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Comparative study of collocation method Spline collocation and Finite Difference method for solving ground water recharge problem

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Abstract - This paper provides an overview of the formulation, analysis and implementation of Spline collocation and finite difference method for the numerical solution of partial differential equation with two space variable which is of parabolic type. The method includes the solution of non-linear equation which can be expressed as in matrix form. The use of spline collocation methods in the solution of initial-boundary value problems for parabolic-type system id described, with emphasis on alternating direction implicit methods. Problem of vertical groundwater recharge solve by spline collocation and finite difference method. Solution is compared with spline collocation and finite difference method.

Keywords - Spline collocation, Finite Difference Method, Partial differential equation.

I. INTRODUCTION

Non-Linear parabolic partial differential equation solves by spline collocation and finite difference method. While numerical solution obtained. Therefore, two common questions are encountered, first is about its acceptance whether it is sufficiently close to true solution or not. If one has an analytic solution then this can be answered very clearly but in either case it is not so easy. One has to be careful while concluding that a particular numerical solution is acceptable when an analytic solution is not available. Normally a method is selected which does not produce an excessive error.

II. NON-LINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

Consider a nonlinear parabolic partial differential equation

$\frac{\partial u}{\partial t} + Au\,\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$ ……….. (1)

Where A is constant.
Subject to certain initial and boundary conditions.
Initial condition: u (u, 0) =f(x)
Boundary condition: u (0, t) = u (a, t) = 0

III. FINITE DIFFERENCE METHOD TO SOLVE NONLINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

Explicit Finite Difference Formula:

$$u_{i+1,j+1} + 4u_{i,j+1} + u_{i-1,j+1} = u_{i+1,j}(1+6r) + u_{i,j}(4-12r) + u_{i-1,j}(1+6r) - \left(\frac{AB}{2}\right)\{u_{i+1,j}\left(u_{i+2,j} - u_{i,j}\right) + 4u_{i,j}\left(u_{i+1,j} - u_{i-1,j}\right) + u_{i-1,j}(u_{i,j} - u_{i-2,j})\}$$ ----------- (3)

Where $r = k/h^2$ & $B = k/h$

Implicit Finite Difference Formula:

$$u_{i+1,j+1}(6r-2) + u_{i,j+1}(-12r-8) + u_{i-1,j+1}(6r-2) = AB\{u_{i+1,j}\left(u_{i+2,j} - u_{i,j}\right) + 4u_{i,j}\left(u_{i+1,j} - u_{i-1,j}\right) + u_{i-1,j}\left(u_{i,j} - u_{i-2,j}\right)\} - \{u_{i+1,j}(6r+2) + u_{i,j}(8 - 12r) + u_{i-1,j}(6r+2)\}$$

------------ (5)

IV. SPLINE FORMULA TO SOLVE NONLINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

Divide the region [0,a] into say N- subintervals of equal length. Denote the points of subdivisions by x_0, x_1, x_2,... ,x_n. Let u_{ij} denote the value of u at i^{th} mesh point at time jΔt. We approximate the function u at time jΔt by cubic spline s(x). Discretizing the left side of equation (1) by forward and central difference formula and replace right side by the second derivative $s''(x_i)$ at j^{th} level like explicit scheme in finite difference, we get,

$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} + Au_{i,j}\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} = s''_{i,j}$ ---------- (2)

Where $s''_{i,j}$ denotes $s''(x_i)$ at j^{th} level

Once the value of u are known at $(j+1)^{th}$ level, we can proceed to compute next level j+2 by repeating the same process. Each set of equations give tri-diagonal matrix. It can be solved by any standard method. Thus, the method can proceed by steps.

The convergence and stability of these methods totally depend upon value of r. Convergence and stability along with small values of r is more accurate. Values much larger than unity are not recommended. These two methods will be discussed later on by taking its actual approximation to a problem.

V. Problem of Vertical Groundwater Recharge

The problem of flow of water through partially saturated porous media has been discussed by Klute [1952] and Verma [1969]. We have obtained a numerical solution of the problem by using spline collocation technique. In the investigated mathematical model we consider that the ground water recharge takes place over the large basin of such geological location that the sides are limited by rigid boundaries and the bottom by a thick layer of water table. In this case the flow may be assumed vertically downwards through unsaturated porous media. Here the average diffusivity coefficient of the whole range of moisture content is regarded as constant and the permeability of the moisture content is assumed to have a parabolic distribution. The theoretical formulation of the problem yields a non-linear partial differential equation for the moisture content.

VI. Formulation of the problem

The equation of continuity for an unsaturated medium is given by

$$\frac{\partial}{\partial t}(\rho s \ \theta) = -\nabla, M$$

Where ρs the bulk density of the medium is, θ is its moisture content on a dry weight basis, and M is the mass flux of moisture.

7.Result:

Table 7.1
Comparison of moisture content at different t level

	t=7/96		t = 8/96		t = 9/96		t=7/96		t = 8/96		t = 9/96	
	θ		θ		θ		θ		θ		θ	
ξ	SE	FE	SE	FE	SE	FE	SI	FI	SI	FI	SI	FI
0.0	0.100	0.1	0.100	0.1	0.100	0.1	0.100	0.1	0.100	0.1	0.100	0.1
0.25	0.0622	0.0625	0.0642	0.0642	0.0653	0.0656	0.0694	0.0690	0.0726	0.0720	0.0753	0.0752
0.50	0.0337	0.0335	0.0352	0.0350	0.0379	0.0378	0.0376	0.0372	0.0419	0.0414	0.0459	0.0456
0.75	0.0161	0.0158	0.0190	0.0189	0.0201	0.0200	0.0179	0.0178	0.0208	0.0204	0.0239	0.0235
1.00	0.0121	0.0120	0.0126	0.0124	0.0152	0.0149	0.0120	0.0118	0.0139	0.0135	0.0162	0.0156

***SE: Explicit Spline method; SI : Implicit Spline method; FE: Finite difference Explicit method ;FI: Finite difference Implicit method**

VII. Conclusion

It is noticed that, spline explicit and implicit solution are more accurate than finite difference explicit and implicit solution. Also both the methods required compact calculations and besides these both methods namely explicit and implicit method, implicit methods have more accurate results than the explicit method. Also ,we can conclude that from the graph the curves indicate the behavior of moisture content corresponding to different values, from the figure it can be observed that moisture content θ decreases considerably throughout the region as ξ increase but time t increases moisture content throughout region as well as at the layer increases.

VIII. References

[1] Ahlberg J H, Nilson E N AndWals J N (1967), The Theory of Spline And Their Application. Academic Press, N.Y..

[2] Bhatt R V (2005): “An Extended Study Of The Spline Collocation Approach To Partial Differential Equations”, Thesis, South Gujarat University, Surat.

[3] Bickley W G (1968) -Piecewise Cubic Interpolation and Two point Boundary Value Problem. Comp. J, 11,206.

[4] Blue J L (1969)-„Spline Function Methods for linear Boundary Value problem.”Communications of the ACM. 12, No.6, 327.

[5] Dr. Grewal B S (2005):- Numerical Methods In Engineering And Science, Fifth ed, Khanna Publishers.

[6] Doctor H D and Kalthia N L (1983) - Spline Approximation of Boundary Value Problems. Presented at Annual Conf.of Indian math.Soc, 49

[7] Doctor H D, Bulsari A B And Kalthia N L (1984)- Spline Collocation Approach To Boundary Value Problems. Int. J. Number, Floids. 4, 6,511.

[8] Fyfe D J (1969) -The Use Of Cubic Splines In Solutions Of Two Point Boundary Value Problems, Com.P.J, 12,188-192.

[9] Shastry S S (2005) - “Introductory Methods of Numerical Analysis. Prentice-Hall of India Private Limited. New Delhi.

[10] P. B. Choksi, A. K. Pathak, “Solve Non-Linear Parabolic Partial Differential Equation by Spline Collocation Method”, June 17 Volume 5 Issue 6 , International Journal on Recent and Innovation Trends in Computing and Communication (IJRITCC), ISSN: 2321-8169, PP: 387 – 391