

$(g\#, s)$ -continuous functions in topology

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Abstract—In this paper, we introduce $(g\#, s)$ -continuous functions between topological spaces, study some of its basic properties and discuss its relationships with other topological functions.

IndexTerms— $g\#$ -closed set, regular open set, $g\#$ -T1/2 space, πg -T1/2 space, $(g\#, s)$ -continuous function.

I. INTRODUCTION

It is well known that the concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [26] initiated the study of generalized closed sets. The concept of $g\#$ -closed sets was introduced by Veerakumar [46]. Recently, this notion is further studied by Ravi et al [40]. Initiation of contra-continuity was due to Dontchev [10]. Many different forms of contra-continuity functions have been introduced over the years by various authors [5,11,14,15,17,19,22,37,40].

In this paper, new generalizations of contra-continuity by using $g\#$ -closed sets called $(g\#, s)$ -continuity are presented. Characterizations and properties of $(g\#, s)$ -continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces

II. PRELIMINARIES

In this paper, spaces X and Y mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X , $cl(A)$ and $int(A)$ represent the closure of A and interior of A respectively.

A subset A of a space X is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$) [43]. The δ -interior [45] of a subset A of X is the union of all regular open sets of X contained in A and it is denoted by $\delta-int(A)$. A subset A is called δ -open [45] if $A = \delta-int(A)$. The complement of δ -open set is called δ -closed. The δ -closure of a set A in a space (X, τ) is defined by $\delta-cl(A) = \{ x \in X: A \cap int(cl(U)) \neq \emptyset, U \in \tau \text{ and } x \in U \}$ and it is denoted by $\delta-cl(A)$.

The finite union of regular open sets is said to be π -open [48]. The complement of π -open set is said to be π -closed. A subset A is said to be semi-open [25] (resp. α -open [31], preopen [30], β -open [1] or semi-preopen [2]) if $A \subset cl(int(A))$ (resp. $A \subset int(cl(int(A)))$, $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$). The complement of semi-open (resp. α -open, preopen, β -open) is said to be semi-closed (resp. α -closed, preclosed, β -closed). The union (resp. intersection) of all α -open (resp. α -closed) sets, each contained in (containing) a set S in a topological space X is called α -interior (resp. α -closure) of S and it is denoted by $\alpha cl(S)$ (resp. $\alpha int(S)$). The union (resp. intersection) of all semi-open (resp. semi-closed) sets, each contained in (containing) a set S in a topological space X is called semi-interior (resp. semi-closure) of S and it is denoted by $scl(S)$ (resp. $sint(S)$). The union (resp. intersection) of all preopen (resp. preclosed) sets, each contained in (containing) a set S in a topological space X is called preinterior (resp. preclosure) of S and it is denoted by $pcl(S)$ (resp. $pint(S)$).

A subset A of a space X is said to be α -generalized closed (briefly, αg -closed)[29] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of αg -closed set is called αg -open. A subset A of a space X is said to be generalized closed (briefly, g -closed) [26] (resp. πg -closed [13], $g\#$ -closed [46]) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open (resp. π -open, αg -open) in X . The complement of g -closed (resp. πg -closed, $g\#$ -closed) is said to be g -open (resp. πg -open, $g\#$ -open, rg -open). The union (resp. intersection) of all $g\#$ -open (resp. $g\#$ -closed) sets, each contained in (containing) a set S in a topological space X is called $g\#$ -interior (resp. $g\#$ -closure) of S and it is denoted by $g\#-int(S)$ (resp. $g\#-cl(S)$).

A point $x \in X$ is said to be a θ -semi-cluster point [23] of a subset A of X if $cl(U) \cap A \neq \emptyset$ for every semi-open set U containing x . The set of all θ -semi-cluster points of A is called the θ -semi-closure of A and is denoted by $\theta-s-cl(A)$. A subset A is called θ -semi-closed [23] if $A = \theta-s-cl(A)$. The complement of a θ -semi-closed set is called θ -semi-open.

The family of all δ -open (resp. $g^\#$ -open, $g^\#$ -closed, πg -open, πg -closed, regular open, regular closed, semi-open, closed) sets of X containing a point $x \in X$ is denoted by $\delta O(X, x)$ (resp. $G^\#O(X, x)$, $G^\#C(X, x)$, $\pi GO(X, x)$, $\pi GC(X, x)$, $RO(X, x)$, $RC(X, x)$, $SO(X, x)$, $C(X, x)$). The family of all δ -open (resp. $g^\#$ -open, $g^\#$ -closed, πg -open, πg -closed, semi-open, β -open, preopen, regular open, regular closed) sets of X is denoted by $\delta O(X)$ (resp. $G^\#O(X)$, $G^\#C(X)$, $\pi GO(X)$, $\pi GC(X)$, $SO(X)$, $\beta O(X)$, $PO(X)$, $RO(X)$, $RC(X)$).

Definition 1: A space X is said to be

- (1) s -Urysohn [3] if for each pair of distinct points x and y in X , there exist $U \in SO(X, x)$ and $V \in SO(X, y)$ such that $cl(U) \cap cl(V) = \phi$.
- (2) weakly Hausdorff [41] if each element of X is an intersection of regular closed sets.

Definition 2 [20]: Let B be a subset of a space X . The set $\bigcap \{ A \in RO(X) : B \subset A \}$ is called the r -kernel of B and is denoted by $r\text{-ker}(B)$.

Proposition 3 [20]: *The following properties hold for subsets A, B of a space X :*

- (1) $x \in r\text{-ker}(A)$ if and only if $A \cap K \neq \phi$ for any regular closed set K containing x .
- (2) $A \subset r\text{-ker}(A)$ and $A = r\text{-ker}(A)$ if A is regular open in X .
- (3) $A \subset B$, then $r\text{-ker}(A) \subset r\text{-ker}(B)$.

Lemma 4 [28]: If V is an open set, then $scl(V) = int(cl(V))$. The subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of a function $f : X \rightarrow Y$ and is denoted by $G(f)$.

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III. CHARACTERIZATIONS OF $G^\#$ -OPEN SETS

Lemma 5: For any subset K of a topological space X , $X \setminus g^\#cl(K) = g^\#int(X \setminus K)$.

Lemma 6: *If a subset A is $g^\#$ -closed in a space X , then $A = g^\#cl(A)$.*

Theorem 7: If A and B are $g^\#$ -closed sets, then $A \cup B$ is also a $g^\#$ -closed set.

Proof It follows from the fact that $cl(A \cup B) = cl(A) \cup cl(B)$.

Theorem 8: If A and B are $g^\#$ -open sets, then $A \cap B$ is also a $g^\#$ -open set.

Theorem 9: A set A is $g^\#$ -open in (X, τ) if and only if $F \subseteq int(A)$ whenever F is αg -closed in X and $F \subseteq A$.

Proof Assume that A is $g^\#$ -open, $F \subseteq A$ and F is αg -closed. Then $X \setminus F$ is αg -open and $X \setminus A \subseteq X \setminus F$. Since $X \setminus A$ is $g^\#$ -closed, $cl(X \setminus A) \subseteq X \setminus F$. It implies that $X \setminus int(A) \subseteq X \setminus F$ and hence $F \subseteq int(A)$.

Conversely, put $X \setminus A = B$. Suppose $B \subseteq U$ where U is αg -open. Now if $X \setminus A \subseteq U$, then $F = X \setminus U \subseteq A$ and F is αg -closed. It implies that $F \subseteq int(A)$ and hence $X \setminus int(A) \subseteq X \setminus F = U$. Therefore $X \setminus int(X \setminus B) \subseteq U$ and consequently $cl(B) \subseteq U$. Hence B is $g^\#$ -closed and therefore A is $g^\#$ -open.

Theorem 10: Suppose that A is $g^\#$ -open in X and that B is $g^\#$ -open in Y . Then $A \times B$ is $g^\#$ -open in $X \times Y$.

Proof Suppose that F is closed and hence αg -closed in $X \times Y$ and that $F \subseteq A \times B$. By the previous theorem, it suffices to show that $F \subseteq int(A \times B)$.

Let $(x, y) \in F$. Then, for each $(x, y) \in F$, $cl(\{x\}) \times cl(\{y\}) = cl(\{x\} \times \{y\}) = cl(\{x, y\}) \subset cl(F) = F \subset A \times B$. Two closed sets $cl(\{x\})$ and $cl(\{y\})$ are contained in A and B respectively. It follows from the assumption that $cl(\{x\}) \subseteq int(A)$ and that $cl(\{y\}) \subseteq int(B)$. Thus $\{x, y\} \in cl(\{x\}) \times cl(\{y\}) \subseteq int(A) \times int(B) \subseteq int(A \times B)$. It means that, for each $(x, y) \in F$, $(x, y) \in int(A \times B)$ and hence $F \subseteq int(A \times B)$. Therefore $A \times B$ is $g^\#$ -open in $X \times Y$.

Theorem 11: *Let A be a subset of Y . The following holds: $g^\#cl(X \times A) = X \times g^\#cl(A)$.*

Proof Since X and $g^\#cl(A)$ are $g^\#$ -closed, the product $X \times g^\#cl(A)$ is a $g^\#$ -closed set containing $X \times g^\#cl(A)$. Using definition we have $g^\#cl(X \times A) \subset X \times g^\#cl(A)$.

To prove the converse containment relation: $X \times g\#-cl(A) \subset g\#-cl(X \times A)$, we suppose that there exists a point $(a, b) \notin g\#-cl(X \times A)$. Then, there exists a $g\#$ -open set W containing (a, b) [similar to 38; Lemma 2] such that $W \cap (X \times A) = \emptyset$ and hence $p(W) \cap A = \emptyset$, where $p: X \times Y \rightarrow Y$ is the projection onto Y . Since $p(W)$ is $g\#$ -open containing $b = p(a, b) \in p(W)$, $b \notin g\#-cl(A)$. Therefore, we show $(a, b) \notin X \times g\#-cl(A)$ and hence $X \times g\#-cl(A) \subset g\#-cl(X \times A)$.

Definition 12 A function $f: X \rightarrow Y$ is called pre $g\#$ -closed [46] if $f(V)$ is $g\#$ -closed set in Y for each $g\#$ -closed set V in X .

Theorem 13 If a function $f: X \rightarrow Y$ is pre $g\#$ -closed, then for each subset B of Y and each $g\#$ -open set U of X containing $f^{-1}(B)$, there exists a $g\#$ -open set V in Y containing B such that $f^{-1}(V) \subset U$.

Proof Suppose that f is pre $g\#$ -closed. Let B be a subset of Y and $U \in G\#O(X)$ containing $f^{-1}(B)$. Put $V = Y \setminus f(X \setminus U)$, then V is a $g\#$ -open set of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.
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IV. ($G\#, s$)-CONTINUOUS FUNCTIONS

Definition 14 A function $f: X \rightarrow Y$ is called ($g\#, s$)-continuous if the inverse image of each regular open set of Y is $g\#$ -closed in X .

Theorem 15 The following are equivalent for a function $f: X \rightarrow Y$:

- (1) f is ($g\#, s$)-continuous,
- (2) The inverse image of a regular closed set of Y is $g\#$ -open in X ,
- (3) $f^{-1}(\text{int}(\text{cl}(V)))$ is $g\#$ -closed in X for every open subset V of Y ,
- (4) $f^{-1}(\text{cl}(\text{int}(F)))$ is $g\#$ -open in X for every closed subset F of Y ,
- (5) $f^{-1}(\text{cl}(U))$ is $g\#$ -open in X for every $U \in \beta O(Y)$,
- (6) $f^{-1}(\text{cl}(U))$ is $g\#$ -open in X for every $U \in SO(Y)$,
- (7) $f^{-1}(\text{int}(\text{cl}(U)))$ is $g\#$ -closed in X for every $U \in PO(Y)$.

Proof

(1) \Leftrightarrow (2): Obvious.

(1) \Leftrightarrow (3): Let V be an open subset of Y . Since $\text{int}(\text{cl}(V))$ is regular open, $f^{-1}(\text{int}(\text{cl}(V)))$ is $g\#$ -closed. The converse is similar.

(2) \Leftrightarrow (4): Similar to (1) \Leftrightarrow (3)

(2) \Leftrightarrow (5): Let U be any β -open set of Y . By Theorem 2.4 of [2] that $\text{cl}(U)$ is regular closed. Then by (2) $f^{-1}(\text{cl}(U))$ is $g\#$ -open in X .

(5) \Rightarrow (6): Obvious from the fact that $SO(Y) \subset \beta O(Y)$.

(6) \Rightarrow (7): Let $U \in PO(Y)$. Then $Y \setminus \text{int}(\text{cl}(U))$ is regular closed and hence it is semi-open. Then we have $X \setminus f^{-1}(\text{int}(\text{cl}(U))) = f^{-1}(Y \setminus \text{int}(\text{cl}(U))) = f^{-1}(\text{cl}(Y \setminus \text{int}(\text{cl}(U))))$ is $g\#$ -open in X . Hence $f^{-1}(\text{int}(\text{cl}(U)))$ is $g\#$ -closed in X .

(7) \Rightarrow (1): Let U be any regular open set of Y . Then $U \in PO(Y)$ and hence $f^{-1}(U) = f^{-1}(\text{int}(\text{cl}(U)))$ is $g\#$ -closed in X .

Lemma 16 [35] For a subset A of a topological space (Y, σ) , the following properties hold:

- (1) $\alpha \text{cl}(A) = \text{cl}(A)$ for every $A \in \beta O(Y)$,
- (2) $p \text{cl}(A) = \text{cl}(A)$ for every $A \in SO(Y)$,
- (3) $s \text{cl}(A) = \text{int}(\text{cl}(A))$ for every $A \in PO(Y)$.

Corollary 17 The following are equivalent for a function $f: X \rightarrow Y$:

- (1) f is ($g\#, s$)-continuous,
- (2) $f^{-1}(\alpha \text{cl}(A))$ is $g\#$ -open in X for every $A \in \beta O(Y)$,
- (3) $f^{-1}(p \text{cl}(A))$ is $g\#$ -open in X for every $A \in SO(Y)$,
- (4) $f^{-1}(s \text{cl}(A))$ is $g\#$ -closed in X for every $A \in PO(Y)$.

Proof It follows from Lemma 16.

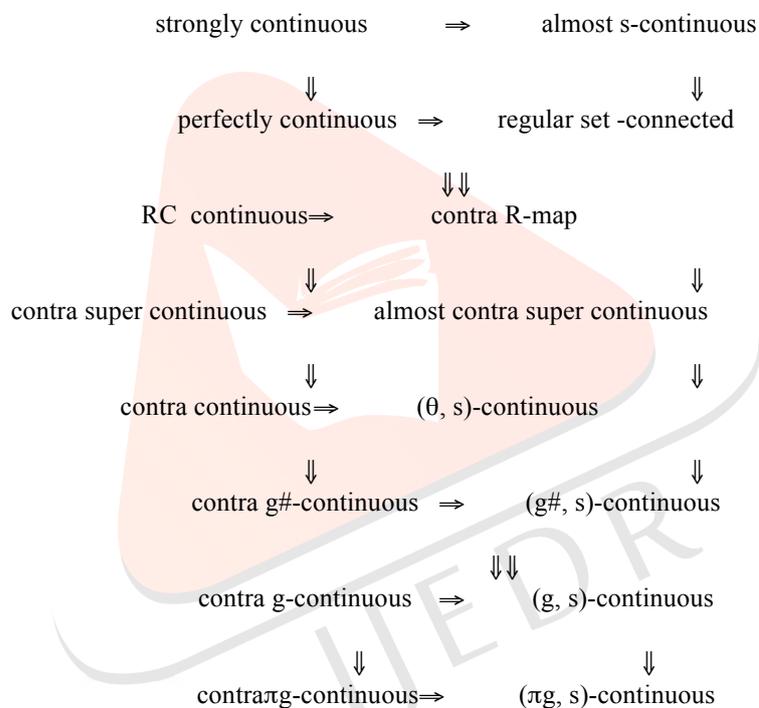
V. THE RELATED FUNCTIONS WITH ($G\#, S$)-CONTINUOUS FUNCTIONS

Definition 20 A function $f: X \rightarrow Y$ is said to be

- (1) perfectly continuous [33] if $f^{-1}(V)$ is clopen in X for every open set V of Y ,
- (2) regular set -connected [12,16] if $f^{-1}(V)$ is clopen in X for every $V \in RO(Y)$,

- (3) almost s-continuous [6,36] if for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists an open set U in X containing x such that $f(U) \subset scl(V)$,
- (4) strongly continuous [24] if the inverse image of every set in Y is clopen in X ,
- (5) RC-continuous [11] if $f^{-1}(V)$ is regular closed in X for each open set V of Y ,
- (6) contra R-map [17] if $f^{-1}(V)$ is regular closed in X for each regular open set V of Y ,
- (7) contra-super-continuous [22] if for each $x \in X$ and for each $F \in C(Y, f(x))$, there exists a regular open set U in X containing x such that $f(U) \subset F$,
- (8) almost contra-super-continuous [15] if $f^{-1}(V)$ is δ -closed in X for every regular open set V of Y .
- (9) contra continuous [10] if $f^{-1}(V)$ is closed in X for every open set V of Y ,
- (10) contra g-continuous [5] if $f^{-1}(V)$ is g-closed in X for every open set V of Y ,
- (11) (θ, s) -continuous [23,37] if for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists an open set U in X containing x such that $f(U) \subset cl(V)$,
- (12) contra πg -continuous [19] if $f^{-1}(V)$ is πg -closed in X for each open set V of Y .
- (13) contra $g\#$ -continuous [40] if $f^{-1}(V)$ is $g\#$ -closed in X for each open set V of Y ,
- (14) $g\#$ -continuous [46] if $f^{-1}(V)$ is $g\#$ -closed in X for each closed set V of Y ,
- (15) (g, s) -continuous [14] if $f^{-1}(V)$ is g-closed in X for each regular open set V of Y ,
- (16) $(\pi g, s)$ -continuous [14] if $f^{-1}(V)$ is πg -closed in X for each regular open set V of Y .

Remark 21 The following diagram holds for a function $f: X \rightarrow Y$:



None of these implications is reversible as shown in the following examples and in the related papers [14,40].

Example 22 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X \setminus \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{b\}, \{a, c\}\}$. Then the identity function $f: X \rightarrow Y$ is (g, s) -continuous but it is not $(g\#, s)$ -continuous.

Example 23 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X \setminus \{a\}, \{b, c\}\}$ and $\sigma = \{\phi, Y, \{a, b\}\}$. Then the identity function $f: X \rightarrow Y$ is $(g\#, s)$ -continuous but it is not contra $g\#$ -continuous.

Example 24 Let $X = Y = \{a, b, c, d\}$, $\tau = \sigma = \{\phi, X = Y, \{c\}, \{a, d\}, \{a, c, d\}\}$. Then the function $f: X \rightarrow Y$ which is defined as $f(a) = c, f(b) = c, f(c) = b, f(d) = b$ is $(g\#, s)$ -continuous but it is not (θ, s) -continuous.

A topological space (X, τ) is said to be extremely disconnected [4] if the closure of every open set of X is open in X .

Definition 25 A function $f: X \rightarrow Y$ is said to be almost $g\#$ -continuous if $f^{-1}(V)$ is $g\#$ -open in X for every regular open set V of Y .

Theorem 26 Let (Y, σ) be extremely disconnected. Then, the following are equivalent for a function $f: (X, \tau) \rightarrow (Y, \sigma)$:

- (1) f is $(g\#, s)$ -continuous,
- (2) f is almost $g\#$ -continuous.

Proof

(1) \Rightarrow (2): Let $x \in X$ and U be any regular open set of Y containing $f(x)$. Since Y is extremely disconnected, by lemma 5.6 of [39] U is clopen and hence U is regular closed. Then $f^{-1}(U)$ is $g^\#$ -open in X . Thus, f is almost $g^\#$ -continuous.

(2) \Rightarrow (1): Let K be any regular closed set of Y . Since Y is extremely disconnected, K is regular open and $f^{-1}(K)$ is $g^\#$ -open in X . Thus, f is $(g^\#, s)$ -continuous.

Definition 27 A space (X, τ) is called:

- (1) $T_b[9]$ if every g_s -closed set is closed.
- (2) $\pi g-T_{1/2}$ [18] if every πg -closed set is closed.

Definition 28 A function $f: X \rightarrow Y$ is called:

- (1) contra sg -continuous [11] if $f^{-1}(V)$ is sg -closed in X for each open set V of Y ,
- (2) contrags-continuous [11] if $f^{-1}(V)$ is g_s -closed in X for each open set V of Y .

Theorem 29 Let $f: X \rightarrow Y$ be a function from an T_b -space X to a topological space Y . The following are equivalent

- (1) f is $(g^\#, s)$ -continuous.
- (2) f is contra $g^\#$ -continuous.
- (3) f is contra sg -continuous.
- (4) f is contra g_s -continuous.
- (5) f is contra-continuous.

Proof Follows by the results in [46].

Theorem 30 Let $f: X \rightarrow Y$ be a function from an $\pi g-T_{1/2}$ -space X to a topological space Y . The following are equivalent.

- (1) f is (θ, s) -continuous.
- (2) f is $(g^\#, s)$ -continuous.
- (3) f is (g, s) -continuous.
- (4) f is $(\pi g, s)$ -continuous.
- (5) f is contra-continuous.

Definition 31 A space is said to be $P\Sigma$ [47] or strongly s -regular [21] if for any open set V of X and each $x \in V$, there exists $K \in$

$RC(X, x)$ such that $x \in K \subset V$.

Definition 32 A space (X, τ) is called $g^\#-T_{1/2}$ if every $g^\#$ -closed set is closed.

Theorem 33 Let $f: X \rightarrow Y$ be a function. Then, if f is $(g^\#, s)$ -continuous, X is $g^\#-T_{1/2}$ and Y is $P\Sigma$, then f is continuous.

Proof Let G be any open set of Y . Since Y is $P\Sigma$, there exists a subfamily Φ of $RC(Y)$ such that $G = \bigcup \{A: A \in \Phi\}$. Since X is $g^\#-T_{1/2}$ and f is $(g^\#, s)$ -continuous, $f^{-1}(G)$ is open in X . Thus, f is continuous.

Theorem 34 Let $f: X \rightarrow Y$ be a function from a $\pi g-T_{1/2}$ -space (X, τ) to an extremely disconnected space (Y, σ) . Then the following are equivalent..

- (1) f is $(\pi g, s)$ -continuous.
- (2) f is (g, s) -continuous.
- (3) f is $(g^\#, s)$ -continuous.
- (4) f is (θ, s) -continuous.
- (5) f is almost contra-super-continuous.
- (6) f is contra R -map.
- (7) f is regular set-connected.
- (8) f is almost s -continuous.

Proof (8) \Rightarrow (7) \Rightarrow (6) \Rightarrow (5) \Rightarrow (4) \Rightarrow (3) \Rightarrow (2) \Rightarrow (1): Obvious.

(1) \Rightarrow (8) Let V be any semi-open and semi-closed set of Y . Since V is semi-open, $cl(V) = cl(int(V))$ and hence $cl(V)$ is open in Y . Since V is semi-closed, $int(cl(V)) \subset V \subset cl(V)$ and hence $int(cl(V)) = V = cl(V)$. Therefore, V is clopen in Y and $V \in RO(Y) \cap RC(Y)$. Since f is $(\pi g, s)$ -continuous, $f^{-1}(V)$ is πg -open and πg -closed in X . Since X is $\pi g-T_{1/2}$ -space, $\tau = \pi gO(X)$. Thus, $f^{-1}(V)$ is clopen in X and hence f is almost s -continuous [36, Theorem 3.1].

Definition35 A space is said to be weakly $P\Sigma$ [34] if for any $V \in RO(X)$ and each $x \in V$, there exists $F \in RC(X, x)$ such that $x \in F \subset V$.

Theorem 36 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $(g^\#, s)$ -continuous function and Let $G^\#C(X)$ be closed under arbitrary intersections. If Y is weakly $P\Sigma$ and X is $g^\#-T^{1/2}$, then f is regular set-connected.

Proof Let V be any regular open set of Y . Since Y is weakly $P\Sigma$, there exists a subfamily Φ of $RC(Y)$ such that $V = \cup\{A: A \in \Phi\}$. Since f is $(g^\#, s)$ -continuous, $f^{-1}(V)$ is $g^\#$ -open in X for each $A \in \Phi$ and $f^{-1}(V)$ is $g^\#$ -open in X . Also $f^{-1}(V)$ is $g^\#$ -closed in X since f is $(g^\#, s)$ -continuous. Since X is $g^\#-T^{1/2}$ space, then $\tau = G^\#O(X)$. Hence $f^{-1}(V)$ is clopen in X and then f is regular set-connected.

Definition 37 A function $f: X \rightarrow Y$ is said to be $g^\#$ -irresolute [46] if $f^{-1}(V)$ is $g^\#$ -open in X for every $V \in G^\#O(Y)$.

Theorem 38 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then, the following properties hold:

- (1) If f is $g^\#$ -irresolute and g is $(g^\#, s)$ -continuous, then $g \circ f: X \rightarrow Z$ is $(g^\#, s)$ -continuous.
- (2) If f is $(g^\#, s)$ -continuous and g is contra R -map, then $g \circ f: X \rightarrow Z$ is almost $g^\#$ -continuous.
- (3) If f is $g^\#$ -continuous and g is (θ, s) -continuous, then $g \circ f: X \rightarrow Z$ is $(g^\#, s)$ -continuous.
- (4) If f is $(g^\#, s)$ -continuous and g is RC continuous, then $g \circ f: X \rightarrow Z$ is $g^\#$ -continuous.
- (5) If f is almost $g^\#$ -continuous and g is contra R -map, then $g \circ f: X \rightarrow Z$ is $(g^\#, s)$ -continuous.

Theorem 39 Let Y be regular space and $f: X \rightarrow Y$ be a function. Suppose that the collection of $g^\#$ -closed sets of X is closed under arbitrary intersections. Then if f is $(g^\#, s)$ -continuous, f is $g^\#$ -continuous.

Proof Let x be an arbitrary point of X and V an open set of Y containing $f(x)$. Since Y is regular, there exists an open set G in Y containing $f(x)$ such that $cl(G) \subset V$. Since f is $(g^\#, s)$ -continuous, there exists $U \in G^\#O(X, x)$ such that $f(U) \subset cl(G)$. Then $f(U) \subset cl(G) \subset V$. Hence, f is $g^\#$ -continuous.

REFERENCES

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