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Flexural Vibrations of Thermoporoelastic Solids in the Presence of Initial Stress with Two Relaxation Times

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Abstract - This paper deals with flexural vibrations of thermoporoelastic plates in the presence of initial stress with two relaxation times are investigated in the framework of Biot's theory. Pertinent constitutive relations and equations of motion are derived. Frequency equation is obtained in the presence of dissipation. In particular case, frequency equation is obtained in the absence of dissipation. Frequency and attenuation is computed as a function of wavenumber and initial stress. Numerical results are presented graphically.

Keywords: Thermoporoelasticity, Flexural Vibrations, Initial Stress, Frequency, Attenuation, Wavenumber.

1. Introduction

The wave propagation in thermoporoelastic media in the presence of initial stress is of much importance in various fields such as earthquake Engineering, Soil dynamics and Geophysics. Hany H. Sherief and Heba A. Saleh [1] investigated a problem on thermoelasticity with two relaxation times for an infinite thermoelastic layer. On the flexural and extensional thermoelastic waves in orthotropic plates with two thermal relaxation times is studied by Verma and Noriohasbe [2]. On the extensional and flexural generalized thermoelastic waves in an anistropic plate is investigated by Abd-alla et al [3]. Nilratan Chakraborty [4] investigated reflection of plane waves at a free surface under initial stress and temperature field. Employing the Biot's theory [5], On flexural vibrations of poroelastic circular cylindrical shells immersed in acoustic medium is investigated by Shah and Tajuddin [6]. Theodorakopoulos and Beskos [7] studied flexural vibrations of poroelastic plates. Flexural vibrations of poroelastic solids in the presence of static stresses is investigated by Rajitha et al [8]. Flexural vibrations of poroelastic solid cylinder in the presence of static stresses is investigated studied by Manjula et al [9]. To the best of author knowledge, flexural vibrations in thermoporoelastic solids in the presence of initial stress with two relaxation times is not yet studied. Therefore, in the present paper same is investigated in the frame work of Biot's theory. The pertinent equations of motion are derived. Frequency is computed as a function of initial stress and wave number.
The rest of the paper is organized as follows. In section 2, governing equations and solution of the problem are discussed. In section 3, numerical results are given. Finally, concluding remarks are given in section 4.

2. Solution of the problem

Consider the thermoporoelastic solid in Cartesian coordinate system [10] and heat conduction [11] are as follows

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}-P\frac{\partial\omega_y}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}u+\rho_{12}U)+b\frac{\partial}{\partial t}(u-U),$$

$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}-P\frac{\partial\omega_x}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V)+b\frac{\partial}{\partial t}(v-V),$$

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}-P(\frac{\partial\omega_y}{\partial x}-\frac{\partial\omega_x}{\partial y})=\frac{\partial^2}{\partial t^2}(\rho_{11}w+\rho_{12}W)+b\frac{\partial}{\partial t}(w-W),$$

$$\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u+\rho_{22}U)-b\frac{\partial}{\partial t}(u-U),$$

$$\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V)-b\frac{\partial}{\partial t}(v-V),$$

$$\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w+\rho_{22}W)-b\frac{\partial}{\partial t}(w-W),$$

$$K\nabla^2T=\rho c_v(T+\tau_0\frac{\partial T}{\partial t})\frac{\partial}{\partial t}+\beta T_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla.u$$

(1)

In eq. (1), $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, and $\omega_x = \frac{1}{2}(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})$, $\omega_y = \frac{1}{2}(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x})$, ρ is the mass density, P is the initial stress, c_v is the specific heat capacity, K is the thermal conductivity, T_0 is the reference temperature, τ_0 is the relaxation time, $\rho_{11}, \rho_{12}, \rho_{22}$ are the mass coefficients, (u, v, w) and (U, V, W) are the displacements of solid and fluid. s is the fluid pressure, σ_{ij} are the stress components are given by [4, 11]

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij} - \beta(T + \tau_1 \frac{\partial T}{\partial t})\delta_{ij},$$
$$s = Qe + R\varepsilon. \tag{2}$$

In eq. (2), e_{ij} 's are strain components, A, N, Q, R are poroelastic constants and $(D = A + 2N)$, β is the thermal stress, τ_1 is the relaxation time, T is the temperature, e and ε are dilatations of solid and fluid. Substitution of eq. (2) in eq. (1) the equations of motion are as follows

$$N\nabla^2 u + (A+N)\frac{\partial e}{\partial x} + Q\frac{\partial \varepsilon}{\partial x} - \beta\frac{\partial T}{\partial x} - \beta\tau_1\frac{\partial^2 T}{\partial x \partial t} - \frac{P}{2}(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x \partial z}) = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U) + b\frac{\partial}{\partial t}(u - U),$$
$$N\nabla^2 v + (A+N)\frac{\partial e}{\partial y} + Q\frac{\partial \varepsilon}{\partial y} - \beta\frac{\partial T}{\partial y} - \beta\tau_1\frac{\partial^2 T}{\partial y \partial t} + \frac{P}{2}(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 w}{\partial y \partial z}) = \frac{\partial^2}{\partial t^2}(\rho_{11}v + \rho_{12}V) + b\frac{\partial}{\partial t}(v - V),$$
$$N\nabla^2 w + (A+N)\frac{\partial e}{\partial z} + Q\frac{\partial \varepsilon}{\partial z} - \beta\frac{\partial T}{\partial z} - \beta\tau_1\frac{\partial^2 T}{\partial z \partial t} - \frac{P}{2}(\frac{\partial^2 u}{\partial z \partial x} - \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z})$$
$$= \frac{\partial^2}{\partial t^2}(\rho_{11}w + \rho_{12}W) + b\frac{\partial}{\partial t}(w - w),$$
$$Q\frac{\partial e}{\partial x} + R\frac{\partial \varepsilon}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U) - b\frac{\partial}{\partial t}(u - U),$$
$$Q\frac{\partial e}{\partial y} + R\frac{\partial \varepsilon}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{12}v + \rho_{22}V) - b\frac{\partial}{\partial t}(v - V),$$
$$Q\frac{\partial e}{\partial z} + R\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{12}w + \rho_{22}W) - b\frac{\partial}{\partial t}(w - W),$$
$$K\nabla^2 T = \rho c_v(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}) + \beta T_0(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}). \tag{3}$$

Now we can assume the solution to the eq. (3) in the following form [8]

$$(u,v,w,U,V,W,T)(x,y,z) = (C_1, C_2, C_3, C_4, C_5, C_6, C_7)e^{j\omega t - j(k_1 x + k_2 y + k_3 z)}. \tag{4}$$

In all the above $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ are arbitrary constants, j is the complex unity and $k_i (i = 1,2,3)$ is the wavenumber in the i^{th} direction such that the wavenumber $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$. Substituting eq. (4) in the eq. (3), we obtain

$$(N(k_1^2 + k_2^2 + k_3^2) + (A+N)k_1^2 - \omega^2\rho_{11} - \frac{P}{2}k_3^2 + bj\omega)C_1 + (A+N)k_1k_2C_2 + ((A+N)k_1k_3 + \frac{P}{2}k_1k_3)C_3$$
$$+ (Qk_1^2 - \omega^2\rho_{12} - bj\omega)C_4 + Qk_1k_2C_5 + Qk_1k_3C_6 - \beta(-jk_1 - \tau_1\omega)k_1C_7 = 0,$$
$$-(A+N)k_1k_2C_1 + (N(k_1^2 + k_2^2 + k_3^2) + (A+N)k_2^2 + \frac{P}{2}k_3^2 - \omega^2\rho_{11} + bj\omega)C_2 + ((A+N)k_2k_3 - \frac{P}{2}k_2k_3)C_3$$
$$+ Qk_1k_2C_4 + (Qk_2^2 - \omega^2\rho_{12} - bj\omega)C_5 + Qk_2k_3C_6 + (\beta jk_2 - \beta\tau_1\omega k_2)C_7 = 0,$$
$$(-(A+N)k_1k_3 + \frac{P}{2}k_1k_3)C_1 + (-(A+N)k_2k_3 + \frac{P}{2}k_2k_3)C_2 + (-N(k_1^2 + k_2^2 + k_3^2) - (A+N)k_3^2 - \frac{P}{2}(k_1^2 + k_2^2)$$
$$- \omega^2\rho_{11} - bj\omega)C_3 - Qk_1k_3C_4 - Qk_2k_3C_5 - (Qk_3^2 + \omega^2\rho_{12} + bj\omega)C_6 - (\beta jk_3 + \tau_1\omega k_3)C_7 = 0,$$

$$(Qk_1^2-\omega^2\rho_{12}+bj\omega)C_1+Qk_1k_2C_2+Qk_1k_3C_3+(Rk_1^2-\omega^2\rho_{22}-bj\omega)C_4+Rk_1k_2C_5+Rk_1k_3C_6=0,$$
$$Qk_1k_2C_1+(Qk_2^2-\omega^2\rho_{12}+bj\omega)C_2+Qk_2k_3C_3+Rk_1k_2C_4+(Rk_2^2-\omega^2\rho_{22}-bj\omega)C_5+Rk_2k_3C_6=0,$$
$$Qk_1k_3C_1+Qk_2k_3C_2+(Qk_3^2-\omega^2\rho_{12}+bj\omega)C_3+Rk_1k_3C_4+Rk_2k_3C_5+(Rk_3^2-\omega^2\rho_{22}-bj\omega)C_6=0,$$
$$(-\beta T_0\omega k_1-\beta T_0 j\tau_0\omega^2k_1)C_1+(-\beta T_0\omega k_2-\beta T_0 j\tau_0\omega k_2)C_2+(-\beta T_0\omega k_3-\beta T_0 j\tau_0\omega^2k_3)C_3$$
$$+(K(k_1^2+k_2^2+k_3^2)+\rho c_v(j\omega-\omega^2\tau_0))C_7=0. \quad (5)$$

3. Numerical results

For the numerical work, the wave propagation is considered along $z-$direction. In this case $k_1=k_2=0$ and eq. (5) reduces to the following matrix form.

$$\begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}=0. \quad (6)$$

Due to the presence of dissipation (b) nature of the medium, waves are attenuated. For a non-trivial solution, the determinant of coefficient matrix is zero. Accordingly we obtain the complex valued frequency equation.

$$\begin{vmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{vmatrix}=0. \quad (7)$$

Where $A_{ij}=|b_{ij}|+i|d_{ij}|, \quad i,j=1,2,3$ and the expression for the b_{ij} and d_{ij} are given below

$$b_{11}=Nk_3^2\omega^2\rho_{22}-\frac{P}{2}k_3^2\omega^2\rho_{22}-\frac{\omega^4N\rho_{22}}{V_s^2},$$
$$b_{22}=Nk_3^2\omega^2\rho_{22}+\frac{P}{2}k_3^2\omega^2\rho_{22}-\frac{\omega^4N\rho_{22}}{V_s^2},$$
$$b_{33}=DR\rho c_vk_3^4\omega^3\tau_0-DRKk_3^6+DKk_3^4\omega^2\rho_{22}-Dk_3^2\rho c_v\omega^5\rho_{22}\tau_0+RKk_3^4\omega^2\rho_{22}-Rk_3^2\rho c_v\omega^5\tau_0$$
$$-\omega^4\rho_{11}\rho_{12}Kk_3^2+\rho_{11}\rho_{22}\omega^6\rho c_v\tau_0-Kk_3^2b^2\omega^2+\rho c_vb^2\omega^5\tau_0-Q^2Kk_3^6+Q^2k_3^4\rho c_v\omega^2\tau_0+Qk_3^4\omega^2\rho_{12}$$
$$-Qk_3^2\rho c_v\omega^4\rho_{12}\tau_0+QKk_3^6\omega^2\rho_{12}-Qk_3^2\rho c_v\omega^4\rho_{12}\tau_0-\omega^4\rho_{12}^2Kk_3^2+\omega^6\rho_{12}^2\rho c_v\tau_0+b^2\omega^2Kk_3^2$$
$$-b^2\rho c_v\omega^4\tau_0+R\beta T_0\omega^2k_3^4\tau_0\tau_1-\beta T_0k_3^2\tau_0\tau_1\omega^4\rho_{22},$$
$$d_{11}=\frac{P}{2}k_3^2b\omega-Nk_3^2b\omega+\omega^2\rho_{11}b\omega+\omega^2\rho_{22}b\omega+2\omega^3\rho_{12}b,$$
$$d_{22}=2\rho_{12}b\omega^3+\rho_{22}b\omega^3-\rho_{12}b\omega^3-\frac{P}{2}b\omega k_3^2-Nk_3^2b\omega,$$
$$d_{33}=Dk_3^2\omega^4\tau_0-DKk_3^4b\omega+Kk_3^2b\omega^3\rho_{11}-\rho c_v\omega^6b\rho_{11}\tau_0+RKk_3^4b\omega-Rk_3^2b\rho c_v\tau_0\omega^4-Kb\omega^3k_3^2\rho_{22}$$
$$-\rho c_v\tau_0b\rho_{22}\omega^6-QKk_3^4b\omega+Qk_3^2b\rho c_v\omega^3\tau_0+2Kk_3^2\rho_{12}b\omega^3-\rho_{12}b\rho c_v\tau_0\omega^3-QKk_3^6b\omega+Qk_3^2b\rho c_v\omega^3\tau_0$$
$$-\rho c_vb\rho_{12}\tau_0\omega^5+\beta T_0\omega^3k_3^2\tau_0\tau_1,$$

and $V_s^2=\dfrac{N\rho_{22}}{\rho_{11}\rho_{22}-\rho_{12}^2}$ is the shear wave velocity.

a. Particular case

In the absence of dissipation coefficient $(b=0)$, we obtain the frequency equation in the following form

$$\begin{vmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{vmatrix}=0.$$

(8)

Where

$$C_{11} = Nk_3^2 - \frac{Pk_3^2}{2} - N\frac{\omega^2}{V_S^2},$$

$$C_{22} = Nk_3^2 + \frac{Pk_3^2}{2} - N\frac{\omega^2}{V_S^2},$$

$$\begin{aligned} C_{33} = {} & DKk_3^4\omega^2\rho_{22} - DRKk_3^6 + DRk_3^4\omega^2\rho c_v\tau_0 - D\rho c_v k_3^2\omega^4\rho_{22}\tau_0 + RKk_3^4\ \omega^2\rho_{11} - Kk_3^2\omega^4\rho_{11}\rho_{22} \\ & - Rk_3^2\rho c_v\rho_{11}\omega^4\tau_0 + \rho c_v\omega^6\tau_0\rho_{11}\rho_{22} + Q^2Kk_3^6 - 2QKk_3^4\omega^2\rho_{12} + Kk_3^2\omega^4\rho_{12}^2 - \rho c_v\rho_{12}^2\omega^6\tau_0 \\ & + 2Qk_3^2\omega^4\tau_0\rho_{12} - R\beta^2T_0k_3^4\omega^2\tau_0\tau_1 + \beta^2T_0k_3\omega^4\tau_0\tau_1\rho_{22}. \end{aligned}$$

In the above, $\tau_0 = \tau_0 - \frac{j}{\omega}$, $\tau_1 = 1 + j\omega\tau_1$. The frequency equation (7) and (8) is investigated for particular solids namely sandstone saturated with kerosene while material-2 is sandstone saturated with water. The values are taken [13, 14, 15]

Material-1

$$\begin{aligned} & A = 0.4436\times10^{10}\,N/m^2, \quad N = 0.2765\times10^{10}\,N/m^2, \quad Q = 0.07635\times10^{10}\,N/m^2, \\ & R = 0.0326\times10^{10}\,N/m^2, \quad \rho_{11} = 1.926137\times10^3\,kg/m^3, \quad \rho_{12} = -0.002137\times10^3\,kg/m^3, \\ & \rho_{22} = 0.21537\times10^3\,kg/m^3, \quad \beta = 3.2\times10^{-4}\,1/^0\,k, \quad K = 0.13\omega/m^0k, \quad \rho c_v = 1.67\times10^6\,J/m^3k, \\ & \tau_0 = 0.1s,\ \tau_1 = 0.2s. \end{aligned} \tag{9}$$

Material-2

$$\begin{aligned} & A = 0.306\times10^{10}\,N/m^2, \quad N = 0.922\times10^{10}\,N/m^2, \quad Q = 0.013\times10^{10}\,N/m^2, \\ & R = 0.0637\times10^{10}\,N/m^2, \quad \rho_{11} = 1.90302\times10^3\,kg/m^3, \quad \rho_{12} = 0, \rho_{22} = 0.2268\times10^3\,kg/m^3, \\ & \beta = 6.6\times10^{-5}\,1/^0\,k, \quad K = 0.607\omega/m^0k, \quad \rho c_v = 4.17\times10^6\,J/m^3k,\ \tau_0 = 0.1s, \tau_1 = 0.2s. \end{aligned} \tag{10}$$

The complex frequency (7) gives frequency and attenuation coefficient as a function of wavenumber. The real part of eq. (7) gives frequency of wave, whereas Q^{-1} = (2imaginary part of eq. (7)) / (real part of eq. (7)) gives attenuation coefficient. Substituting the values of eqs. (9), (10) in the frequency equations (7) and (8) frequency, attenuation coefficient are computed as a function of wavenumber and initial stress. Frequency and attenuation are computed using the bisection method implemented in MATLAB, and results are depicted in figures 1-3 graphically. Figure-1 shows the plot of frequency against wavenumber at initial stress (IS=1) and dissipation (b=1). From the figure it is observed that frequency of material-2 values is greater than that of material-1. It is also observed that as the wavenumber increases frequency increases for both materials. Figure-2 shows the plot of attenuation against wavenumber at initial stress (IS=1) and dissipation (b=1). From the figure it is clear that material-1 values are greater than that of material-2, this inconsistency due to the presence of initial stress and fluid present in the materials. Figure-3 shows the plot of frequency against wavenumber at initial stress (IS=1). From the figure it is clear that frequency of material-2 is greater than that of material-1. It is also clear that as the wavenumber increases frequency increases for both materials.

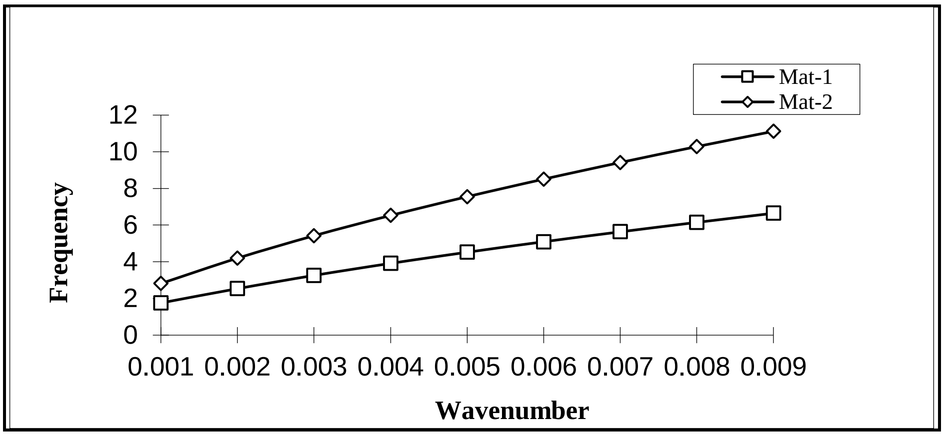

Figure-1 Variation of frequency with wavenumber at dissipation (b=1)

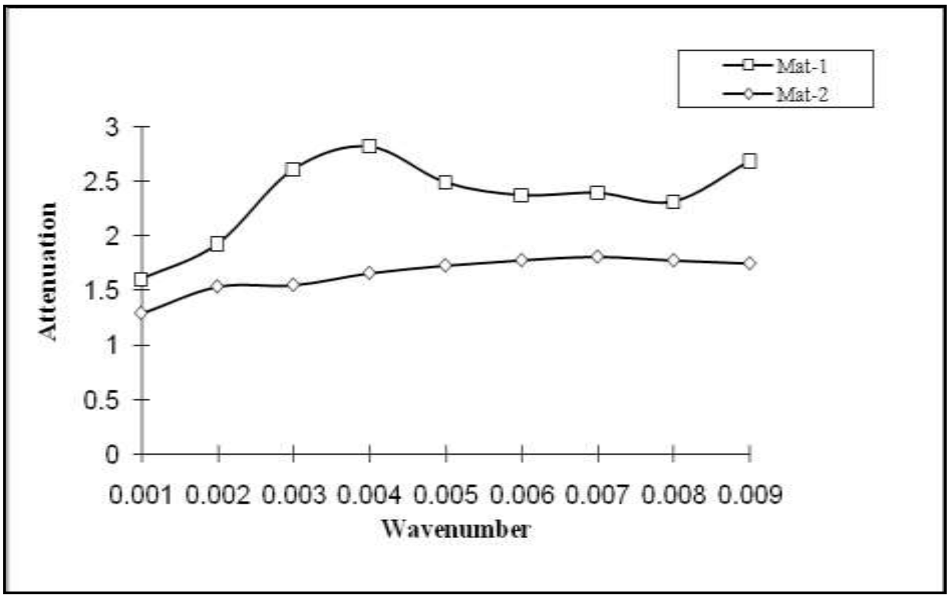

Figure-2 Variation of attenuation with wavenumber at dissipation (b=1)

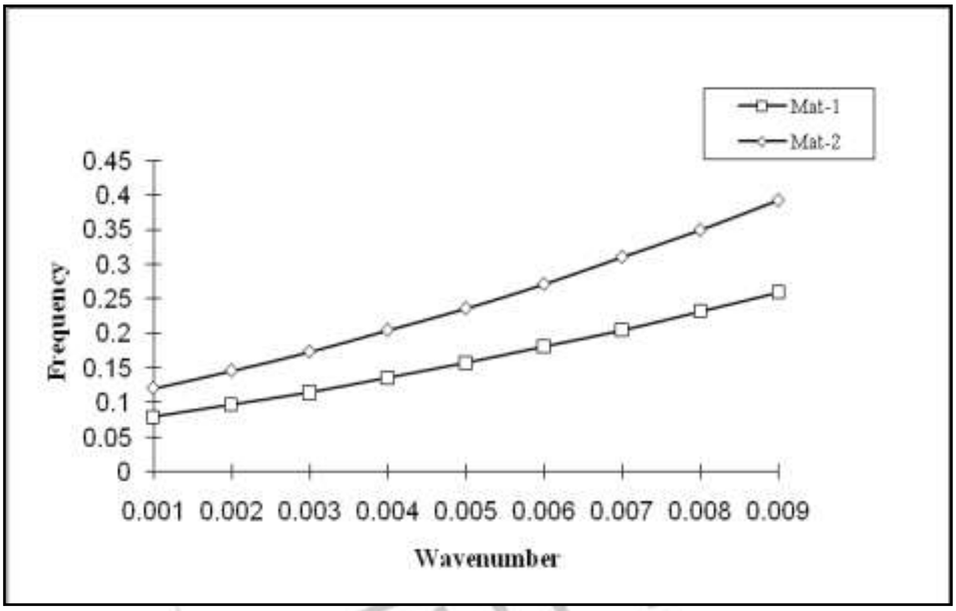

Figure-3 Variation of frequency with wavenumber

4. Conclusion

Employing Biot's theory, flexural vibrations of thermoporoelastic solids in the presence initial stress with two relaxation times. Pertinent constitutive relations and equations of motion are derived. Frequency and attenuation is computed for two poroelastic solids. The complex valued frequency equation is reduced to real valued equations which gives that frequency and attenuation. Attenuation values are greater than that of frequency. In the absence of dissipation as the wavenumber increases frequency increases for two materials.

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