# On Strongly αg\*p-Irresolute Functions in Topological Spaces

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Abstract - In this paper, we introduce and investigate the notion of strongly  $\alpha g^* p$  -irresolute functions. We obtain fundamental properties and characterization of strongly  $\alpha g^* p$  -irresolute functions and discuss the relationships between strongly  $\alpha g^* p$  -irresolute functions and other related functions.

Keywords: strongly ag\*p -irresolute, strongly α -irresolute, strongly β- ag\*p irresolute.

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# **1.INTRODUCTION**

Dontchev[4] introduced the notion of contra continuous functions in 1996. Jafari and Noiri[9] introduced contra precontinuous functions. Ekici.E[6] introduced almost contra precontinuous functions in 2004.. Dontchev and Noiri [5] introduced and investigated contra semi-continuous functions and RC continuous functions between topological spaces.Veerakumar [25] also introduced contra pre semi-continuous functions. Recently, S.Sekar and P.Jayakumar[20] introduced contra gp\*-continuous functions. In this paper we introduce and study the new class of functions called contra ag\*p-continuous and almost contra  $\alpha$ g\*p-continuous functions in topological spaces. Also we define the notions of contra  $\alpha$ g\*p-locally indiscrete space and study some of their properties.

# **2. PRELIMINARIES**

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  (or simply X, Y, and Z) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X, the closure of A and interior of A will be denoted by cl (A) and int(A) respectively. The union of all  $\alpha g^*p$ -open sets of X contained in A is called  $\alpha g^*p$ -interior of A and it is denoted by  $\alpha g^*p$ -int (A). The intersection of all  $\alpha g^*p$ -closed sets of X containing A is called  $\alpha g^*p$ -closure of A and it is denoted by  $\alpha g^*p$ -cl(A).

Also the collection of all  $\alpha g^*p$ -open subsets of X containing a fixed point x is denoted by  $\alpha g^*p$ -O(X,x).

**Definitions 2.1:** A subset A of a topological space  $(X, \tau)$  is called

(i) preopen [13] if  $A \subseteq int (cl (A))$  and preclosed if  $cl (int(A)) \subseteq A$ .

(ii) semi-open [11] if  $A \subseteq cl$  (int (A)) and semi-closed if int (cl (A))  $\subseteq A$ .

(iii)  $\alpha$ -open [14] if  $A \subseteq$  int (cl (int (A))) and  $\alpha$ -closed if cl(int(cl(A))) \subseteq A.

(iv) semi-preopen [1] ( $\beta$ -open) if  $A \subseteq cl(int(cl(A)))$  and semi-preclosed ( $\beta$ -closed) if int (cl (int (A)))  $\subseteq A$ .

**Definition 2.2**: A subset A of a topological space  $(X, \tau)$  is called

(i) generalized preclosed (briefly,gp-closed)[12] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in X.

(ii) generalized semi-preclosed (briefly, gsp-closed)[3] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in X.

(iii) generalized pre regularclosed (briefly, gpr-closed)[7] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is regular open in X.

(iv) generalized star preclosed (briefly, g\*p-closed set) [23] if pcl (A)  $\subseteq$  U whenever A  $\subseteq$  U and U is g-open in X.

(v) generalized pre star closed (briefly, gp\*-closed set) [10] if cl (A)  $\subseteq$  U whenever A  $\subseteq$  U and U is gp-open in X.

(vi) pre semi-closed [24] if spcl (A)  $\subseteq$  U whenever A  $\subseteq$  U and U is  $\alpha$ g-open in X.

**Definition 2.3:**[18] A subset A of a topological space  $(X, \tau)$  is called alpha generalized star preclosed set (briefly,  $\alpha g^*p$ -closed) if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is  $\alpha g$ -open in X.

**Definition 2.4:**[17] A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $\alpha g^*p$ -continuous if f - 1(V) is  $\alpha g^*p$ -closed set in  $(X, \tau)$  for every closed set V in  $(Y, \sigma)$ .

**Definition 2.5:**[17] A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $\alpha g^*p$ -irresolute if f - 1(V) is  $\alpha g^*p$ -closed set in  $(X, \tau)$  for every  $\alpha g^*p$ -closed set V in  $(Y, \sigma)$ .

**Definition 2.6:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called i) contra continuous [4] if f-1(V) is closed set in X for each open set V of Y. ii) contra precontinuous [9] if f - 1(V) is preclosed set in X for each open set V of Y.

iii) contra semi-continuous [5] if f-1(V) is semi-closed set in X for each open set V of Y.

iv) contra  $\alpha$  continuous [8] if f-1(V) is  $\alpha$ -closed set in X for each open set V of Y.

v) contra pre semi-continuous [25] if f - 1(V) is pre semi-closed set in X for each open set V of Y.

vi) contra gp-continuous if f-1(V) is gp-closed set in X for each open set V of Y.

vii) contra gpr-continuous if f-1(V) is gpr-closed set in X for each open set V of Y.

viii) contra gsp-continuous if f - 1(V) is gsp-closed set in X for each open set V of Y.

ix) contra gp\*-continuous [20] if f-1(V) is gp\*-closed set in X for each open set V of Y.

x) contra g\*p-continuous [16] if f-1(V) is g\*p-closed set in X for each open set V of Y.

**Definition 2.7:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

i) perfectly continuous [15] if f -1(V) is clopen in X for every open set V of Y.

ii) almost continuous [21] if f-1(V) is open in X for each regular open set V of Y.

iii) almost  $\alpha g^*p$ -continuous [17] if f-1(V) is  $\alpha g^*p$ -open in X for each regular open set V of Y.

iv) almost contra g\*p-continuous [16] if f-1(V) is g\*p-closed in X for each regular open set V of Y.

v) preclosed [13] if f(U) is preclosed in Y for each closed set U of X.

vi) contra preclosed [2] if f (U) is preclosed in Y for each open set U of X.

Definition 2.8: Let A be a subset of a space (X ,  $\tau$  ) .

(i) The set  $\cap \{U \in \tau \mid A \subseteq U\}$  is called the kernel of A and is denoted by ker (A).

(ii) The set  $\cap$  {F  $\in X / A \subseteq F$ , F is preclosed} is called the preclosure of A and is denoted by pc1(A)

Lemma 2.9: The following properties hold for subsets A, B of a space X:

(1)  $x \in ker(A)$  if and only if  $A \cap F \neq \phi$  for any  $F \in C(X, x)$ .

(2)  $A \subset ker(A)$  and A = ker(A) if A is open in X.

(3) If  $A \subset B$ , then ker (A)  $\subset$  ker(B).

**Definition 2.10[18]:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called contra  $\alpha g^*p$ -continuous if  $f^{-1}(V)$  is  $\alpha g^*p$ -closed set in X for every open set V in Y.

# .3. Strongly ag\*p-irresolute functions.

# Definition:3.1

A function f:  $X \rightarrow Y$  is said to be strongly  $\alpha g^* p$ -irresolute if  $f^1(V)$  is open in X for every  $\alpha g^* p$ -open set V of Y.

# Definition:3.2

A function f:  $X \rightarrow Y$  is said to be strongly  $\alpha$ - irresolute if  $f^{1}(V)$  is open in X for every  $\alpha$ - open set V of Y.

# Theorem:3.3

If f:  $X \rightarrow Y$  is a strongly  $\alpha g^*p$  -irresolute ,then f is strongly  $\alpha$ -irresolute.

**Proof:**Let V be  $\alpha$ - open set in Y and hence V is  $\alpha g^*p$ -open in Y. Since f is strongly  $\alpha g^*p$ -irresolute, then  $f^1(V)$  is open in X. Therefore  $f^1(V)$  is open in X for every  $\alpha$ - open set V in Y. Hence f is strongly  $\alpha$ - irresolute.

# Theorem:3.4

If f:  $X \rightarrow Y$  is a continuous and Y is a  $\alpha g^* p - T_{1/2}$ -space, then f is strongly  $\alpha g^* p$ -irresolute.

**Proof:**Let V be  $\alpha g^*p$  -open in Y. Since Y is  $\alpha g^*p$  -T<sub>1/2</sub>-space, V is  $\alpha$ -open in Y and hence open in Y. Since f is continuous, f<sup>1</sup>(V) is open in X. Thus, f<sup>1</sup>(V) is open in X for every  $\alpha g^*p$  -open set V in Y. Hence f is strongly  $\alpha g^*p$  - irresolute.

# Theorem:3.5

If f:  $X \rightarrow Y$  is a  $\alpha g^*p$ -irresolute, X is a  $\alpha g^*p$ -T<sub>1/2</sub>-space, then f is strongly  $\alpha g^*p$ -irresolute.

**Proof:**Let V be  $\alpha g^* p$ -open in Y. Since f is  $\alpha g^* p$ -irresolute,  $f^1(V)$  is  $\alpha g^* p$ -open in X. Since X is a  $\alpha g^* p$ - $T_{1/2}$ -space,  $f^1(V)$  is  $\alpha$ -open in X and hence open in X. Thus,  $f^1(V)$  is open in X for every  $\alpha g^* p$ -open set V in Y. Hence f is strongly  $\alpha g^* p$ -irresolute.

# Theorem:3.6

Let f:  $X \rightarrow Y$  and g:  $Y \rightarrow Z$  be any functions. Then

- (i) g o f:  $X \rightarrow Z$  is  $\alpha g^* p$ -irresolute if f is  $\alpha g^* p$ -continuous and g is strongly  $\alpha g^* p$ -irresolute.
- (ii) g o f:  $X \rightarrow Z$  is strongly  $\alpha g^* p$  -irresolute if f is strongly  $\alpha g^* p$  -irresolute and g is  $\alpha g^* p$  -irresolute.

#### **Proof:**

(i)Let V be a  $\alpha g^*p$  -open set in Z. Since g is strongly  $\alpha g^*p$  -irresolute,  $g^{-1}(V)$  is open in Y. Since f is  $\alpha g^*p$  -continuous,  $f^{-1}(g^{-1}(V))$  is  $\alpha g^*p$  -open in X.

- $\Rightarrow$  (g o f)<sup>-1</sup>(V) is  $\alpha$ g\*p -open in X for every  $\alpha$ g\*p -open set V in Z.
- $\Rightarrow$  (g o f) is  $\alpha$ g\*p -irresolute.

(ii) Let V be a  $\alpha g^* p$ -open set in Z. Since g is  $\alpha g^* p$ -irresolute,  $g^{-1}(V)$  is  $\alpha g^* p$ -open in Y. Since f is strongly  $\alpha g^* p$ -irresolute,  $f^{-1}(g^{-1}(V))$  is open in X.

 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is open in X for every  $\alpha g^* p$  -open set V in Z.

 $\Rightarrow$  (g o f) is strongly  $\alpha g^* p$  -irresolute.

#### Theorem:3.7

The following are equivalent for a function f:  $X \rightarrow Y$ :

- (i) f is strongly  $\alpha g^* p$  -irresolute.
- (ii) For each  $x \in X$  and each  $\alpha g^*p$  -open set V of Y containing f(x), there exists an open set U in X containing x such that  $f(U) \subset V$ .
- (iii)  $f^{1}(V) \subset int (f^{1}(V))$  for each  $\alpha g^{*}p$  -open set V of Y.
- (iv)  $f^{1}(F)$  is closed in X for every  $\alpha g^{*}p$  -closed set F of Y.

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Proof: (i)⇒(ii):
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Let  $x \in X$  and V be a  $\alpha g^*p$  -open set in Y containing f(x). By hypothesis,  $f^1(V)$  is open in X and contains x.

Set  $U=f^{1}(V)$ . Then U is open in X and  $f(U) \subset V$ .

(ii)⇒(iii):

Let V be a  $\alpha g^* p$  -open set in Y and  $x \in f^1(V)$ .

By assumption, there exists an open set U in X containing x, such that  $f(U) \subset V$ .

Then  $x \in U \subset int(U)$ 

 $\subset$  int (f<sup>1</sup>(V)). Then f<sup>1</sup>(V)  $\subset$  int(f<sup>1</sup>(V))

 $(iii) \Rightarrow (iv):$ 

Let F be a  $\alpha g^*p$  -closed set in Y. Set V= Y – F. Then V is  $\alpha g^*p$  -open in Y.

By (iii),  $f^1(V) \subset int(f^1(V))$ .

Hence  $f^{1}(F)$  is closed in X.

 $(iv) \Rightarrow (i):$ 

Let V be  $\alpha g^*p$  -open set in Y. Let F = Y - V. That is F is  $\alpha g^*p$  -closed set in Y. Then  $f^1(F)$  is closed in X,(by (iv)). Then  $f^1(V)$  is open in X. Hence f is strongly  $\alpha g^*p$  -irresolute.

#### Theorem:3.8

A function f:  $X \rightarrow Y$  is strongly  $\alpha g^*p$  -irresolute if A is open in X, then f/A:  $A \rightarrow Y$  is strongly  $\alpha g^*p$  -irresolute.

**Proof:**Let V be a  $\alpha g^* p$  -open set in Y. By hypothesis,  $f^1(V)$  is open in X. But  $(f/A)^{-1}(V) = A \cap f^1(V)$  is open in A and hence f/A is strongly  $\alpha g^* p$  -irresolute.

# Theorem:3.9

Let f:  $X \rightarrow Y$  be a function and  $\{A_i: i \in \Lambda\}$  be a cover of X by open sets of  $(X,\tau)$ . Then f is strongly  $\alpha g^*p$  -irresolute if  $f/A_i: (A_i, \tau/A_i) \rightarrow (Y,\sigma)$  is strongly  $\alpha g^*p$  -irresolute for each  $i \in \Lambda$ .

**Proof:**Let V be a  $\alpha g^* p$  -open set in Y. By hypothesis,  $(f/A_i)^{-1}(V)$  is open in  $A_i$ . Since  $A_i$  is open in X,  $(f/A_i)^{-1}(V)$  is open in X for every  $i \in \Lambda$ .

 $f^{1}(V) = X \cap f^{1}(V)$ 

 $= \cup \{ A_i \cap f^1(V) : i \in \Lambda \}$ 

=U{ $(f/A_i)^{-1}(V)$ : i  $\in \Lambda$ } is open in X.

Hence f is strongly  $\alpha g^* p$  -irresolute.

# Theorem:3.10

Let f:  $X \rightarrow Y$  be a strongly  $\alpha g^*p$  -irresolute surjective function. If X is compact, then Y is  $\alpha G^*PO$ -compact.

**Proof:**Let  $\{A_i: i \in \Lambda\}$  be a cover of sg $\alpha$  -open sets of Y. Since f is strongly sg $\alpha$  -irresolute and X is compact, we get  $X \subset \bigcup \{f^{1}(A_i): i \in \Lambda\}$ . Since f is surjective,  $Y = f(X) \subset \bigcup \{A_i: i \in \Lambda\}$ . Hence Y is  $\alpha G^*PO$ -compact.

# Theorem:3.11

If  $f:X \to Y$  is strongly  $\alpha g^*p$ -irresolute and a subset B of X is compact relative to X, then f(B) is  $\alpha G^*PO$ -compact relative to Y.

# Proof: Obvious.

# **Definition: 3.12**

A function f:  $X \rightarrow Y$  is said to be

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(i) a strongly \alpha- \alpha g^* p-irresolute function if f^1(V) is \alpha- open in X for every \alpha g^* p-open set V in Y.
(ii) a strongly \beta- \alpha g^* p-irresolute function if f^1(V) is \beta-open in X for every \alpha g^* p-open set V in Y.
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# Theorem:3.13

- (i) If f:  $X \rightarrow Y$  is strongly  $\alpha$   $\alpha g^* p$ -irresolute, then f is strongly  $\alpha g^* p$ -irresolute.
- (ii) If f:  $X \rightarrow Y$  is strongly  $\alpha$   $\alpha g^* p$  -irresolute, then f is strongly  $\beta$   $\alpha g^* p$  -irresolute.

**Proof:** (i)Let f be a strongly  $\alpha$ -  $\alpha g^*p$  -irresolute function and let V be a  $\alpha g^*p$  -open set in Y. Then  $f^1(V)$  is  $\alpha$ - open in X and hence open in X.

 $\Rightarrow$  f<sup>1</sup>(V) is open in X for every  $\alpha g^* p$  -open set V in Y.

Hence f is strongly  $\alpha$ -  $\alpha$ g\*p -irresolute.

(ii) Let f be a strongly  $\alpha$ -  $\alpha$ g\*p -irresolute function and let V be a  $\alpha$ g\*p -open set in Y. Then

 $f^{-1}(V)$  is  $\alpha$ -open in X and hence open in X.

 $\Rightarrow$  f<sup>1</sup>(V) is open in X for every αg\*p -open set V in Y.  $\Rightarrow$  f<sup>1</sup>(V) is β-open in X for every αg\*p -open set V in Y.

Hence f is strongly  $\beta$ -  $\alpha g^*p$  -irresolute.

# Remark: 3.14

Converse of the above need not be true as seen in the following examples.

# Example: 3.15

(i)Let  $X = Y = \{a,b,c\}$ ,  $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$  and  $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}\}$ .

Let  $f:X \rightarrow Y$  be an identity map. Here for every  $\alpha g^* p$  -open set V in Y,  $f^1(V)$  is open and  $\beta$ -open in X. Hence f is strongly  $\alpha g^* p$  -irresolute and strongly  $\beta$ -  $\alpha g^* p$  -irresolute.

But for every  $\alpha g^* p$  -open set V in Y,  $f^1(V)$  is not  $\alpha$ - open in X. Thus, f is not strongly  $\alpha$ -  $\alpha g^* p$  -irresolute .Hence strongly  $\alpha g^* p$  -irresolute function need not be strongly  $\alpha$ -  $\alpha g^* p$  -irresolute function and strongly  $\beta$ -  $\alpha g^* p$  -irresolute function .

#### Theorem:3.16

If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$ , then g o f:  $X \rightarrow Z$  is

- (i) strongly  $\alpha g^*p$  -irresolute if f is strongly  $\alpha$   $\alpha g^*p$  -irresolute and g is  $\alpha g^*p$  -irresolute.
- (ii) strongly  $\beta$   $\alpha g^*p$  –irresolute if f is strongly  $\alpha g^*p$  irresolute and g is  $\alpha g^*p$  -irresolute.

**Proof:**Let V be an  $\alpha g^*p$  -open set in Z. Since g is  $\alpha g^*p$  -irresolute,  $g^{-1}(V)$  is  $\alpha g^*p$  -open in Y. Since f is strongly  $\alpha$ -  $\alpha g^*p$  -irresolute,  $f^{-1}(g^{-1}(V))$  is  $\alpha$ - open in X.

 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is regular open in X and hence open in X.

Hence (g o f) is strongly  $\alpha g^* p$  -irresolute.

(i) Let V be an  $\alpha g^*p$  -open set in Z. Since g is  $\alpha g^*p$  -irresolute,  $g^{-1}(V)$  is  $\alpha g^*p$  -open in Y. Since f is strongly  $\alpha g^*p$  - irresolute,  $f^{-1}(g^{-1}(V))$  is open in X and hence  $\beta$ -open in X.

 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is  $\beta$ - open in X for every  $\alpha$ g\*p –open set V in Z.

Hence (g o f) is strongly  $\beta$ -  $\alpha$ g\*p -irresolute.

#### Theorem:3.17

If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$ , then g o f:  $X \rightarrow Z$  is

- (i) strongly  $\alpha$   $\alpha g^* p$  -irresolute if f is regular irresolute and g is strongly  $\alpha$   $\alpha g^* p$  -irresolute.
- (ii) strongly  $\alpha$   $\alpha$ g\*p -irresolute if f is  $\alpha$  continuous and g is strongly  $\alpha$ g\*p -irresolute.
- (iii) strongly  $\beta$   $\alpha g^* p$  -irresolute if f is continuous and g is strongly  $\alpha g^* p$  -irresolute.

**Proof:**Let V be a  $\alpha g^*p$ -open set in Z. Since g is strongly  $\alpha$ -  $\alpha g^*p$ -irresolute,  $g^{-1}(V)$  is  $\alpha$ - open in Y. Since f is  $\alpha$ - irresolute,  $f^{-1}(g^{-1}(V))$  is  $\alpha$ - open in X.

 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is  $\alpha$ - open in X.

Hence (g o f) is strongly  $\alpha$ -  $\alpha$ g\*p -irresolute.

(i) Let V be an  $\alpha g^* p$  -open set in Z. Since g is strongly  $\alpha g^* p$  -irresolute,  $g^{-1}(V)$  is open in Y. Since f is  $\alpha$ - continuous,  $f^{-1}(g^{-1}(V))$  is  $\alpha$ -open in X.

 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is  $\alpha$ - open in X.

Hence (g o f) is strongly  $\alpha$ -  $\alpha$ g\*p -irresolute.

(ii) Let V be an  $\alpha g^* p$  -open set in Z. Since g is strongly  $\alpha g^* p$  -irresolute,  $g^{-1}(V)$  is open in Y.

Since	f	is	continuous	,	$f^{-1}(g^{-1}(V))$	is	open	in	Х.
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 $\Rightarrow$  (g o f)<sup>-1</sup>(V) is open in X and hence  $\beta$ -open in X.

Hence (g o f) is strongly  $\beta$ -  $\alpha$ g\*p -irresolute.

# Theorem :3.18

The following are equivalent for a function f:  $X \rightarrow Y$ :

- (i) f is strongly  $\alpha$   $\alpha$ g\*p -irresolute.
- (ii) For each  $x \in X$  and each  $\alpha g^*p$  -open set V of Y containing f(x), there exists a  $\alpha$  open set U in X containing x such that  $f(U) \subset V$ .
- (iii)  $f^{1}(V) \subset Cl(Int (f^{1}(V)))$  for each  $\alpha g^{*}p$  -open set V of Y.
- (iv)  $f^{1}(F)$  is regular closed in X for every  $\alpha g^{*}p$  -closed set F of Y.

**Proof:** Similar to that of Theorem 3.7

# Theorem:3.19

The following are equivalent for a function f:  $X \rightarrow Y$ :

- (i) f is strongly  $\beta$   $\alpha$ g\*p -irresolute.
- (ii) For each  $x \in X$  and each  $\alpha g^*p$  -open set V of Y containing f(x), there exists a  $\beta$  open set U in X containing x such that  $f(U) \subset V$ .
- (iii)  $f^{1}(V) \subset Cl(Int(f^{1}(V)))$  for each  $\alpha g^{*}p$  -open set V of Y.
- (iv)  $f^{1}(F)$  is  $\beta$ -closed in X for every  $\alpha g^{*}p$ -closed set F of Y.

**Proof:** Similar to that of Theorem 3.7.

# Lemma: 3.20

If f:  $X \rightarrow Y$  is strongly  $\alpha$ -  $\alpha g^*p$  -irresolute and A is a  $\alpha$ - open subset of X, then f/A :  $A \rightarrow Y$  is strongly  $\alpha$ -  $\alpha g^*p$  -irresolute.

# **Proof:**

Let V be a  $\alpha g^*p$ -open in Y. By hypothesis,  $f^1(V)$  is  $\alpha$ - open in X. But  $(f/A)^{-1}(V) = A \cap f^1(V)$  is regular open in A. Hence f/A is strongly  $\alpha$ -  $\alpha g^*p$ -irresolute.

# Theorem:3.21

Let f:  $X \rightarrow Y$  and  $\{A_{\lambda}: \lambda \in \Lambda\}$  be a cover of X by  $\alpha$ - open set of  $(X,\tau)$ . Then f is a strongly  $\alpha$ -  $\alpha g^*p$  -irresolute function if  $f/A_{\lambda}$ :  $A_{\lambda} \rightarrow Y$  is strongly  $\alpha$ -  $\alpha g^*p$  -irresolute for each  $\lambda \in \Lambda$ .

**Proof:**Let V be any  $\alpha g^* p$  -open set in Y. By hypothesis,  $(f/A_{\lambda})^{-1}(V)$  is  $\alpha$ - open in  $A_{\lambda}$ . Since  $A_{\lambda}$  is regular open in X, it follows that  $(f/A_{\lambda})^{-1}(V)$  is  $\alpha g^* p$  -open in X for each  $\lambda \in \Lambda$ .

$$\begin{split} f^{1}(V) = & X \cap f^{1}(V) \\ = & \cup \{A_{\lambda} \cap f^{1}(V): \lambda \in \Lambda\} \\ = & \cup \{(f/A_{\lambda})^{-1}(V): \lambda \in \Lambda\} \text{ is regular open in } X. \end{split}$$

Hence f is strongly  $\alpha$ -  $\alpha$ g\*p -irresolute.

# Lemma:3.22

If f: X $\rightarrow$ Y is strongly  $\beta$ -  $\alpha$ g\*p -irresolute and A is a  $\alpha$ -open subset of X, then f/A : A $\rightarrow$ Y is strongly  $\beta$ -  $\alpha$ g\*p -irresolute.

**Proof:**Let V be a  $\alpha g^* p$  -open in Y. By hypothesis,  $f^1(V)$  is  $\beta$ -open in X. But  $(f/A)^{-1}(V) = A \cap f^1(V)$  is  $\beta$ - open in A. Hence f/A is strongly  $\beta$ -  $\alpha g^* p$  -irresolute.

# Theorem:3.23

Let f:  $X \rightarrow Y$  and  $\{A_{\lambda}: \lambda \in \Lambda\}$  be a cover of X by  $\beta$ - open sets of  $(X,\tau)$ . Then f is a strongly  $\beta$ -  $\alpha g^*p$  -irresolute function if  $f/A_{\lambda}$ :  $A_{\lambda} \rightarrow Y$  is strongly  $\beta$ -  $\alpha g^*p$  -irresolute for each  $\lambda \in \Lambda$ .

**Proof:**Let V be any  $\alpha g^* p$  -open set in Y. By hypothesis,  $(f/A_{\lambda})^{-1}(V)$  is  $\beta$ - open in  $A_{\lambda}$ . Since  $A_{\lambda}$  is  $\beta$ - open in X, it follows that  $(f/A_{\lambda})^{-1}(V)$  is  $\beta$ -open in X for each  $\lambda \in \Lambda$ .

 $f^{1}(V) = X \cap f^{1}(V)$ 

 $= \cup \{A_{\lambda} \cap f^{1}(V): \lambda \in \Lambda\}$ 

= $\cup \{ (f/A_{\lambda})^{-1}(V) : \lambda \in \Lambda \}$  is  $\beta$ - open in X. Hence f is strongly  $\beta$ -  $\alpha g^*p$  -irresolute.

# Theorem:3.24

If a function f:  $X \rightarrow Y$  is strongly  $\beta \cdot \alpha g^* p$ -irresolute, then  $f^{-1}(B)$  is  $\beta$ -closed in X for any nowhere dense set B of Y.

**Proof:** Let B be any nowhere dense subset of Y. Then Y–B is regular in Y and hence  $\alpha g^*p$  -open in Y. By hypothesis, f <sup>1</sup>(Y–B) is  $\beta$ -open in X. Hence f<sup>1</sup>(B) is  $\beta$ -closed in X.

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