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Minimizing Balanced Transportation Cost Problems of Homogenous Products

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Abstract - This paper presents a model for analyzing the generalized transport cost of distributing homogeneous products from three sources to four destinations. A line with this model an associated objective value is to minimize the transportation cost using mathematical methods or linear programming methods such as: North west-corner, Least-cost and Vogel approximation methods. The objective of this paper is necessity of using the methods to be having a minimum transportation cost better than of random use of distributing the homogeneous products from the sources to the shops. Transportation cost problem has been considered as one of the important applications of linear programming problems. The transportation problem model is to determine the schedule for transportation of goods from sources to destinations in such a way that minimize the road cost and satisfies all the demand and supply constraints. In this study, the above three methods for solving transportation cost problem has been discussed and compared for finding an optimal solution for a one problem formation which are using northwest-corner method is 2400 birr, Least-cost method is 2450 birr and using Vogel approximation method is 2300 birr but using random distribution of products from the sources to the shop is more than 2580 birr. Hence the preferable method of minimizing transportation cost is Vogel's approximation method.

Key words: Linear programming, Transportation problem, Transportation cost, Minimizing Transportation cost

I. INTRODUCTION

The transportation problem is one of the sub-classes of linear programming problems in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum [1]. Transportation problem is a special kind of linear programming problems in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination respectively, such that the total cost of transportation is minimized [2].

To achieve this objective we must know the amount and location of availability supplies and the quantities demanded in addition to the involvement of cost associated with each sending. Transportation tableau representation assumes that model is balanced, meaning that the total demand equals to the total supply. If the model is unbalanced we can add a dummy source or dummy destination to restore balance [3].

Cost refers to the trade-off between uses of resources. This can involves money, time, land or the loss of an opportunity to enjoy a benefit. Costs and benefits have a mirror-image relationship: a cost can be defined as a reduction in benefits and a benefit can be defined in terms of reduced costs. The transportation problem is a special class of linear programming problem which deals with road commodities from source to destination. It is mainly concerned in determining the schedule for transportation of goods to a specified place in such a way that minimizes the road cost and satisfies all the supply and demand constraints [4].

Transportation system makes goods and products movable and provides timely and safely to promote value added under the least-cost principle. Transportation affects the results of logistics activities and, of course, it influences production and sale. The value of transportation varies with different industries. Transportation occupies a very big part and affects profit more and therefore it is more regarded [5]. Transportation problem was first studied by F.L.Hitchock in 1941 and he presented a study entitled "The Distribution of a product from several source to numerous localities" [6]. Vehicle routing problem is an important problem in the field of transportation, distribution and logistics. Effective management of logistics incurs less transportation cost which can be done through vehicle routing problem [7].

Transportation is a key logistic function involving removable and relocation of material asset. A promising solution of the established problem is development of the methodological support including relevant models and methods of evaluating the transport and logistics operations. However, the methods considered in research publications are fragmentary and incomplete while mathematical methods used for optimizing the cost in transport and logistics systems need rework and concretization that would take in to account the specifics of the supply networks [8].

II. MODEL FORMULATION

A manufacturer wishes to transport m number of units of homogeneous products from several origins to n number of destinations. Each store requires, b_j by the j^{th} store, a certain number of units of the products while each ware house, a_i by the i^{th} warehouse, can supply up to a certain amount. The cost of transporting a unit from the i^{th} origin to the j^{th} destination is C_{ij} and is known for all combinations(ij). The problem is to determine the amount x_{ij} to be transported over all routes(i, j), so as to minimize the total cost of transportation.

This problem can be expressed in the tabular form as below:
Here the amount products from origin i to destination j is x_{ij} the total product from origin i is $a_i \geq 0$ and the received by destination j is $b_j \geq 0$, the total amount product is equal to the total amount received . That is, $\sum a_i = \sum b_j$.
The total cost of product x_{ij} units is $C_{ij}.x_{ij}$. Since negative products has no meaning, that is $x_{ij} \geq 0$.

	$j \downarrow$	Destinations					a_i
		$i \rightarrow$ 1	2	3	… … ..	n	
Sources (Origin)	1	C_{11}	C_{12}	C_{13}	… … ..	C_{1n}	a_1
	2	C_{21}	C_{22}	C_{23}	… … ..	C_{2n}	a_2
	3	C_{31}	C_{32}	C_{33}	… … ..	C_{3n}	a_3
	.	… … ..	… … ..	… … ..	… … ..	… … ..	… … ..
	.	… … ..	… … ..	… … ..	… … ..	… … ..	… … ..
	.	… … ..	… … ..	… … ..	… … ..	… … ..	… … ..
	m	C_{m1}	C_{m2}	C_{m3}	… … ..	C_{mn}	a_m
b_j		b_1	b_2	b_3	… … ..	b_n	

We can state the above problem mathematically as follow:
The value of x_{ij} for which the transportation cost is minimized. That is

Minimize $T = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}.x_{ij}$

Subject to

$\sum_{j=1}^{n} x_{ij} \leq a_i \quad , \quad i = 1, 2, 3, \ldots . . m$ (Availability constraints)

$\sum_{i=1}^{m} x_{ij} \geq b_j \quad , \quad j = 1, 2, 3, \ldots . . n$ (Requirement constraints)

And $x_{ij} \geq 0$

Here, $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Where

$m =$ Number of origin points to supply
$n =$ Number of destination points to receive
a_i be the supply at the source i
b_j be the demand at the destination j
C_{ij} be the cost of transportation per unit from the source i to the destination j
x_{ij} be the number of units to be transported from the source i to the destination j

The transportation model seeks the determination of a transportation plan of a single commodity or product from a number of sources to a number of destinations. The model has the following basic assumptions: [9].

a) Availability of products (commodity): The supply available at different sources is equal to or more than the total demand of the different destinations when it is equal it is called balanced problem.
b) Transportation of commodity or items: The model assumes that all item is can be conveniently transported from sources to destinations.
c) Certainty of per unit transportation cost: There is a definite cost of transportation of products from sources to destinations.
d) Independent cost per unit: The per unit cost is independent of the quantity transported from sources to destinations.
e) Transportation cost on any given route is proportional to the number of units transported.
f) Objective function: The objective is to minimize the total transportation cost for the entire organization.

III. Problem Formulation

An industry has three bakeries so, breads have distributed to four shops daily from the bakeries as of the transport cost per distribution per unit bread monthly as the table below. Considering of balanced transporting problem:

Table3.1: Distribution of breads from bakeries to shops with a unit cost

	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop2 (T_2) (Unit cost/birr)	Shop3 (T_3) (Unit cost/birr)	Shop4 (T_4) (Unit cost/birr)	
Source (bakeries)	№ 1	3	5	7	6	150
	№ 2	2	5	8	2	250
	№ 3	3	6	9	2	100
Demand		50	150	150	150	500

Here are, the transporting cost of the breads from Bakery № 1 to the destinations T_1, T_2, T_3, and T_4 has needed $3\ birr, 5\ birr, 7\ birr$ and $6\ birr$, respectively. Again the cost from bakery № 2 to the destinations T_1, T_2, T_3, and T_4 has needed $2\ birr, 5\ birr, 8\ birr$ and $2\ birr$, respectively. Similarly from bakery № 3 to the destinations T_1, T_2, T_3, and T_4 has needed $3\ birr, 6\ birr, 9\ birr$ and $2\ birr$, respectively. The supply of products of breads from Bakeries of № 1, № 2 and № 3 are 150, 250 and 100 in number respectively and the demands of T_1, T_2, T_3, and T_4 are 50, 150, 150 and 150 in number, respectively.

IV. Results And Discussions

The transportation problem is a distribution type linear programming problem, concerned with transferring goods from sources to destinations. In case its main goal is to minimize the costs, the problem is known as minimizing. So, here below is the comparison of transport cost of distributing products from sources to destinations by random distribution method and using programming models such as: Northwest-corner method, Least-cost method and Vogel approximation method. Hence the result of the same problem has compared below using these methods.

1.1. *Computing transportation cost using random distribution method*

Table 4.1.1: Distribution of breads from sources to the destination per unit cost using random distribution

Source (bakeries)	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop2 (T_2) (Unit cost/birr)	Shop3 (T_3) (Unit cost/birr)	Shop4 (T_4) (Unit cost/birr)	
	№ 1	3	5	7	6	150
	№ 2	2	5	8	2	250
	№ 3	3	6	9	2	100
Demand		50	150	150	150	500

Here, the products which are distributed from the source № 1 to all shops T_1, T_2, T_3, and T_4 and the source № 2 also distributes to all the four shops. Similarly the source № 3 distributes the breads to all shops with the balanced transportation problem. Hence to arrive all the products to the shops it needs transportation cost as follow:

Table 4.1.1.1: Distribution of breads from sources to the destination of balanced transportation problem using random distribution.

Source (bakeries)	Source name	Destination				Supply
		Trader 1 (T_1) (Unit cost/birr)	Trader 2 (T_2) (Unit cost/birr)	Trader 3 (T_3) (Unit cost/birr)	Trader 4 (T_4) (Unit cost/birr)	
	№ 1	3 20	5 30	7 50	6 50	150
	№ 2	2 10	5 100	8 80	2 60	250
	№ 3	3 20	6 20	9 20	2 40	100
Demand		50	150	150	150	500

Transportation cost $= 20*3+30*5+50*7+50*6+10*2+100*5+80*8+60*2+20*3+20*6+20*9+40*2$

$= 2580\ birr$

If there is using the transporting road out of the above which distributes randomly it may increase the transportation cost. Thus, it is better to use a linear programming model to minimize the transportation cost of the products to distribute from the sources to the destinations by applying concept of modeling as follow.

1.2. *Methods to solve transportation problem with a minimum cost*

A general transportation model with m sources and n destinations has $m+n$ constraints equations, one for each source and each destination. However, because the transportation model is always balanced (that is, $sum\ of\ supply = sum\ of\ demand$). One of the equation is redundant, reducing the model to $m+n-1$ independent equations and $m+n-1$ basic variables [2].

The special structure of the transportation problem allows securing a non artificial starting basic solution using one of the following three methods:

I. Northwest-corner method
II. Least-cost method
III. Vogel approximation method (VAM)

Northwest-corner method

The method starts at the north west-corner cell of the tableau variable C_{11}

Step1: Allocate as much as possible to the selected cell and adjust the associated amounts of supply and demand by subtracting the allocated amount.

Step2: Cross out the row or column with zero supply or demand to indicate that not further assignments can be made in that row or column.

Step3: If exactly one row or column is left uncrossed out stop. Otherwise move to the cell to the right if a column has just been crossed out or below. If a row has been crossed out go to step 1

[3, 6].

Table 4.2.1: The transportation cost for which a product distributes from three sources to four destinations with a minimum cost using North west-corner method.

So ur		Destination	Supply

	Source name	Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)	
	№ 1	3	5	7	6	150
	№ 2	2	5	8	2	250
	№ 3	3	6	9	2	100
Demand		50	150	150	150	500

Here is balanced transporting problem because of sum of supplies and sum of demands are equal. So, the objective is minimizing the transportation cost during the distribution of the products from the three sources to the four destinations. Consider $C_{11} = 3$ and allocate the cell and adjust the supply based on the demand. Hence it crossed out the first column with zero demand. This indicates that there no need of further assignments in that column.
Again the remaining supply should assign to $C_{12} = 5$ with the consideration of the values of demand. So that crossed out the first row with zero supply. This also not need of further assignment. Because of this we have to proceed to $C_{22} = 5$ and assign the supply based on the remaining demand. So, the second column crossed out with zero demand. Similarly proceed to $C_{23} = 8$ and assign the remaining supply to the cell according the demand and it becomes crossed out the third column with zero demand. Next consider $C_{24} = 2$ and assign the remaining supply and crossed out the second row with zero supply. Lastly allocate the remaining cell $C_{34} = 2$ and assign the supply to the cell according the demand and becomes crossed out the third row and fourth column with zero supply and demand.
Finally the transportation problem becomes balanced, that is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
and for the sake of minimizing the cost of transportation, when the product arrives from the sources to the destinations it should goes through:

$$C_{11} \longrightarrow C_{12} \longrightarrow C_{22} \longrightarrow C_{23} \longrightarrow C_{24} \longrightarrow C_{34} \longrightarrow$$

Therefore, we can summarize the above transportation problem in table as follow:

Table 4.2.1.1: Summarized transportation cost using Northwest-corner method

	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)	
Source (bakeries)	№ 1	3 50	5 100	7	6	150 50 100 0
	№ 2	2	5 50	8 150	2 50	250 50 150 50 0
	№ 3	3	6	9	2 100	100 100 0
Demand		50 50 0	150 100 50 0	150 150 0	150 50 100 0	500 500 0

Thus, the transportation cost which minimize using the Northwest-corner method is:

$$\text{Minimize } T = \sum_{i=1}^{4}\sum_{j=1}^{4} C_{ij}.x_{ij} = C_{11}.x_{11} + C_{12}.x_{12} + C_{22}.x_{22} + C_{23}.x_{23} + C_{24}.x_{24} + C_{34}.x_{34}$$

$$= 3*50 + 5*100 + 5*50 + 8*150 + 2*50 + 2*100$$

$$= 2400\ birr$$

Least-cost method

The least cost method finds a better starting solution by targeting the cheapest routes. It assigns as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted according. If both a row and a column are satisfied simultaneously, only one crossed out, the same as in the northwest-corner method. Next select the uncrossed out cell with the smallest unit cost and repeat the process until exactly one row or column is self uncrossed out [2].

Table 4.2.2: Product distributes from sources to destinations with a minimum cost using Least-cost method.

	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)	
Source (bakeries)	No 1	3	5	7	6	150
	No 2	2 [start]	5	8	2	250
	No 3	3	6	9	2	100
Demand		50	150	150	150	500

To compute the minimum transportation cost using the least cost method is as follow:

- Cell C_{21} , $cell\ C_{24}\ and\ cell\ C_{34}$ has the least unit cost in the tableau ($= 2\ birr$). The most that can be shipped through $C_{21}\ is\ x_{21} = 50$. So, cross-out column one because it is satisfied.
- C_{24} has the smallest uncrossed out unit cost ($= 2birr$). Assign $x_{24} = 150$ and crossed out column four because it is satisfied.
- C_{12} has the least unit cost ($= 5birr$). Assign $x_{12} = 150$ and crossed out column two because it is satisfied.
- C_{23} has the smallest uncrossed out unit cost ($= 8\ birr$) and again C_{33} has the next smallest uncrossed out unit cost ($= 9\ birr$). So, crossed out column three because it is satisfied.

The resulting starting solution is summarized in the table below. The rows show the order in which the allocations are made.

Table 4.2 2. 1: The least-cost starting solution table

	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)	
Source (bakeries)	№ 1	3	5 [3] 150	7	6	150
	№ 2	2 [start] [1] 50	5	8 [4] 50	2 [2] 150	250
	№ 3	3	6	9 [End] 100	2	100
Demand		50	150	150	150	500

For the sake of minimizing the cost of transportation for which the product arrives from the sources to the destinations it should goes through:

$$C_{21} \longrightarrow C_{24} \longrightarrow C_{12} \longrightarrow C_{23} \longrightarrow C_{33}$$

The associated objective value that is the minimum transportation cost using this method is:

$$\text{Minimize } T = \sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}.x_{ij} = C_{11}.x_{11} + C_{13}.x_{13} + C_{12}.x_{12} + C_{22}.x_{22} + C_{33}.x_{33}$$

$$= 2 * 50 + 2 * 150 + 5 * 150 + 8 * 50 + 9 * 100$$

$$= 2450\ birr$$

Vogel Approximation method (VAM)

VAM is an improved version of the least –cost method that generally, but not always, produces starting better solutions.

Steps in Vogel's approximation method: [6].

Step1: Find the penalties of each row and each column by finding the difference between the two minimum values of that particular row or column.
Step2: Select the highest penalty from the rows and columns.
Step3: Choose the row or column corresponding to the chosen highest penalty.
Step4: Along the selective row or column select the cell with minimum value.
Step5: Allocate the particular cell from the demand and supply. If the demand or supply is fully allocated delete the particular row or column.
Step6: Now again find the penalties of the remaining rows or columns.
Step7: Continue the process until all the demand and supply are allocated.

Table 4.2.3: Transportation cost from the sources to the destinations

	Source name	Destination				Supply
		Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop r 4 (T_4) (Unit cost/birr)	
Source (bakeries)	№ 1	3	5	7	6	150
	№ 2	2	5	8	2	250
	№ 3	3	6	9	2	100
Demand		50	150	150	150	500

The transportation cost of the above using Vogel's approximation method to achieve a minimum cost when we compared without using a model and other methods.

Table 4.2.3.1: Transportation cost of products from source to destination using Vogel approximation method.

	Source name	Destination				Supply	Penalty
		Shop 1 (T_1) (Unit cost/birr)	Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)		
Source (bakeries)	№ 1	3 50	5	7	6	150 50	2 (highest)
	№ 2	2	5	8	2	250	0
	№ 3	3	6	9	2	100	1
Demand		50 50	150	150	150	500	
Penalty		1	1	1	0		

$Transportation\ cost\ = 50 * 3 + - - -$

Here, the first column should be deleted since the demand of it fully allocated.
Now again,

Table 4.2.3.2: Result after deleting column one

	Source name	Destination			Supply	Penalty
		Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)		
Source (bakeries)	№ 1	5	7	6	150 50	1
	№ 2	5	8	2	250	3
	№ 3	6	9	2 100	100 100	4 (highest)
Demand		150	150	150 100	500	
Penalty		0	1	0		

$Transportation\ cost\ = 100 * 2 + - - -$

The third row is deleted because the supply of it fully allocated.

Table 4.2.3.3: Result after deleting row three

	Source name	Destination			Supply	Penalty
		Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)	Shop 4 (T_4) (Unit cost/birr)		
Source (bakeries)	№ _1	5	7	6	150 50	1
	№ 2	5	8	2 50	250 50	3
Demand		150	150	150 100 50	500	
Penalty		0	1	4 (highest)		

Transportation cost = $50 * 2 + - - -$

Column four should delete because it's the demand is fully allocated.

Table 4.2.3.4: Resulting after deleting column four

	Source name	Destination		Supply	Penalty
		Shop 2 (T_2) (Unit cost/birr)	Shop 3 (T_3) (Unit cost/birr)		
Source (bakeries)	№ 1	5	7	150 50	2
	№ 2	5 150	8	250 50 150	3 (highest)
Demand		150 150	150	500	

Penalty	0	1		

Transportation cost $= 150 * 5 + - - - -$
Here also column one should delete because it's demand is fully allocated.
Table 4.2.3.5: Resulting after deleting column one

Source (bakeries)	Source name	Destination	Supply	Penalty
		Shop 3 (T_3) (Unit cost/birr)		
	№ 1	7 100	150 50 100	
	№ 2	8 50	250 50 150 50	
Demand		150 100 50	500	
Penalty		1		

Transportation cost $= 100 * 7 + 50 * 8 + - - - -$
Therefore, the total transportation cost using the Vogel's approximation method is :

$$= 50 * 3 + 100 * 2 + 50 * 2 + 150 * 5 + 100 * 7 + 50 * 8$$
$$= 2300 \, birr$$

Thus, the solution for the transportation cost problem of table 1, when we compare the three methods that are Northwest-corner method, Least -cost method and Vogel approximation methods, the Vogel approximation method happens to have a better objective value than the other methods. That is the transportation cost is minimum using the Vogel approximation method.

V. Conclusions And Recommendations

The total transportation cost has been calculated without any method that is random distribution and using different methods for the validation and comparison of the result. The transportation cost without using any method or the cost of simple distribution of all products from the sources to the destinations is greater than of $\mathbf{2580}\ \boldsymbol{birr}$. But the transportation cost using the methods: Northwest-corner method, Least-cost method and Vogel approximation method have minimum transportation cost and their values are $\mathbf{2400}\ \boldsymbol{birr}$, $\mathbf{2450}\ \boldsymbol{birr}$ and $\mathbf{2300}\ \boldsymbol{bir}$, respectively.

Table 5.1: Comparison of transportation cost for different methods.

Types of methods	Transportation cost (in birr)
Random distribution	≥ 2580
Northwest-corner	2400
Least- cost method	2450
Vogel approximation method	2300

With these results of present study, to achieve the minimum transportation cost of distributing products from the sources to destinations is preferable use of the above methods better than of distributing randomly. And the Vogel approximation method is reliable and appropriate method to minimize the transportation cost when it compared with the other methods. Thus, the obtained result substantiates the effectiveness of the application of the proposed method. This work can also refine further for different factories.

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