Analytical Solution for Ultimate bearing capacity of strip footing seated on inclined backfill

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Abstract- With the increase in infrastructure facilities in hilly regions on earth, simultaneous construction of buildings started. Low height residential and commercial buildings preferred shallow foundations. Footings rested on inclined ground is totally different then it is rested on flat surface. Meyerhof first proposed analytical solutions for footing rested on inclined ground, which is further extended by various researchers. In this paper, analytical solution have been developed for obtaining the ultimate bearing capacity of a footing on slope. Rankine's earth pressure theory was used to perform the one-sided equilibrium analysis. The results have been presented for different values of slope angles for the c-φ soil. Analytical results have been compared to the Meyerhof's theoretical solution that determined the ultimate bearing capacity of a shallow foundation located on the face of a slope.

Index Terms - Footing on inclined slope, Analytical method, Meyerhof theory, Slope angle β, C-Ø soil, Ø –soil.

I. INTRODUCTION

This paper presents analytical solution for calculation of the ultimate bearing capacity of footing with varying angle of θ by Rankine's earth Pressure theory. Rankine's earth Pressure theory, one of the most important earth pressure theories, is still used because of its rigorous theory, clear concept and simple calculation. It was used to perform the one-sided equilibrium analysis for the calculation of ultimate bearing capacity of the footing. With graphic method about Mohr's circle of Stresses [3] we have developed an analytical solution [1][4] to determine the active and passive lateral earth pressure distribution on the footing when a cohesive backfill is inclined.

Rankine's earth pressure theory assumes the failure surface on which the soil moves to be planar. It assumes that failure will occur when the maximum principal stress at any point reaches a value equal to the tensile stress in a simple tension specimen at failure. This theory does not take into account the effect of the other two principal stresses.

II. ANALYSIS

Using the Rankine's Earth Pressure Theory, we can derive the expression for the bearing capacity of a strip footing placed on a sloping ground.
As shown in the Figure 1, the bearing capacity can be calculated from the surcharge from the side of the footing facing the slope[4].

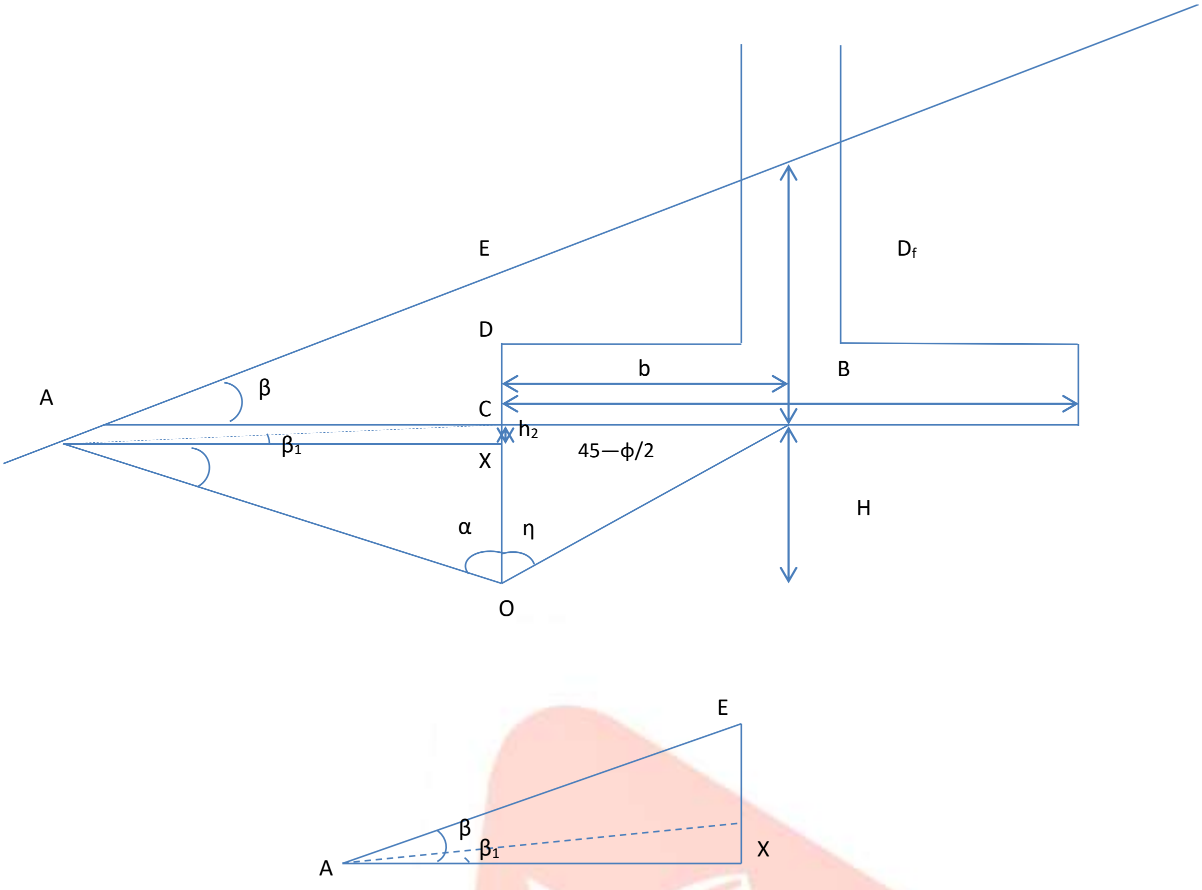


Figure 1: Failure Surface of the soil using Rankine's Theory

Assumptions:

- The soil mass is semi-infinite and homogeneous
- Failure surfaces are assumed to be linear
- β_1 is assumed to be proportional to β and is positive
- Water table is much below the zone of influence
- Active zone is considered from center of the footing

III. FOR THE SIDE FACING THE SLOPE:

The surcharge can be calculated using the weight of the soil in the triangular region CAE and the length AC.

$$q=\frac{W}{AC} \quad \ldots\ldots\ldots\ldots (1)$$

From the Rankine's theory, making use of the vertical and normal stresses, we can calculate the resistance coefficients for active and passive case as shown in Figure 2.

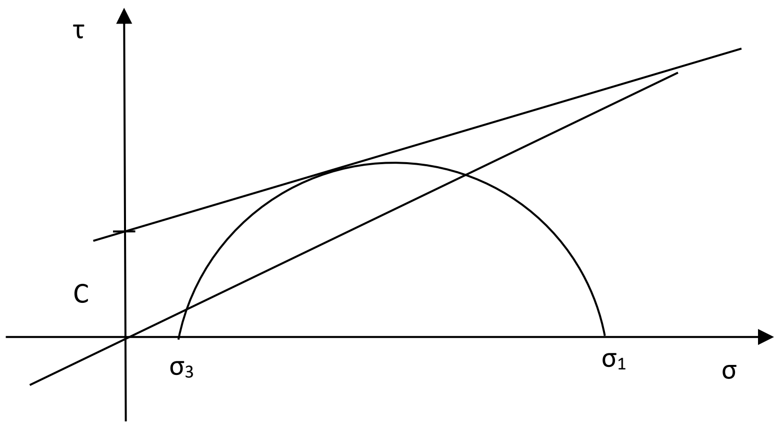


Figure 2: Mohr-Coulomb Failure

$$\left.\begin{aligned}\sigma_n &= \sigma_v \cos\beta \\ \sigma_v &= \gamma z \cos\beta \\ \sigma_n &= \gamma z \cos^2\beta\end{aligned}\right\} \quad \text{......... (2)}$$

According to Rankine's Theory,

For c- ϕ soil:

$$\left.\begin{aligned}K_a &= \frac{\cos\beta\{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}\}}{\{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}\}} \\ K_p &= \frac{\cos\beta\{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}\}}{\{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}\}}\end{aligned}\right\} \quad \text{....... (3)}$$

For cohesive soil (ϕ =0):

$$\left.\begin{aligned}K_a &= \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45 - \frac{\phi}{2}) \\ K_p &= \frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45 + \frac{\phi}{2})\end{aligned}\right\} \quad (4)$$

From the geometry of the figure we can calculate the different dimensions required for the force calculation, as shown below:

$\beta_1 = k.\beta$ .... (5)

$$H = \frac{b}{\tan\alpha} = \frac{B}{2.\tan(45 - \frac{\phi}{2})}$$

$$CX = \frac{D_f + h_2}{\tan\beta} - \frac{B}{2}$$

$$H_1 = \tan\beta(\frac{D_f}{\tan\beta} - \frac{B}{2})$$

$$a = \frac{H_1}{1 - \frac{\tan\beta_1}{\tan\beta}}$$

$$h_2 = a\tan\beta$$

$$AC = \sqrt{h_2^2 + CX^2}$$

The area of the region under which the weight of the soil is supposed to be acting is :

$$\Delta = \frac{1}{2}.CX.CX\tan\beta - \frac{1}{2}.CX.h_2 \quad \text{....... (6)}$$

Now, we can determine the expression of ultimate stress, as shown below:

$$q_n = \frac{\gamma.\Delta}{AC}$$

$$q_v = \frac{q_n}{\cos\beta_1}$$

$$q_h = K_p.q_v$$

$$F_a = \frac{K_p.\gamma.\Delta}{\cos\beta_1.AC}.H + \frac{1}{2}.K_p.\gamma.H^2$$

$$q_u = \frac{K_p.\gamma.\Delta}{AC\cos\beta_1}.\frac{1}{K_a} + \frac{1}{2}.\frac{K_p.\gamma.H}{K_a} - \frac{1}{2}.\gamma.H + \frac{2(\sqrt{K_p} - \sqrt{K_a})c}{K_a}$$

As stated in our assumptions, the point on the footing from where the soil failure surface is assumed to be starting is the center point.

IV. FOR THE SIDE AWAY FROM THE SLOPE:

In the similar way the geometry and the surcharge can be calculated as follows:

$$b' = \frac{B}{2}$$

$$x = b'\tan^2(45 + \frac{\phi}{2})$$

$$h' = \tan\beta\{\frac{(h' + D_f)}{\tan(45 - \frac{\phi}{2})} + b'\frac{\tan^2(45 + \frac{\phi}{2})}{\tan(45 - \frac{\phi}{2})}\}$$

$$x' = \frac{h'}{\tan(45 - \frac{\phi}{2})} + x + \frac{D_f}{\tan(45 - \frac{\phi}{2})}$$

$$AC' = [\{h' + D_f\}^2 + \{x + \frac{D_f + h'}{\tan(45 - \frac{\phi}{2})}\}^2]^{\frac{1}{2}}$$

$$q_n = \frac{\gamma.\Delta}{AC'}$$

The earth pressure coefficients remain the same on this side of the footing also. Hence, the expression for the bearing capacity stress can be as shown below:

$$q_v = \frac{q_n}{\cos(45 - \frac{\phi}{2})}$$

$$q_u = \frac{K_p.\gamma.\Delta}{AC'\cos(45 - \frac{\phi}{2})}.\frac{1}{K_a} + \frac{1}{2}.\frac{K_p.\gamma.H}{K_a} - \frac{1}{2}.\gamma.H + \frac{2(\sqrt{K_p} - \sqrt{K_a})c}{K_a}$$

From the equations, the ultimate bearing capacity from left part of the foundation as shown in figure is less than that of the right side. So the governing value of the capacity comes from the left side of foundation that has lesser value of surcharge.

V. RESULTS

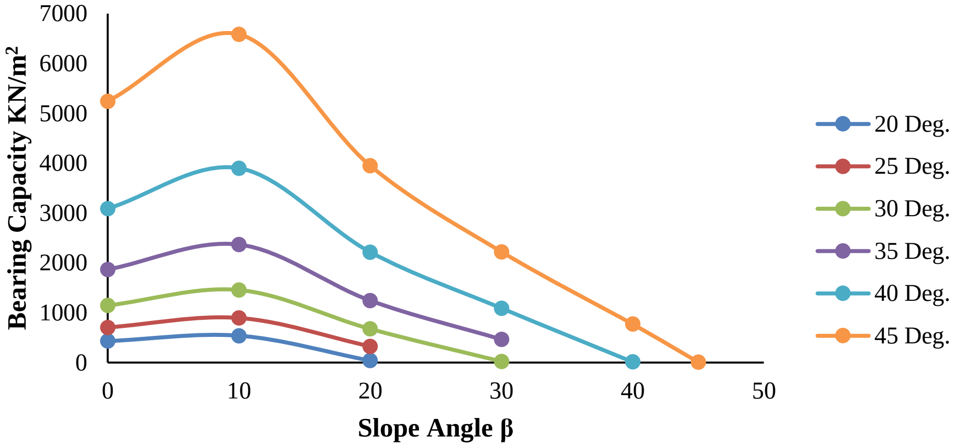
The following is the table showing the ultimate bearing capacity, q values for different β, φ and k values for the soil properties as given below:

(1) For, c= 0, γ= 18 kN/m^3, D_f =2 m, B=2m, k=0.3

Table 1 Ultimate Bearing Capacity in kN/m^2 for cohesionless soil

β	**φ**						
	20°	**25°**	**30°**	**35°**	**40°**	**45°**	**50°**
0°	430.4	705.3	1145.4	1867.3	3088.9	5242.1	9242.6
10°	539.1	894.8	1455.6	2367.2	3899.5	6584.3	11543.8
20°	39.1	325.8	675.2	1244.0	2216.1	3950.0	7206.7
30°			21.7	465.6	1090.8	2223.1	4397.0
40°					12.4	771.0	2051.4
45°						9.0	1046.3
50°							6.2

Slope angle β vs Bearing Capacity for different angle of internal friction (Ø-Soil)



(2) For, c= 60 kPa, γ= 18 kN/m³, D_f =2 m, B=2m, k=0.3

Table 2 Ultimate Bearing Capacity in kN/m² for c-φ soil

β	φ						
	20°	**25°**	**30°**	**35°**	**40°**	**45°**	**50°**
0°	430.4	705.3	1145.4	1867.3	3088.9	5242.1	9242.6
10°	539.1	894.8	1455.6	2367.2	3899.5	6584.3	11543.8
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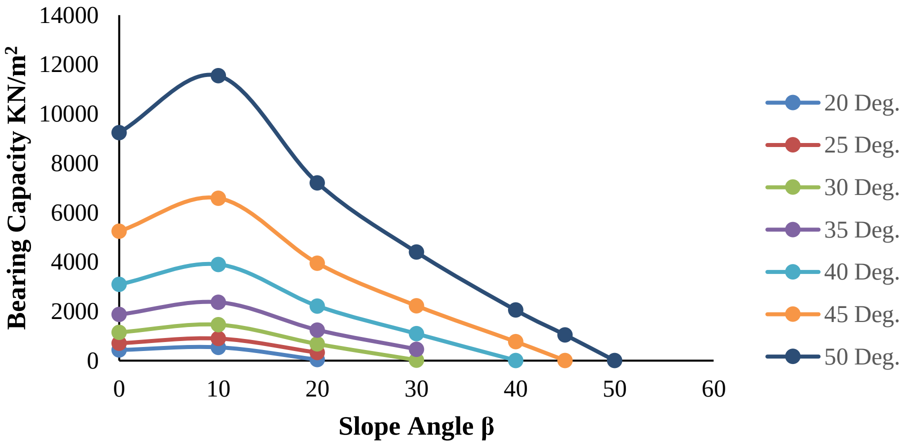
Slope angle β vs bearing capacity for different angle of internal friction

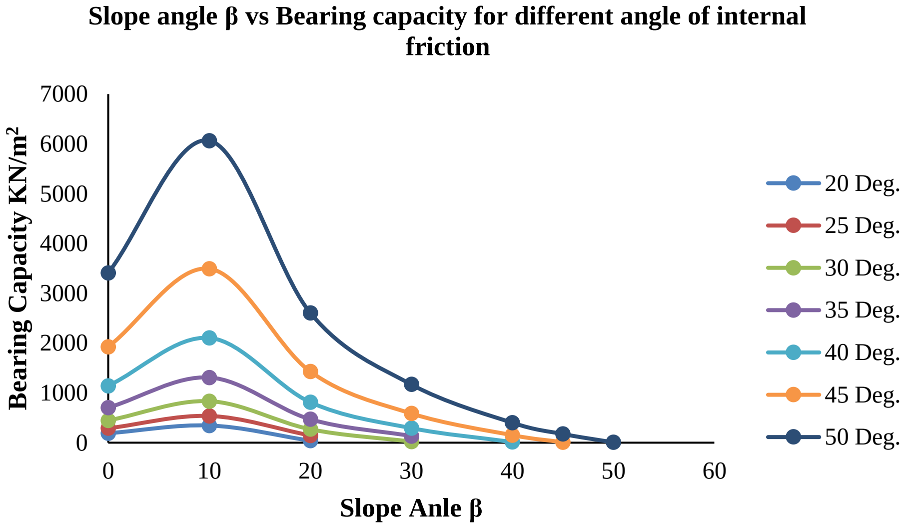


(3) For, c= 0, γ= 18 kN/m³, D_f =2 m, B=2m, k=0.2

Table 3 Ultimate Bearing Capacity in KN/m2 for cohesionless soil

β	φ						
	20°	**25°**	**30°**	**35°**	**40°**	**45°**	**50°**
0°	189.5	289	446	704.2	1142.7	1924.9	3406.4
10°	345.3	537	833	1309	2103.7	3493.3	6066.5
20°	39.9	145	270	469.9	812.4	1429.1	2607.8
30°			22	133.5	292	585.9	1169.8
40°					12.4	151	400.5

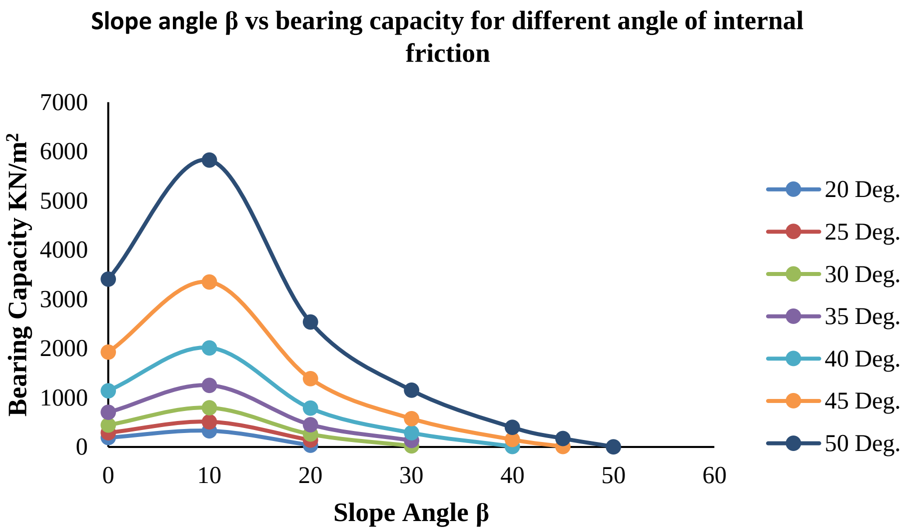
45°						9	174.1
50°							6.1



(4) For, c= 0, γ= 18 kN/m³, D_f =2 m, B=2m, k=0.4

Table 4 Ultimate Bearing Capacity in KN/m² for cohesionless soil

β	**φ**						
	20°	**25°**	**30°**	**35°**	**40°**	**45°**	**50°**
0°	190	288.7	446.3	704.2	1142.7	1924.9	3406.4
10°	329	512.5	796.3	1253	2014.9	3349.5	5823.7
20°	38.1	140.1	260.7	455.3	788.3	1388.7	2537.9
30°			21.3	131	287	576.6	1152.8
40°					12.3	150.3	398.8
45°						9	174.1
50°							6.2



VI. COMPARISON OF RESULTS WITH EXISTING METHODS

To ensure the accuracy of this method, we have compared it with the Meyerhof method [7] for footing on slope. Following are the results that were obtained using the Meyerhof method,

In Meyerhof method:

For c=0 (cohesionless soil):

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

For, γ= 18 kN/m3, D_f =2 m, b=2m, k=0.3

Comparing the values obtained from our analysis to the Meyerhof method:

Table 3: Bearing Capacity Values by Meyerhof method

β	Bearing Capacity (KN/m²)		
	Ø= 30°	Ø= 40°	Ø= 45°
0°	315	1800	5040
10°	248.4	1260	2880
20°	144	675	1440
30°	54	315	594

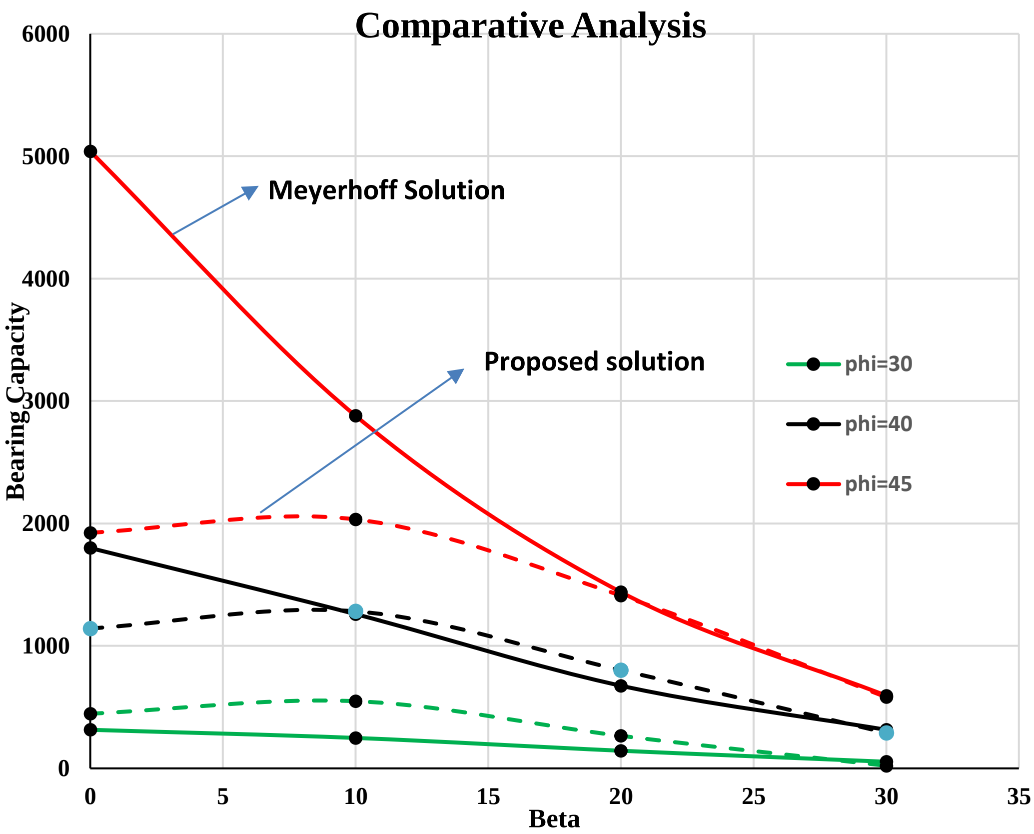


Figure 3: Comparative Analysis of Meyerhof method and proposed method

VII. Conclusion

Using the Rankin's theory, proposed equations for the ultimate bearing capacity values for cohesionless soil and c-ϕ soil as,

For β >10°, $q_u = \frac{K_p.\gamma.\Delta}{ACcos\beta_1}.\frac{1}{K_a}+\frac{1}{2}.\frac{K_p.\gamma.H}{K_a}-\frac{1}{2}.\gamma.H+\frac{2(\sqrt{K_p}-\sqrt{K_a})c}{K_a}$ (Side facing the slope)

For β <10°, $q_u = \frac{K_p.\gamma.\Delta}{AC'\cos(45-\frac{\phi}{2})}.\frac{1}{K_a}+\frac{1}{2}.\frac{K_p.\gamma.H}{K_a}-\frac{1}{2}.\gamma.H+\frac{2(\sqrt{K_p}-\sqrt{K_a})c}{K_a}$ (Side away from the slope)

This bearing capacity varies with c, ϕ and an assumed constant k (β_1=kβ, a factor depending on the failure surface). Also the comparison of our analytical solution with the existing Meyerhof's theory for foundation on slope, (as shown in Figure 3) it can be observed that for slope angle(β) less than 10°, our theory is not very efficient whereas for β >10°, our solution is converging Meyerhof's results. For β =30°, the error in our values is nearly 1%.

LIMITATIONS

The limitations of our theory is that

- For low values of slope, our analysis is not giving good results
- This theory is valid only for $\beta < \phi$
- This formula is not valid for purely cohesive soil
- Our analysis is based on the assumption that angle of failure surface from the slope edge is a function of a parameter β_1 which is directly proportional to slope angle β

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APPENDIX

NOTATIONS

B= Base of footing in meters
c= Cohesion of soil (kPa)
γ= Unit weight of soil (kN/m^3),
D_f= Depth of foundation in meters
β = Angle of slope
φ = Angle of internal friction
q_u = Ultimate bearing capacity of soil (kPa)
K_a = Coefficient of active earth pressure
K_p = Coefficient of Passive earth pressure