The Application of stochastic differential equation on rate of budget implementation: the case of Adigrat University

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***Abstract* - This paper is devoted for the application of stochastic differential equation in the rate of budget implementation the case of Adigrat University. Here I consider a data of five years budget implemented in the university. I developed a model in forecasting a rate of budget implementation using continuous stochastic process which is the sum of a predictable and an unpredictable part. However, the model does not take into account the breakup of budget exploited in the given institution. I also included monte carlo simulation of rate of budget implementation and estimated the probability of the rate of budget implementation in the given institution which will foster the execution of the budget activities in the entire institution based on the planned budget.**

***Key words* - Implementation rate, stochastic, Brownian motion, forecasting.**

I. INTRODUCTION

Today, the state of condition entirely or partly become idealistic and being altered. Environmental changes of several classes are hurrying globally and trick significant challenges for humankind. The failure in implementing the approved budget is one of the encounters in which institutions faced difficulties to carry out their activities since it is a means to achieve their objectives. These challenges might be occasioned because of irregular expenditures, overspending, lack of proper monitoring in revenues and expenses, price fluctuations, feeble responsibility and accountability of principals, Subjectivity in the process of drafting the budget, weak commitment of individuals and more. Consequently, the rate of budget implementation varies from year to year in which eventually affects the development of the institution. Besides, it is clear that the level of strength and achievement of the *implementation rate varies* from one unit to another. Hence, everybody is probably not relying and uncertain on the instrument executed in the entire organization.

Accordingly, it is with the sort of stochastic model that we have been looking, a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. The random variables are usually based on fluctuations observed in historical data for a selected periods. We are not very interested in forecasting what the amount of budget implemented in the next year, but we are interested in knowing the variability in likely rates of budget implementation over certain years or so. The forecasting using stochastic model refers to a random variable $X(t)$ and predicts future events for $X(t)$ depending on the initial conditions given.

Henceforth, this research paper is an application for the development of a stochastic model which will enable us to forecast rate of budget implementation taking into consideration the case of Adigrat University in five years execution periods. For predicting the rate of budget implementation we demand seeking the best possible estimate of what the rate of implementation of the budget looks like in the next few years. To this end, we use the notion of each service year measured the rate of the implemented budget in the given institution.

II. OBJECTIVE OF THE STUDY

2.1. General objective

The overall objective of this study is to apply the stochastic differential equation in rate of budget implementation in the case of Adigrat University.

2.2. Specific objectives

The specific objectives are;

- ✓ To develop stochastic model in rate of budget implementation.
- ✓ To detect the volatility of rate of budget implementation.
- ✓ To forecast the rate of budget implementation using Brownian motion.

III. BUDGET IMPLEMENTATION

An Approved budget, irregular expenditure (not in accordance with), overspending (means causing the operational or capital expenditure incurred by the municipality during a financial year to exceed the total amount appropriated in that year's budget for its operational or capital expenditure, as the case may be), "past financial year. "Service delivery and budget implementation

plan" means a detailed plan approved by the mayor of a municipality in terms of section. Unauthorized expenditure (any expenditure incurred by a municipality otherwise than in accordance).
The municipality shall not budget for a deficit and should also ensure that revenue projections in the budget are realistic taking into account actual collection levels. Expenses may only be incurred in terms of the approved annual budget. There will be instances where unforeseen and unavoidable expenditure is required, as well as other circumstances which could necessitate the submission of an Adjustments Budget.

Monitoring
The accounting officer with the assistance of the chief financial officer and other senior managers is responsible for the implementation of the budget, and must take reasonable steps to ensure that:

- ✓ funds are spent in accordance with the budget
- ✓ expenses are reduced if expected revenues are less than projected
- ✓ Revenues and expenses are properly monitored.

Faces difficulties of internal nature, characterized by increased costs, poor discipline in the implementation of tasks, lack of transparency and clarity in the delivery system of the entity, which directly determines the worsening of financial indicators.

Difficulties and advantages

- ✓ Lack of budget coordination with the strategy of the entire entity,
- ✓ Heads of departments do not participate in the approval and control of budgets,
- ✓ Reduced data specification and control. Often, developing a budget requires more time, sometimes even a year
- ✓ Subjectivity in the process of drafting the budget. Some managers involved in the development budget increases or decreases some indicators intended to have a backup if the owners will require their reduction or growth.

The implementation phase is characterized by the adoption of the budget plans that are known to the responsible persons, managers. In this context, the staff shall carry out measures to implement the plan, setting results through various tasks and documents.
It is worth mentioning that there is a clear link between the planning phase and the implementation phase because depending on the existing real circumstances certain corrections can be made in planned actions. First of all, control and correction may occur in cash flow budget.
The traditional role of the central budget office is incompatible with the management reforms enfolding in various OECD Member countries. These reforms are grounded on the principle that managers must be permitted to run their operations without undue outside interference. The logic of reform is that only when managers are free to use money and other organizational resources within agreed budgets can they be responsible for the organization's successes or failures. In countries where a culture of reform has taken hold, there is consensus that halfway measures do not suffice, that managers either are free to act or are not. It is not a matter of relaxing one or another restriction, but of reshaping the operations of public institutions and the behavior of those who work in them. The budget process is one of the main areas in which the machinery of government is undergoing fundamental transformation.
Governments facing these tensions cannot effectively control the budget by operating the detailed controls that served them so well in the past. Trying to do so may weaken their control of the totals, hobble efforts to reprioritize the budget, and degrade the efficiency of public expenditure. This is the paradox of contemporary budgeting: central control of the items of expenditure may not give the center effective control of the budget [1].
As [5], in implementing the budget, managers enjoy great flexibility in the use of appropriations. This is partly due to the institutional organization of public administration in Sweden.

Budget
A budget is simply the statement of expected income and expenditure over a time period, usually a year of the government. Governments at all levels do envisage how much they are likely to generate from all source available to them. At the same time, they visualize what the expenditure will be. The income side of a budget normally does include loans sourced both internally and externally. In essence, budget has become the means by which governments achieve their objectives. Beyond the state, budgeting concerns virtually everybody.

Constraints
Most of the plans were over-ambitions and unattainable, considering the resources (human and natural) that abide in the country. Second, weak database is a major hindrance too. In Nigeria and several other third world countries, paucity of data is prevalent. In most cases forecasts are done based on grossly inadequate information, thereby, resulting into distorted growth. Thirdly, there are a number of political constraints on public policy implementations. First, there must be political will-to-rule and plan. Thirdly, because of lack of managerial effectiveness in developing countries.
Furthermore, budget indiscipline is a factor that needs to be emphasized too. The chief objective of drawing up budget is to achieve national spending through careful consideration of priorities and options.
Today it is realized that the fully and effective market is unrealistic: price fluctuations sometimes cannot immediately reflect all the information, traders and investors are probably not relying only on the current price when they are doing transactions.
The intensity and success of their implementation varies from one unit to another. However, participants acknowledged that the level of implementation varies considerably between countries and some regions, in particular Africa, are less advanced than others. The level of implementation varies significantly across the United Nations Economic Commission for Europe (UNECE)

region, depending, inter alia, upon countries' legal traditions, experiences in governance and socio-economic conditions. Progress in implementation varies, and generally correlates with proximity to the EU, in both a geographic sense and in terms of the accession process [1].

IV. Some Mathematical Preliminaries

Definition: A triple $(\Omega, \mathcal{F}, P)$ is called a probability space provided Ω is any set, $\mathcal{F}$ is a σ-algebra of subsets of Ω, and P is a probability measure on $\mathcal{F}$ [4].

Definition: Let $(\Omega, \mathcal{F}, P)$ be a probability space. A mapping $X: \Omega \to R^n$ is an n- dimensional random variable if X is $\mathcal{F}$-measurable.

Definition: Let X be a continuous random variable with probability density function $f(x)$, then

i. The expected value of is defined as $E[X] = \mu = \int_{-\infty}^{\infty} xf(x)dx$, and
ii. The variance, $Var[X] = \sigma^2 = E\{[X - E(X)]^2\}$

Definition: A stochastic process.is a parameterized collection of random variables $\{X(t): t \geq 0\}$, defined on a probability space $(\Omega, \mathcal{F}, P)$ and assumes values in R^n.

An important example of a stochastic process is a Brownian motion defined below.

Definition: A real valued stochastic process $\{B(t) : t \geq 0\}$ is called a (linear) Brownian motion with start in $x \in R$ if the following holds:

I. $B(0) = x$
II. The process has independent increments, i.e., for all times $0 \leq t_1 \leq t_2 \ldots \leq t_n$ the increments $B(t_n) - B(t_{n-1})$, $B(t_{n-1}) - B(t_{n-2})$, $\ldots B(t_2) - B(t_1)$ are independent random variables,
III. For all $t \geq 0$ and $h > 0$, the increments $B(t+h) - B(t)$ are normally distributed with expectation zero and variance h,
IV. The process $B(t)$ is continuous in t.

We say that $\{B(t) : t \geq 0\}$ is a standard Brownian motion if $x = 0$ [7].

Ito's Formula: Suppose $f(X,t)$ is an infinitely differentiable function then,

$$df(X,t) = \frac{\partial f(X,t)}{\partial X} dX + \frac{1}{2}\frac{\partial^2}{\partial X^2} f(X,t)dt + \frac{\partial}{\partial t} f(X,t)dt \qquad (1)$$

An informal derivation starts by expanding df in Taylor series in dX and dt up to second order in dX and first order in t , where $(dX)^2 = dt$ [3].

Here, we can extend the idea of Ito's formula for Brownian motion as;

$$df(B,t) = \partial_B f(B,t)dB + \frac{1}{2}\partial_B{}^2 f(B,t)dt + \partial_t f(B,t)dt$$

V. Modelling Rate of Budget Implementation

In this section we develop a model in forecasting a rate of budget implementation using continuous stochastic process which is the sum of a predictable and an unpredictable part. However, the model does not take into account the breakup of budget exploited in the given institution.

Let R_t denote the rate of a budget implementation at time t. We consider R_t is a continuous process. The change on the rate of the budget implementation during the time interval Δt measures a percentage in the rate of the budget implementation between instances t and $t + \Delta t$ which is given by $\frac{R_{t+\Delta t} - R_t}{R_t}$. When Δt is infinitely small, we obtain the instantaneous change $\frac{dR_t}{R_t}$. This is supposed to be the sum of two components: The predictable part μdt and the noisy part due to unexpected news σdW_t.

Adding these parts yields

$$\frac{dR_t}{R_t} = \mu dt + \sigma dW_t \,,$$

Which leads to the stochastic equation

$$dR_t = \mu R_t dt + \sigma R_t dW_t$$

The parameters μ and σ are positive constants which represent the drift and volatility of the rate of budget implementation. This equation can be solved using method of variation of parameters.

Let's divide the entire equation by R_t and obtain the expression

$$\frac{dR_t}{R_t} = \mu dt + \sigma dW_t$$

Switch to the integral form

$$\int \frac{dR_t}{R_t} = \int \mu dt + \int \sigma dW_t$$

Integrating blindly and get

$$\ln R_t = \mu t + \sigma W_t + c$$

Where c is a constant integration. We arrive at the following "pseudo-solution"

$$R_t = e^{\mu t + \sigma W_t + c}$$

Assume the constant c is replaced by a function $c(t)$, so we are looking for a solution of the form

$$R_t = e^{\mu t + \sigma W_t + c(t)} \qquad (2)$$

Apply Ito's formula (Eq.1) we get

$$dR_t = R_t\left[\mu + c'(t) + \frac{\sigma^2}{2}\right]dt + \sigma R_t dW_t$$

Subtracting the initial equation yields

$$\left(c'(t) + \frac{\sigma^2}{2}\right)dt = 0$$

Which satisfied for $c'(t) = -\frac{\sigma^2}{2}$, with the solution $c(t) = -\frac{\sigma^2}{2}t + k$, $k \epsilon R$. Substituting into (2) yields the solution

$$R_t = e^{\mu t + \sigma W_t - \frac{\sigma^2}{2} + k} = e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t + k} = R_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t} \qquad (3)$$

Where R_0 denotes the rate of budget implementation at time $t = 0$. It is worth nothing that rate of the budget implementation is $\mathcal{F}_t$ – adapted, positive and it has a log-normal distribution. At each time the Geometric Brownian Motion has lognormal distribution with parameters $\left(\ln(R_0) + \mu t - \frac{\sigma^2}{2}t\right)$ and $\sigma\sqrt{t}$.

By definition, the mean and variance are

$$E[R_t] = R_0 e^{\mu t}$$

$$Var[R_t] = {R_0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$$

Table 1: The historical rates of budget implementation in the University

year	2004	2005	2006	2007	2008
Rate of Implementation (%)	80	94.3	95.2	96	74.9

We first compute estimation of the expected value and variation of the ln of the rate of implementation ratios.

Table 2: variance, drift, expected value and volatility

$\ln\left(\frac{R_t}{R_{t-1}}\right)$		-0.16445	-0.0095	-0.00837	0.248194
Variance	0.02924				
drift	0.031088				
Expected value	0.016468				
Volatility	0.170998				

The Expected Value

$$V = \frac{1}{4}\sum_{i=1}^{4} \ln(R_t/R_{t-1}) = 0.016468$$

And the variance

$$S^2 = \frac{1}{3}\sum_{i=1}^{3}[\ln\left(\frac{R_t}{R_{t-1}}\right) - V]^2 = 0.02924$$

Thus, the annual volatility and drift are given respectively as

$$\sigma = \sqrt{S^2} = \sqrt{0.02924} = 0.170998$$

And

$$\mu = V + \frac{S^2}{2} = 0.031088$$

The following is the possible rate of budget implementation for the next five years in the given institution which is the brownian of the monte carlo simulation generated using random number.

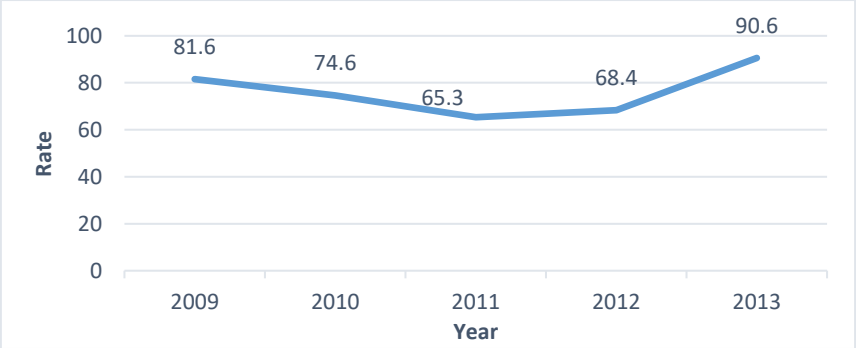

Fig 1: Monte carlo simulation of rate of budget implementation

The expected rate of implementation three years from now that is in 2011 is given by:

$$E[R_t] = R_0 e^{\mu t} = (74.9)e^{0.031088*3} = (74.9)*(1.1) = 82.22\%$$

Besides, we can calculate the time when the institution would implemented a rate of 100% using

$$E[R_t] = R_0 e^{\mu t}$$

$$100 = (74.9)e^{(0.031088)t}$$

$$1.34 = e^{(0.031088)t}$$

Which leads to $\ln(1.34) = (0.031088)t$ or $t = 9.414231 \approx 9$ years and 4 months.

As a consequence we can predict the probability of the rate of budget implementation will be at least 90% three years from now as:

Since R_t is a geometric Brownian motion, $\ln(R_t)$is a regular Brownian motion with 0.031088 drift and $\sigma = 0.170998$. We want to know the probability that $\ln(R(3)) \geq \ln(90)$ given that $\ln(R(0)) \geq \ln(74.9)$. This means;

$$\ln R(3) - \ln R(0) \geq \ln(90) - \ln(74.9) = \ln\left(\frac{90}{74.9}\right) = \ln(1.2) = 0.18$$

In this case $Z = \frac{(\ln R(3) - \ln R(0)) - (0.031088\,)(3)}{0.170998\sqrt{3}} = \frac{0.18 - 0.093}{0.296} = \frac{0.087}{0.296} = 0.293$ is a standard normal random variable. So

$$Pr\left\{\frac{(\ln R(3) - \ln R(0)) - (0.031088\,)(3)}{0.170998\sqrt{3}} > 0.18\right\} = Pr\{Z > 0.293\} = 1 - Pr\{Z \leq 0.293\}$$

$$= 1 - 0.6141 = 0.3859 \approx 39\%$$

This indicates that the rate of budget implementation at least 90% in the given institution three years later (2011) would be less chance (39%) of execution.

VI. Conclusion

To summarize, the paper presented the application of stochastic differential equation rate of budget implementation in Adigrat University. The stochastic model is developed and is given by

$$R_t = R_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

Where R_t denote the rate of a budget implementation at time t , parameters μ and σ are positive constants which represent the drift and volatility of the rate of budget implementation. With the help of this model it projected and estimated the rate of budget implementation in the next years. The budget allocated for the institution comprised capital budget and regular budgets. Capital budget is mainly used for construction works and for foreign investment, Teachers' salaries and procurement of capital goods for the university. There is also a transferring the transactions from one account to another according to the law if the payment could not be executed because it was not done. That is why the flow of money was made which may lead to vary the rate of budget implementation.

Besides, the study shows that the budget utilization of the university is not consistent and that annual budget performance is reflected in the changes that are attributable to the low commitment of the administration, lack of experts in finance and inappropriate planned budget. It is known that rumors will swarm when budget falls are announced. The chatter lines will be working overtime both within the department and on the campus. Hence, it is recommended that one of the most useful ways to help colleagues in a time of uncertainty is to provide clear information to them as rapidly as possible. Even if you do not yet fully know what is going on, let staff and faculty members in the unit know that. When the strategy being employed by the institution is known, share that information with all members of the unit in as clear and concise a manner as possible. Eventually, this model gives an evidence for the rate of budget implementation whether it drives practical or not.

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