$\alpha\pi$g- Closed sets in Intuitionistic Fuzzy Topological spaces

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Abstract: This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper intuitionistic fuzzy $\alpha\pi$g closed sets are introduced. We study some of their basic properties.

Key words and phrases: Intuitionistic fuzzy topology, Intuitionistic fuzzy open sets, Intuitionistic fuzzy closed sets, Intuitionistic fuzzy α closed sets and Intuitionistic fuzzy $\alpha\pi$- generalized closed sets.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [9] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Sarsak and Rajesh[7] introduced π-generalized semi-Preclosed sets. In this paper we introduce intuitionistic fuzzy $\alpha\pi$-generalized closed sets and study some of their properties.

II. PRELIMINARIES

in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. We denote the set of all intuitionistic fuzzy sets in X, by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form
$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$.
Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [2] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by
$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5:[7] A subset of A of a space (X, τ) is called:
(i) regular open if A = int (cl(A)).
(ii) π open if A is the union of regular open sets.

Definition 2.6:[4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$.

Definition 2.7:[4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq cl(int(A))$.

Definition 2.8:[4] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$.

Definition 2.9:[4] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy α-open set (IFαOS in short) if $A\subseteq int(cl(int(A)))$,
(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

Definition 2.10:[5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy γ-open set (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$,
(ii) intuitionistic fuzzy γ-closed set (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

Definition 2.11:[4] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy semi pre open set (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

Definition 2.12:[4] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy semi pre closed set (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X, τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Definition 2.13:[7] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)),
(ii) intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)).

Definition 2.14:[7] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.15:[6] Let an IFS A of an IFTS (X, τ). Then the semi closure of A (scl(A) in short) is defined as $scl (A) = \cap\{ K / K$ is an IFSCS in X and $A \subseteq K \}$.

Definition 2.16:[6] Let A be an IFS of an IFTS (X, τ). Then the semi interior of A (sint(A) in short) is defined as $sint(A) = \cup\{ K / K$ is an IFSOS in X and $K \subseteq A\}$.

Definition 2.17:[8] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Result 2.18 : [8] Let (X, τ) be an IFS. If A is an IFS of X then $scl(A^c) = (sint(A))^c$

Result 2.19: Let A be an IFS in (X, τ), then
(i) $scl(A) = A \cup int(cl(A))$,
(ii) $sint(A) = A \cap cl(int(A))$.

III. **Intuitionistic fuzzy $\alpha\pi$ g closed sets**

In this section, we have introduced intuitionistic fuzzy alpha π generalized closed sets and studied some of their properties.

Definition 3.1: An IFS A in (X, τ) is said to be an intuitionistic fuzzy alpha π generalized closed set (IFαπGCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ). Here The family of all IFαπGCS of an IFTS (X, τ) is denoted by IFαπGC(X).

Example 3.2: Let X = { a, b } and let τ = { $0_\sim$, G, $1_\sim$ } be an IFT on X, where G = ⟨ x, (0.7, 0.6), (0.2, 0.3) ⟩. Here $\mu_G(a)= 0.8$, $\mu_G(b)= 0.7$, $\nu_G(a)= 0.2$ and $\nu_G(b)= 0.3$. Let us consider the IFS A = ⟨ x, (0.2, 0.2), (0.7, 0.6) ⟩. Clearly if A ⊆ $1_\sim$, then αcl(A) ⊆ $1_\sim$. Now let us consider the IFOS G in (X, τ). Clearly A ⊆ G. Now αcl(A) = A ⊆ G . Hence αcl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ). Therefore, the IFS A is an IFαπGCS in (X, τ).

Theorem 3.3: Every IFCS in (X, τ) is an IFαπGCS (X, τ) but not conversely.
Proof: Assume that A is an IFCS in (X, τ). Let us consider an IFS A ⊆ U and U be an IFπOS in (X, τ). Since αcl(A) ⊆ cl(A) and A is an IFCS in X, αcl(A) ⊆ cl(A) = A ⊆ U. That is αcl(A) ⊆ U. Therefore, A is an IFαπGCS in X.

Example 3.4: Let X = { a, b } and let τ = { $0_\sim$, G, $1_\sim$ } where G = ⟨ x, (0.4, 0.2), (0.5, 0.6) ⟩. Consider the IFS A = ⟨ x, (0.5, 0.3), (0.5, 0.6) ⟩. Clearly A ⊆ $1_\sim$ and αcl(A) = ⟨ x, (0.5, 0.6), (0.4, 0.2) ⟩ ⊆ $1_\sim$. Hence A is an IFαπGCS. But A is not an IFCS in X.

Theorem 3.5: Every IFαCS in (X, τ) is an IFαπGCS in (X, τ) but not conversely.

Proof: Let us consider an IFS A ⊆ U and U be an IFOS in (X, τ). Also let A is an IFαCS in X. This implies αcl(A) = A. Hence αcl(A) ⊆ U. Therefore A is an IFαπGCS in X.

Example 3.6: Let X = { a, b } and let τ = { $0_\sim$, G_1, G_2, $1_\sim$ }, where G_1 = ⟨ x, (0.3, 0.2), (0.6, 0.7) ⟩, G_2 = ⟨ x, (0.8, 0.8), (0.2, 0.2) ⟩. Then the IFS A = ⟨x, (0.4, 0.4), (0.5, 0.6) ⟩ is an IFαπGCS in X. But A is not an IFαCS in X because cl(int(cl(A))) = ⟨ x, (0.6, 0.7), (0.3, 0.2) ⟩ ⊈ A.

Theorem 3.7: Every IFRCS in (X, τ) is an IFαπGCS in (X, τ) but not conversely.

Proof: Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, A is an IFCS in X. Hence by Theorem 3.3, A is an IFαπGCS in X.

Example 3.8: Let X = { a, b } and let τ = { $0_\sim$, G, $1_\sim$ } be an IFT on X, where G = ⟨ x, (0.5, 0.6), (0.2, 0.3) ⟩. Then the IFS A = ⟨ x, (0.2, 0.2), (0.8, 0.7) ⟩ is an IFαπGCS in X but not an IFRCS in X.

Theorem 3.9: Every IFGCS in (X, τ) is an IFαπGCS in (X, τ) but its converse may not be true in general.

Proof: Assume that A be an IFGCS in (X, τ). Let A ⊆ U and U be an IFπOS in X. By hypothesis cl(A) ⊆ U. Clearly αcl(A) ⊆ cl(A). This implies αcl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in X. Hence A is an IFαπGCS in X.

Example 3.10: Let X = { a, b } and let τ = { $0_\sim$, G, $1_\sim$ } be an IFT on X, where G = ⟨ x, (0.2, 0.8), (0.3, 0.1) ⟩. Then the IFS A = ⟨ x, (0.1, 0), (0.4, 0.8) ⟩ is an IFαπGCS but A is not an IFGCS in X, as cl(A) ⊈ G even though A ⊆ G and G is an IFπOS in X.

Theorem 3.11: Every IFαπGCS in (X, τ) is an IFGSCS in (X, τ) but its converse may not be true in general.

Proof: Assume that A is an IFαπGCS in (X, τ). Let an IFS A ⊆ U and U be an IFπOS in (X, τ). By hypothesis αcl(A) ⊆ U. That is A ∪ cl(int(cl(A))) ⊆ U. This implies A∪ int(cl(A)) ⊆ U. Therefore scl(A) = A ∪ int(cl(A))) ⊆ U. Hence A is an IFGSCS in X.

Example 3.12: Let X = { a, b } and let τ = { $0_\sim$, G, $1_\sim$ } be an IFT on X, where G = ⟨ x, (0.2, 0.3), (0.4, 0.5) ⟩. Then the IFS A = ⟨ x, (0.1, 0), (0.4, 0.5) ⟩ is an IFGSCS in X but A is not an IFαπGCS in X, since αcl(A) ⊈ G even though A ⊆ G and G is an IFπOS in (X, τ).

Remark 3.13: An IFGSPCS in (X, τ) is need not be an IFαπGCS in (X, τ)

Example 3.14: Let X = { a, b } and G = ⟨ x, (0, 0.8), (0.5, 0.1) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Then the IFS A = ⟨ x, (0, 0.3), (0.7, 0.7) ⟩ is an IFGSPCS but A is not an IFα𝜋GCS in X since αcl(A) ⊈ G even though A ⊆ G and G is an IF𝜋OS in (X, τ).

Remark 3.15: An IFP closedness is independent of an IFα𝜋G closedness.

Example 3.16: Let X = { a, b } and G = ⟨ x, (0, 0.9), (0.5, 0.1) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Then the IFS A = ⟨ x, (0, 0.3), (0.6, 0.6) ⟩ is an IFPCS but not an IFα𝜋GCS in X, as αcl(A) ⊈ G even though A ⊆ G and G is an IF𝜋OS in X.

Example 3.17: Let X = { a, b } and G = ⟨ x, (0.3, 0.3), (0.6, 0.7) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Then the IFS A = ⟨ x, (0.4, 0.3), (0.5, 0.6) ⟩ is an IFα𝜋GCS but not an IFPCS in X, as cl(int(A)) ⊈ A.

Remark 3.18: An IFSP closedness is independent of an IFαG closedness.

Example 3.19: Let X = { a, b } and G = ⟨ x, (0.1, 0.9), (0.6, 0.1) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Consider an IFS A = ⟨ x, (0, 0.4), (0.6, 0.6) ⟩ in X. Since αcl(A) ⊈ G even though A ⊆ G and G is an IFOS in X, A is not an IFα𝜋GCS in X. But A is an IFSPCS in (X, τ).

Example 3.20: Let X = { a, b } and G = ⟨ x, (0.6, 0.7), (0.3, 0.2) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Then the IFS A = ⟨ x, (0.7, 0.8), (0.1, 0.2) ⟩ is an IFα𝜋GCS but not an IFSPCS in X since int(cl(int(A))) = 1~ ⊈ A.

Remark 3.21: An IFα𝜋G closedness is independent of IFγ closedness.

Example 3.22: Let X = { a, b } and G = ⟨ x, (0.4, 0.6), (0.2, 0.2) ⟩ and let τ = {0~, G, 1~} be an IFT on X. Then the IFS A = ⟨ x, (0.4, 0.3), (0.6, 0.2) ⟩ is an IFγCS but not an IFα𝜋GCS in X, as αcl(A) ⊈ G even though A ⊆ G and G is an IF𝜋OS in X.

Example 3.23: Let X = { a, b } and G = ⟨ x, (0.5, 0.1), (0.5, 0.9) ⟩ and let τ = {0~, G, 1~} be an IFT on X. Then the IFS A = ⟨ x, (0.7, 0.8), (0.2, 0.1) ⟩ is an IFα𝜋GCS but not an IFγCS in X, as cl(int(A)) ∩ int(cl(A)) ⊈ A.

Remark 3.24: The intersection of any two IFα𝜋GCS is not an IFα𝜋GCS in general as seen from the following example.

Example 3.25: Let X = { a, b } and G = ⟨ x, (0.5, 0), (0.1, 1) ⟩ and let τ = { 0~, G, 1~ } be an IFT on X. Then the IFS A = ⟨ x, (0.2,1), (0.7, 0) ⟩, B = ⟨ x, (0.5, 0), (0.3, 1) ⟩ are IFα𝜋GCS. Now A ∩ B = ⟨ x, (0.2,0), (0.7, 1) ⟩. Since αcl(A ∩ B) ⊈ G even though A ∩ B ⊆ G and G is an IF𝜋OS in X, A ∩ B is not an IFα𝜋GCS in X.

Theorem 3.26: Let (X, τ) be an IFTS. Then for every A ∈ IFα𝜋GC(X) and for every B ∈ IFS(X), A ⊆ B ⊆ αcl(A) implies B ∈ IFα𝜋GC(X).

Proof: Let an IFS B ⊆ U and U be an IFOS in X. Since A ⊆ B, A ⊆ U and A is an IFα𝜋GCS, αcl(A) ⊆ U. By hypothesis B ⊆ αcl(A), αcl(B) ⊆ αcl(A) ⊆ U. Therefore αcl(B) ⊆ U. Hence B is an IFα𝜋GCS of X.

Theorem 3.27: If A is an IFOS in (X, τ) and an IFα𝜋GCS in (X, τ), then A is an IFαCS in X.
Proof: Let A be an IFOS in X. Since A ⊆ A, by hypothesis αcl(A) ⊆ A. But A ⊆ αcl(A). Therefore αcl(A) = A. Hence A is an IFαCS of X.

Theorem 3.28: The union of IFα𝜋GCS A and B is an IFα𝜋GCS in (X, τ), if A and B are IFCS in (X, τ).

Proof: Since A and B are IFCS in X, cl(A) = A and cl(B) = B. Assume that A and B are IFα𝜋GCS in (X, τ). Let A∪B ⊆ U and U be IF𝜋OS in X. Then cl(int(cl(A∪B))) = cl(int(A∪B)) ⊆ cl(A∪B) = A∪B ⊆ U. That is αcl(A∪B) ⊆ U. Therefore the union of A and B is an IFα𝜋GCS in (X, τ).

Theorem 3.29: Let (X, τ) be an IFTS and A be an IFS in X. Then A is an IFα𝜋GCS if and only if A $\bar{q}$ F implies αcl(A) $\bar{q}$ F for every IFCS F of X.

Proof: Necessity: Let F be an IFCS in X and let A $\bar{q}$ F. Then A ⊆ F^c, where F^c is an IF𝜋OS in X. Therefore αcl(A) ⊆ F^c, by hypothesis. Hence αcl(A) $\bar{q}$ F.
Sufficiency: Let F be an IFCS in X and let A be an IFS in X. Then by hypothesis, A $\bar{q}$ F implies αcl(A) $\bar{q}$ F. Then αcl(A) ⊆ F^c whenever A ⊆ F^c and F^c is an IFOS in X. Hence A is an IFα𝜋GCS in X.

Theorem 3.30: Let (X, τ) be an IFTS. Then IFαO(X) = IFαC(X) if and only if every IFS in (X, τ) is an IFα π GCS in X.

Proof: Necessity: Suppose that IFαO(X) = IFαC(X). Let A ⊆ U and U be an IFOS in X. This implies αcl(A) ⊆ αcl(U). Since U is an IFOS, U is an IFαOS in X. Since by hypothesis U is an IFαCS in X, αcl(U) = U. This implies αcl(A) ⊆ U. Therefore A is an IFα π GCS of X.

Sufficiency: Suppose that every IFS in (X, τ) is an IFα π GCS in X. Let U ∈ IFO(X), then U ∈ IFαO(X). Since U ⊆ U and U is IFOS in X, by hypothesis αcl(U) ⊆ U. But clearly U ⊆ αcl(U). Hence U = αcl(U). That is U ∈ IFαC(X). Hence IFαO(X) ⊆ IFαC(X).

Let A ∈ IFαC(X) then A^c is an IFαOS in X. But IFαO(X) ⊆ IFαC(X). Therefore A^c ∈ IFαC(X). Hence A ∈ IFαO(X). This implies IFαC(X) ⊆ IFαO(X). Thus IFαO(X) = IFαC(X).

Theorem 3.31: If A is an IFOS and an IFα π GCS in (X, τ), then
(i) A is an IFROS in X
(ii) A is an IFRCS in X.

Proof: (i): Let A be an IF π OS and an IFαGCS in X. Then αcl(A) ⊆ A. This implies cl(int(cl(A))) ⊆ A. That is int(cl(A)) ⊆ A. Since A is an IFOS, A is an IFPOS in X. Hence A ⊆ int(cl(A)). Therefore A = int(cl(A)). Hence A is an IFROS in X.

(ii): Let A be an IFOS and an IFα π GCS in X. Then cl(int(cl(A))) ⊆ A. That is cl(int(A)) ⊆ A. Since A is an IFOS, A is an IFSOS in X. Hence A ⊆ cl(int(A)). Therefore A = cl(int(A)). Hence A is an IFRCS in X.

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