Solution to the Fuzzy Transportation Problem using a new Method of Ranking of Trapezoidal Fuzzy Numbers

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Abstract - This paper proposes a new method for ranking of fuzzy numbers. Ranking of fuzzy numbers play a vital role in decision making problems, data analysis, socio economics systems, optimization, forecasting etc. Ranking fuzzy numbers is a necessary step in many mathematical models. Many of the methods proposed so far are non-discriminating. This paper presents a new ranking method which converts the fuzzy transportation problem to a crisp valued transportation problem which then can be solved using MODI Method to find the fuzzy optimal solution. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

Key Words: Trapezoidal fuzzy numbers, Ranking function, Transportation Problem, Fuzzy Transportation Problem, Optimal solution.

1.INTRODUCTION

Transportation problem is used globally in solving certain concrete world problems. A transportation problem plays a vital role in production industry and logistics and supply chain management for reducing cost and time for better service. The transportation problem is a special case of Linear programming problem, which permit us to regulate the optimum shipping patterns between origins and destinations. The solution of the problem will empower us to determine the number of entities to be transported from a particular origin to a particular destination so that the cost obtained is minimum or the time taken is minimum or the profit obtained is maximum. A fuzzy transportation problem is a transportation problem in which the transportation expenditures, supply and demand quantities are fuzzy quantities. Ranking fuzzy number is a necessary step in many mathematical models. The concepts of fuzzy sets were first introduced by Zadeh [1]. Since its inception several ranking procedure have been developed. There onwards many authors presented various approaches for solving the FTP problems [2], [4], [5], [14], [15]. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani [3]. Ranking normal fuzzy number were first introduced by Jain [7] for decision making in fuzzy situations. Chan stated that in many situations it is not possible to restrict the membership function to the general form and proposed the concept of generalized fuzzy numbers. The development in ordering fuzzy numbers can even be found in [6], [7], [8], [9] .To illustrate this proposed method, an example is discussed. As the proposed ranking method is very direct and simple it is very easy to understand and using which it is easy to find the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations. This paper is organized as follows: Section 2, briefly introduced the basic definitions of fuzzy numbers. In section 3, a new ranking procedure is proposed. In section 4, MODI method is adopted to solve Fuzzy transportation problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition:

If x is a set of objects denoted generally by X, then a fuzzy set A in X is defined as a set of ordered pairs A= {(x, $\mu A(x)$/ x Є X}, where $\mu A(x)$ is called the membership function for the fuzzy set A. The membership function maps each element of X to a membership value between 0 and 1.

2.2 Definition:

A fuzzy set A ,is defined on universal set of real numbers, is said to be a generalized fuzzy number if its membership function has the following characteristics-

(i) $\mu A(x) : R \rightarrow [0, 1]$ is continuous
(ii) $\mu A(x) = 0$ for all x Є A $(-\infty, a] \cup [d, \infty)$
(iii) $\mu A(x)$ is strictly increasing on [a, b] and strictly decreasing on [c, d]
(iv) $\mu A(x) = \omega$ for all x Є [b, c], where $0 < \omega \le 1$

2.3 Definition:

A generalized fuzzy number A= (a, b, c, d, ω) is said to be a generalized trapezoidal fuzzy number if its membership function is given by,

$$\mu_A(x)=\begin{cases}\frac{w(x-a)}{b-a} & a\le x\le b\\ \omega & b\le x\le c\\ \frac{w(x-a)}{d-c} & c\le x\le d\\ o & otherwise\end{cases}$$

If ω =1, then A = (a, b, c, d; 1) is a normalized trapezoidal fuzzy number and A is a generalized or non-normal trapezoidal fuzzy number if 0 < ω < 1.

As a particular case if b=c, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by A= (a, b, d; 1)

2.4 Definition:

Let A1= (a1, b1, c1, d1; ω1) and A2= (a2, b2, c2, d2; ω2) be generalized trapezoidal fuzzy number defined on real numbers R then,

(i) A1 + A2 = (a1+a2, b1+b2, c1+c2, d1+d2; min (ω1, ω2))
(ii) A1 - A2 = (a1-d2, b1-c2, c1-b2, d1-a2; min (ω1, ω2))

2.5 Definition:

Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

The transportation problem applies to situations where a single commodity is to be transported from various sources of supply **(origins)** to various demands **(destinations).** Let there be *m* sources of supply s_1, s_2, ……..........s_m having ai (i = 1,2,......m) units of supplies respectively to be transported among *n* destinations d_1, d_2………d_n with b_j (j = 1,2…..n) units of requirements respectively.

Let c_{ij} be the cost for shipping one unit of the commodity from source i, to destination j for each route. If xij represents the units shipped per route from source i, to destination j, then the problem is to determine the transportation schedule which minimizes the total transportation cost of satisfying supply and demand conditions.

The transportation problem can be stated mathematically as a linear programming problem as below:

Minimise Z $=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$

$\sum_{i=1}^{m}x_{ij}\le a_i$ for i = 1, 2, . . . , m

$\sum_{j=1}^{n}x_{ij}\ge b_j$ for j = 1, 2, . . . , n xij ≥ 0 for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

This is a linear program with m × n decision variables, m + n functional constraints, and m × n nonnegativity constraints.

3. RANKING OF TRAPEZOIDAL FUZZY NUMBERS

In this section, a new approach for ranking of generalized trapezoidal number is proposed using trapezoid as reference point. Ranking methods map fuzzy number directly in to the real line. That is, M: F→ R which associate every fuzzy number with a real number and then use the ordering ≥ on the real line.

Let A= (a, b, c, d; ω) be generalized trapezoidal fuzzy numbers then R(A) is calculated as follows:

$$R(A)=\frac{d(b-a)+a(d-c)}{b-a+d-c}$$

4. NUMERICAL EXAMPLE

A company has four sources S1, S2, S3and S4 and four destinations D1, D2, D3 and D4;the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is c_{ij} ,where

$$[c_{ij}]=\begin{pmatrix}(1,2,3,4) & (1,3,4,6) & (9,11,12,14) & (5,7,8,11)\\ (0,1,2,4) & (-1,0,1,2) & (5,6,7,8) & (0,1,2,3)\\ (3,5,6,8,) & (5,8,9,12) & (12,15,16,19) & (7,9,10,12)\end{pmatrix}$$

And fuzzy availability of the product at source are

((1,6,7,12), (0,1,2,3), (5,10,12,17),) and the fuzzy demand of the product at destinations are ((5,7,8,10), (1,5,6,10), (1,3,4,6) (1,2,3,4)) respectively.

Then the problem becomes,

	FD1	FD2	FD3	FD4	SUPPLY
FS1	(1,2,3,4,)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
FS2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
FS3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
DEMAND	(5,7,8,10	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

Table1: Fuzzy Transportation Problem

By using definition, Fuzzy Transportation Problem is balancedi.e. Sum of supply = Sum of demand

Step 1: By using our new approach given in SECTION 3 the fuzzy transportation problem is changed in to a crisp transportation problem as in table 2

	FD1	FD2	FD3	FD4	SUPPLY
FS1	2.5	3.5	11.5	7.4	6.5
FS2	1.33	0.5	6.5	1.5	1.5
FS3	5.5	8.6	15.5	9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Table2: Crisp Transportation Problem

Step 2: Using VAM method we obtain the initial solution as –

	FD1	FD2	FD3	FD4	SUPPLY
FS1	1 2.5	5.5 3.5	11.5	7.4	6.5
FS2	1.33	0.5	6.5	1.5 1.5	1.5
FS3	6.5 5.5	8.6	3.5 15.5	1 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Hence, the IBFS is,

1×2.5+5.5×3.5+1.5×1.5+6.5×5.5+3.5×15.5+1×9.5=123.5

The IBFS for the fuzzy transportation problem obtained above is the same as compared to the IBFS obtained by Shugani Poonam [15], P. Pandian and G. Natarajan [13].

But the solution obtained is not optimal hence it can be further improved using MODI METHOD (UV-METHOD)-

Step 3: Hence by using the MODI method we shall get the optimal solution as

	FD1	FD2	FD3	FD4	SUPPLY
FS1	1 2.5	5.5 3.5	11.5	7.4	6.5
FS2	1.33	0.5	1.5 6.5	1.5	1.5
FS3	6.5 5.5	8.6	2 15.5	2.5 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY
FS1	2.5	5.5 3.5	1 11.5	7.4	6.5
FS2	1.33	0.5	1.5 6.5	1.5	1.5
FS3	7.5 5.5	8.6	1 15.5	2.5 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is an optimal solution.

Hence the optimum cost is -

3.5×5.5+1×11.5+5.5×7.5+15.5×1+1.5×6.5+9.5×2.5=121

CONCLUSION

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers hence it is not possible to mention which fuzzy ranking method is the best. This paper proposed a new ranking method which is simple and efficient. Most of the time choosing a method rather than another is a matter of preference. The ranking of trapezoidal fuzzy numbers done by the Shugani Poonam [15], P. Pandian and G. Natarajan[13] gives the same values for symmetric fuzzy numbers while it changes slightly for non-symmetric fuzzy numbers but maintains the order of the fuzzy numbers.

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