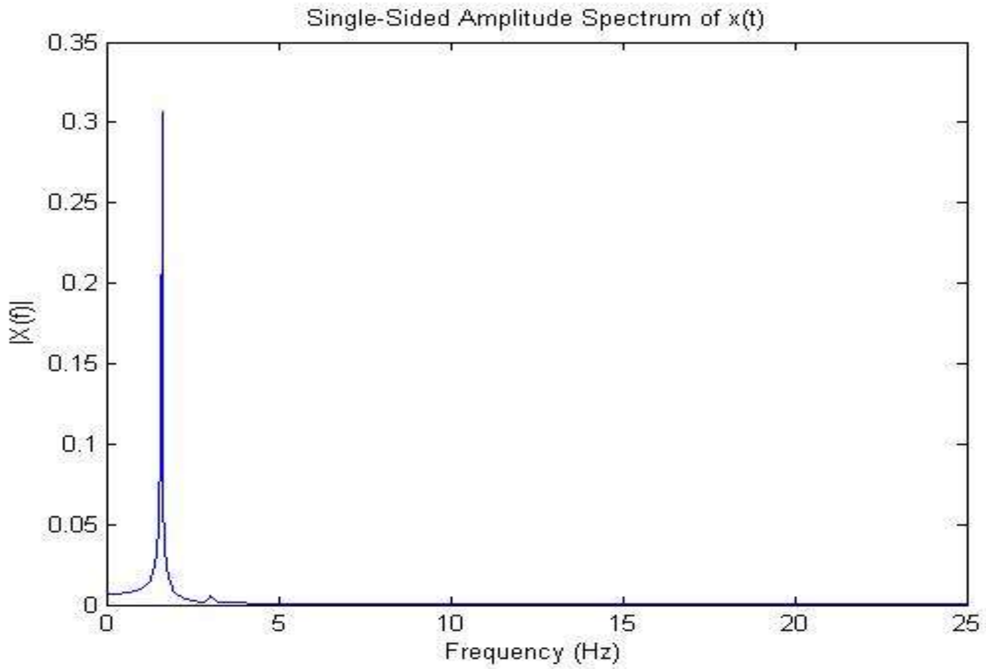

Fig. 7 Single-sided Amplitude spectrum of x(t) (Nonlinear damper)

V. CONCLUSION

In this paper, the nonlinear analysis of a vibration system with weakly non-linear damper is studied. A simple perturbation method known as straight forward expansion is used to find the approximate analytical solutions. Neglecting the higher order terms in the expansion, the approximate analytical solutions for the given model was derived. On the basis of this solution, a time with displacement graph is obtained. Using the same system parameters, a computer generated graph is also formed. Both graphs are studied. . It is found that the plots are almost identical in various situations which ensure the validity of the analytical solutions. The slight discrepancy seen in the graphs are due to the approximations that already taken earlier in Eq.8.

As the graphs are found to be valid, further studies are being conducted with the help of frequency response curves, time plots and phase planes. It is found that the nonlinear system with nonlinear damper will reduce the amplitude of the primary system in comparison with that of the linear system. The stability and the response of the system is also studied with the help of phase plane. It is also found that the entire trajectory in the phase plane approaches to zero due to damping & the system is stable and it is also observed that smaller nonlinear damping values will have no effect as the phase plane remains in the same fashion.

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