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Application of Geometric Series to Health Aspects

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Abstract - In this paper, I had tried to make use of the convergence concept of famous Geometric Series to study about the time period (in weeks) of recovering from infected diseases. Analyzing for various common ratios, I try to determine the best period for healing and determine the best possible threshold limit corresponding to the critical ratio.

Keywords - Common Ratio, Geometric Series, Convergence, Critical Ratio, Threshold Value

1. Introduction

In modern days, people suffer from various health issues beginning from small infection caused by virus or some bacteria. If unnoticed for longer period, this will eventually cause decline in their health, sometimes even ending their life.
Hence, a systematic mathematical analysis is carried out to specify the remedial measures which will eventually help the patients suffering from a particular infection, thereby making their life better.

2. Mathematical Set-Up of the Situation

Let us assume the case when a person is affected by unknown infection caused either by unidentified virus or by specific harmful bacteria. Let the infection rate be r% where $0 < r < 1$. The values for r are given in accordance with $r = 0$ denoting no infection and $r = 1$ denoting 100 percent infected. We make assumption that if a person is affected by an infection, after taking proper treatment the infection reduces over period of specific time and he/she gets well subsequently.
If in the first period, a person's health condition is 100 percent good and suddenly he gets infected with infection rate r%. During the second period, the person's health condition will be declined by r% and so his health condition is r percent affected and $1 - r$ percent unaffected. Thus, if $r = 0.6$, then during the second period, the person's health condition will be 60% affected and 40% unaffected.
Now during the third period, the person's health condition is further declined by r% and so at this instant, the person's health condition would be $r \times r = r^2\%$ percent affected and $(1-r^2)\%$ percent unaffected. Thus, if as before, $r = 0.6$, then at the third period, the person's health will be 36% affected and 64% unaffected. The steep decrease in the affected rate from second period to third period is because of the treatment that the person undergoes.
Continuing in same fashion, we see that, during successive periods, the affected percent will come down drastically leading eventually to complete cure from the infection. This is mathematically possible because the sequence r^n converges to 0, when n is large enough. That is, $\lim_{n\to\infty} r^n = 0$, since $0 < r < 1$.

3. The Critical Ratio

From the previous section, we see that the person's infected rate in n successive periods of time is given by $1, r, r^2, r^3, r^4, \ldots, r^{n-1}$. These are the first n terms of a sequence called Geometric Progression abbreviated as G.P. whose first term is 1 and common ratio is r.
Though, as the terms converge to 0, practically it is impossible to conceive the periods extending up to infinity. To overcome this and provide a practically viable solution, we first consider the total infected rate during the time from period m to period n where $m < n$.
The sum of first k terms of the Geometric Series is given by the formula

$$1 + r + r^2 + r^3 + r^4 + \cdots + r^{k-1} = \frac{1-r^k}{1-r} \qquad (1)$$

This is because of the fact $\left(1 + r + r^2 + r^3 + r^4 + \cdots + r^{k-1}\right) \times (1-r) = 1 - r^k$.

Now, using (1), we obtain the total infected rate during the time from period m to period n where $m < n$ as follows:

$$r^m + r^{m+1} + \cdots + r^{n-1} = \left(1 + r + r^2 + r^3 + r^4 + \cdots + r^{n-1}\right) - \left(1 + r + r^2 + r^3 + r^4 + \cdots + r^{m-1}\right)$$
$$= \left(\frac{1-r^n}{1-r}\right) - \left(\frac{1-r^m}{1-r}\right)$$
$$= \frac{r^m - r^n}{1-r}$$

We can now call this final result $\frac{r^m - r^n}{1-r}$ as the Critical Ratio as this value is the key factor for the conclusion of this work.

4. Simulation for different periods of time

Using the value of Critical Ratio from previous section, we now try to simulate the infected rate values for various choices of *r* as well as values of *m*, *n* which we may refer as number of weeks to get rid of the infection.

For the simulation experiment, we consider eight values of *r* given by

$0\cdot 28, 0\cdot 36, 0\cdot 50, 0\cdot 68, 0\cdot 75, 0\cdot 76, 0\cdot 80, 0\cdot 84$

For these eight values, we consider the periods from *m* to *n* weeks where *m* < *n* in four possible cases given by $(m,n) = (2,5), (3,6), (4,7), (3,5).$ Using the Critical ratio which gives the total infected rate from period *m* to period *n*, we construct the following tabular columns.

r	m	n	$\frac{r^m - r^n}{1-r}$
$0\cdot 28$	2	5	$0\cdot 10649856$
$0\cdot 28$	3	6	$0\cdot 02981959$
$0\cdot 28$	4	7	$0\cdot 00834948$
$0\cdot 28$	3	5	0.02809856

r	m	n	$\frac{r^m - r^n}{1-r}$
$0\cdot 36$	2	5	$0\cdot 19305216$
$0\cdot 36$	3	6	$0\cdot 06949877$
$0\cdot 36$	4	7	$0\cdot 02501955$
$0\cdot 36$	3	5	0.06345216

r	m	n	$\frac{r^m - r^n}{1-r}$
$0\cdot 50$	2	5	$0\cdot 4375$
$0\cdot 50$	3	6	$0\cdot 21875$
$0\cdot 50$	4	7	$0\cdot 109375$
$0\cdot 50$	3	5	$0\cdot 1875$

r	m	n	$\frac{r^m - r^n}{1-r}$
$0\cdot 68$	2	5	$0\cdot 99064576$
$0\cdot 68$	3	6	$0\cdot 67363911$
$0\cdot 68$	4	7	$0\cdot 45807459$
$0\cdot 68$	3	5	$0\cdot 52824576$

r	m	n	$\frac{r^m - r^n}{1-r}$

5.

r	m	n	$\frac{r^m - r^n}{1-r}$
0·75	2	5	1·30078125
0·75	3	6	0·97558593
0·75	4	7	0·73168945
0·75	3	5	0·73828125

r	m	n	$\frac{r^m - r^n}{1-r}$
0·80	2	5	1·5616
0·80	3	6	1·24928
0·80	4	7	0·999424
0·80	3	5	0·9216

r	m	n	$\frac{r^m - r^n}{1-r}$
0·76	2	5	1·35019776
0·76	3	6	1·02615029
0·76	4	7	0·77987422
0·76	3	5	0·77259776

r	m	n	$\frac{r^m - r^n}{1-r}$
0·84	2	5	1·79617536
0·84	3	6	1·50878730
0·84	4	7	1·26738133
0·84	3	5	1·09057536

Threshold Value

From the tabular values of previous section, we find that for $r = 0\cdot 28, 0\cdot 36, 0\cdot 50, 0\cdot 68, 0\cdot 75$ the period from 4 weeks to 7 weeks gives the least value of the critical ratio. Hence the best option for these values of *r* is to consider the treatment from 4 to 7 weeks.

Now, for the values of $r = 0\cdot 76, 0\cdot 80, 0\cdot 84$ the period from 3 weeks to 5 weeks provide the least value of the critical ratio. Hence for these values of *r*, appropriate decision would be from 3 to 5 weeks.

Now, does there exist, a value of *r*, for which these two decisions are equally possible? We call such value of *r* as the Threshold Value of the infected rate *r*.

We find from our calculations, that such a Threshold value of *r* must be some number between $0\cdot 75$ and $0\cdot 76$. To be more precise, the Threshold value of *r* is $0\cdot 755$, since for this value of *r*, the critical ratios agree to first decimal places. Also we note that this is precisely the midpoint of $0\cdot 75$ and $0\cdot 76$.

6. Conclusion

From the discussions of the previous sections, we make the following conclusions.

(i) The Threshold value of the infected rate $r = 0\cdot 755$

(ii) If the infected rate is below the Threshold Value, then the best way to recover from the infection would be to take proper treatment from 4 to 7 weeks.

(iii) If the infected rate is above the Threshold Value, then the best way to recover from the infection would be to take proper treatment from 3 to 5 weeks.

(iv) For any affected rate, if the person undergo proper medical treatment for specific period of time, the infection rate will gradually decrease leading to the recovery of the health.

(v) This result does not hold true for infections or diseases which doesn't have proper vaccinations or medicines to cure, say like HIV virus.

Thus using simple mathematical procedure, we have found ways to improve the health condition under available and suitable environments.

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