Hydromagnetic Instability of visco-elastic Walter's (modal B´) nanofluid layer heated from below

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Abstract - **In this paper, Hydromagnetic instability of visco-elastic Walter's (modal B´) nanofluid layer heated from below is numerically and analytically described. For this Perturbation method, Normal mode technique and the dispersion relation has been used.. The effects of the various physical parameters of the system, namely Lewis number, modified diffusivity ratio, nano particle Rayleigh number and magnetic field on the stationary convection have been analyzed both analytically and graphically. The Lewis number, modified diffusivity ratio and nano particle Rayleigh number are found to have destabilizing effect, whereas magnetic field has a stabilizing effect for stationary convection.**

keywords - **Nanofluid; Walter's (modal B´) visco-elastic fluid; Normal mode technique; Oscillatory convection; Rayleigh number; Lewis number; modified diffusivity ratio; magnetic field.**

NOMENCLATURE

a	dimensionless resultant wave number
d	thickness of nanofluid layer
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
ρ	Density of nanofluid
g	acceleration due to gravity
η	Fluid electrical resistivity
n	growth rate of disturbances
k_1	medium permeability
q	velocity vector
R_a	Rayleigh number
R_m	Density Rayleigh number
R_n	Nano particle Rayleigh number
T	Temperature
T_1	reference temperature
t	time
(u, v, w)	Velocity component
(x, y, z)	space co-ordinate
H	magnetic field
L_e	Lewis number
N_A	Modified diffusivity ratio
N_B	Modified particle-density increment
P_{r_1}	Prandtl number
P_{r_2}	Magnetic Prandtl number
p	Hydrostatic pressure

Greek symbols

α	Thermal expansion coefficient
μ	Viscosity
ε	Porosity
μ_e	Magnetic permeability
μ'	Kinematic visco-elasticity
$(\rho_c)_p$	Heat capacity of nanoparticles
$(\rho_c)_f$	Heat capacity of base fluid
φ	volume fraction nanoparticle
ρ_p	density of nanoparticles
ρ_f	density of base fluid
k	Thermal diffusivity
ω	dimensionless frequenccy
Q	Chandrasekhar number

Superscripts

\`	non-dimensionless variables
″	perturbed quantities

Subscripts

p	particle
r	fluid
0	lower boundary
1	upper boundary

I. INTRODUCTION

A colloidal mixture of nano sized particles in base fluid makes nanofluid. Choi [1] was the first person to use the term nanofluid. The thermal instability of nanofluid is enhanced, when a small amount of nano-sized particles are added to the base fluid. Masuda et al. [2] first showed that the thermal conductivity of nanofluids was enhanced due to the presence of nano particles. Eastman et al. [3] investigated that if 0.3% of copper nano particles were added in ethylene glycol would increase 40%. Xuan and Li [4] observed that the suspended nano particles remarkably enhance heat transfer and the Cu-water nanofluid has larger heat transfer coefficient than that of the original base liquid under the same Reynolds number. Due to the thermal conductivity enhancement of the nanofluids, they have a wide range of industrial applications especially in the process where cooling is of primary interest. Nano particles materials may be taken as metal carbides (SiC), oxide ceramics (Al_2O_3, CuO), nitrides (AlN, SiN) or metals (Cu, Al) etc. and base fluids are water, ethylene or tri-ethylene-glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common visco-elastic fluids. The dimension of nano particles is about to 100 nm.Tzou [5] was the first scientist to use the modal of Buongiorno [6] to investigate the instability problems in nanofluid using the method of eigen function expansion. He observed that the regular fluids were more stable than nanofluids. Nield and Kuznetsov [7] discussed the thermal instability problem for the nanofluid layer, and it is found that the stability of the nanofluids depended on the distribution of the nano particles on the boundaries of the layer. The onset of convection in a horizontal layer of nanofluids uniformly heated from below (Bénard convection) based upon Buongiorno's modal under various assumptions have been discussed by Nield and Kuznetsov [8], Kuznetsov and Nield [9], Yadav et al. [10], Chand et al. [11], Chand [12], Chand and Rana [13], and Rana et al. [14]. There are a large number of technological applications in geophysics, food processing, oil reservoir modeling, petroleum industry, bio-mechanics, building of thermal insulations and nuclear reactors of thermal instability in a porous medium. Lapwood [15] had studied the convective flow in a porous medium using linearized stability theory. Wooding [16] had discussed the Rayleigh instability of a thermal boundary layer in the flow through a porous medium. Nield and Kuznetsov [17] investigated thermal instability in a horizontal nanofluid layer in porous medium by Darcy modal. Kuznetsov [18] studied the same by Brinkman modal. The oscillating convection in a Darcy porous medium have discussed by Chand and Rana [19] and found that "Principal of exchange of stabilities" is not valid.The above literature related with the study of nanofluids as Newtonian fluids. But the growing importance of non-Newtonian fluids in technology and industries, the discussion of such types of fluids are desirable. Convection of non-Newtonian fluids has important part in various processes in the chemical and material industries, in the extrusion of polymer fluids, in geophysical fluid dynamics, chemical technology. Bhatia and Steiner [20] have discussed the thermal instability of visco-elastic fluids. There are many visco-elastic fluids which cannot be characterized by Maxwell constitutive relations. One such class of visco-elastic fluids is Walter's (modal B´) fluid. Walter's [21] noted that the mixture of polymethyl methacrylate and pyridine at 25^0 C containing 30.5gm of polymer per liter with density 0.98 gm per liter behaves nearly as the Walter's (modal B´) fluid.The important role of magnetic field, in the applications of geophysics (e.g. enhanced oil recovery from underground reservoirs) gives the motivation to investigate the thermal instability of visco-elastic nanofluids in a magnetic field. Gupta et al. [22] investigated the effects of magnetic fields on the thermal instability of bottom heavy nanofluids, and found that the magnetic field increased the thermal instability of nanofluid layer. Yadav et al. [23] investigated numerically the effect of magnetic field on the onset of nanofluid convection and found that the volumetric fraction of nano particles, the Lewis number, the modified diffusivity and the density ratios have a destabilizing effect, while the magnetic field has stabilizing effect on the system.In the present paper, the hydromagnetic Instability of visco-elastic Walter's (modal B´) nanofluid layer heated from below has been studied.

II. MATHEMATICAL FORMULATIONS

Suppose an infinite horizontal layers of Walter's (modal B´) visco-elastic nanofluid of thickness d bounded by the plane $z = 0$ and $z = d$ and heated from below. Fluid layer is working in upward direction under gravity force g (0, 0, -g). The temperature T and volumetric fraction φ of nano particles at $z = 0$ taken to be T_0 and φ_0 at $z = 0$ and T_1 and φ_1 at$z = d$, $(T_0 > T_1)$. For the analytical formulation the thermophysical properties of the nanofluid are constant and these properties are not constant and depend upon the volume fraction of the nano particles.

The governing equations for visco-elastic nanofluid Walter's (modal B´) under the oberback Boussinesq approximation are:

$$\nabla \boldsymbol{q} = 0 \qquad \ldots(1)$$

$$\rho \frac{d\boldsymbol{q}}{dt} = -\nabla \mathrm{p} + \rho \mathrm{g} + \left(\mu - \mu' \frac{\partial}{\partial \mathrm{t}}\right) \nabla^2 \mathbf{q} + \frac{\mu_e}{4\pi} \left(\mathbf{H}.\nabla\right)\mathbf{H} \qquad \ldots(2)$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + (\boldsymbol{q}.\nabla)$ stands for convection derivative, **q**(u,v,w) is the velocity vector, p is the hydrostatic pressure, μ and μ' are the viscosity and kinematic visco-elasticity respectively and g(0, 0, -g) is acceleration due to gravity, μ_e is the fluid magnetic permeability and H is the magnetic field. The density ρ of nanofluid can be written as

$$\rho = \varphi\, \rho_p + (1-\varphi)\rho_f \qquad \ldots(3)$$

where φ is the volume fraction of nano particles, ρ_p and ρ_f are the densities of nano particles and base fluid.

The equation of motion for visco-elastic Walter's (modal B´) nanofluid is given as:

$$\rho \frac{d\boldsymbol{q}}{dt} = -\nabla \mathrm{p} + \left(\varphi\, \rho_p + (1-\varphi)\{\rho\,(1 - \alpha(T - T_0))\}\right)\mathrm{g} + \left(\mu - \mu' \frac{\partial}{\partial \mathrm{t}}\right) \nabla^2 \mathbf{q} + \frac{\mu_e}{4\pi} \left(\mathbf{H}.\nabla\right)\mathbf{H} \qquad \ldots(4)$$

where α is the coefficient of thermal expansion and μ_e is the fluid magnetic permeability.

The continuity equation for the nano particles is

$$\frac{\partial \varphi}{\partial t} + \boldsymbol{q}\, \nabla \varphi = D_B\, \nabla^2 \varphi + \frac{D_T}{T_1} \nabla^2 T \qquad \ldots(5)$$

where D_B is the Brownian diffusion coefficient and D_T is the Thermoporetic diffusion coefficient of the nano particles.

The energy equation in nanofluid is

$$\rho_c \left(\frac{\partial T}{\partial t} + \boldsymbol{q}\, \nabla T\right) = k \nabla^2 \mathrm{T} + (\rho_c)_\mathrm{p} \left(D_B \nabla \varphi . \nabla T + \frac{D_T}{T_1}\, \nabla T . \nabla T\right) \qquad \ldots(6)$$

Where ρ_c is the heat capacity of fluid, $(\rho_c)_p$ is the heat capacity of nano particles and k is the thermal conductivity.

The Maxwell equation being

$$\frac{\partial \boldsymbol{H}}{\partial t} + (\boldsymbol{q}\, \nabla)\boldsymbol{H} = (\boldsymbol{H}\, \nabla)\boldsymbol{q} + \eta\, \nabla^2 \mathbf{H} \qquad \ldots(7)$$

$$\nabla \mathbf{H} = 0 \qquad \ldots(8)$$

Where η is the fluid electrical resistivity.

Introducing non-dimensional variables as:

$$(x', y', z') = \left(\frac{x,\ y,\ z}{d}\right), q'(u', v', w') = q\left(\frac{u,\ v,\ w}{k}\right) d\,,\ t' = \frac{tk}{d^2}\,,\ p' = \frac{p}{\rho k^2}\, d^2\,,\ \varphi' = \frac{\varphi - \varphi_0}{\varphi_1 - \varphi_0}\,,\ \mathrm{T}' = \frac{T - T_0}{T_0 - T_1}\,,$$

where $\frac{k}{\rho_c}$ = k is the thermal diffusivity of the fluid. Equations (1), (4), (5), (6), (7) and (8), in non dimensional form can be written as:

$$\nabla \boldsymbol{q} = 0 \quad \dots(9)$$

$$\frac{1}{p_{r_1}}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla \mathrm{p} + (1-\mathrm{nF})\nabla^2 \mathbf{q} - \mathrm{R_m}\hat{\mathrm{e}}_\mathrm{z} - \mathrm{R_n}\varphi\hat{\mathrm{e}}_\mathrm{z} - \mathrm{R_a}\mathrm{T}\hat{\mathrm{e}}_\mathrm{z} + \mathrm{Q}\,\frac{p_{r_1}}{p_{r_2}}\,(\boldsymbol{H}.\nabla)\boldsymbol{H} \quad \dots(10)$$

$$\frac{\partial \varphi}{\partial t} + \boldsymbol{q}\,\nabla\varphi = \frac{1}{L_e}\,\nabla^2\,\varphi + \frac{N_A}{L_e}\nabla^2\,T \quad \dots(11)$$

$$\frac{\partial T}{\partial t} + \boldsymbol{q}\,\nabla T = \nabla^2\mathrm{T} + \frac{\mathrm{N_B}}{L_e}\nabla\varphi.\nabla T + \frac{N_A N_B}{L_e}\,\nabla T\,.\nabla T \quad \dots(12)$$

$$\frac{\partial H}{\partial t} + (\boldsymbol{q}\,\nabla)\boldsymbol{H} = (\boldsymbol{H}\,\nabla)\boldsymbol{q} + \frac{p_{r_1}}{p_{r_2}}\,\nabla^2\boldsymbol{H} \quad \dots(13)$$

$$\nabla \boldsymbol{H} = 0 \quad \dots(14)$$

[The dashes (`) have been dropped for simplicity]

Here non-dimensional parameters are:

Lewis number $L_e = \frac{k}{D_B}$, Prandtl number $p_{r_1} = \frac{\mu}{\rho k}$, Magnetic Prandtl number $p_{r_2} = \frac{\mu}{\rho\eta}$, Rayleigh number $\mathrm{R_a} = \frac{\rho g\alpha d^3}{\mu k}(\mathrm{T_0} - \mathrm{T_1})$,Basic- density Rayleigh number $\mathrm{R_m} = \frac{[\rho_p\varphi_0 + \rho\,(1-\varphi_0)]g\,d^3}{\mu k}$, Nano particle Rayleigh number $\mathrm{R_n} = \frac{(\rho_p - \rho)(\varphi_1 - \varphi_0)g\,d^3}{\mu k}$, Kinematic visco-elasticity parameter $\mathrm{F} = \frac{\mu'}{\rho d^2}$, Modified diffusivity ratio $N_A = \frac{D_T}{D_B \mathrm{T_1}(\varphi_1 - \varphi_0)}(\mathrm{T_0} - \mathrm{T_1})$, Modified particle density increment $N_B = \frac{(\rho_c)_p\,(\varphi_1 - \varphi_0)}{(\rho_c)_f}$, Chandrasekhar number $\mathrm{Q} = \frac{\mu_e H_0^2\, d^2}{4\pi\nu\rho\eta}$

We assume that temperature and volumetric fraction of nano particles are constant on boundaries. Thus the dimensionless boundaries conditions are

$$w = 0,\, T = 1,\; \varphi = 0\;\; at\;\; z = 0 \quad \dots(15)$$

$$\text{and}\quad w = 0,\, T = 0,\; \varphi = 1\;\; at\;\; z = 1 \quad \dots(16)$$

2.1 Basic states and its solutions

The basic state of nanofluid is assumed to be time independent and is described by

$q'(u,v,w) = 0,\; p' = p(z),\; T' = T_b(z),\; \varphi' = \varphi_b(z),\; \boldsymbol{H} = (0,0,1)$

The subscript b represents the primary variable.

Equations (9) to (12) using boundary conditions (15) and (16) give solution as:

$$T_b = 1 - z \;\text{ and }\; \varphi_b = z \quad \dots(17)$$

2.2 Perturbation solutions

To study the stability of the system, let us introduced small perturbations to primary flow, and write

$q'(u,v,w) = 0 + q''\,(u,v,w), T' = T_b + T'', \varphi' = \varphi_b + \varphi'', p' = p_b + p''$,with $T_b = 1 - z$ and $\varphi_b = z$ …(18)

Using equation (18) in equation (9) to (14) and linearise by neglecting the product of the prime quantities, we obtain the following equations:

$$\nabla \boldsymbol{q} = 0 \quad \dots(19)$$

$$\frac{1}{p_{r_1}}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla \mathrm{p} + (1-\mathrm{nF})\nabla^2\mathbf{q} - \mathrm{R_n}\varphi\hat{\mathrm{e}}_\mathrm{z} + \mathrm{R_a}\mathrm{T}\hat{\mathrm{e}}_\mathrm{z} + \mathrm{Q}\,\frac{p_{r_1}}{p_{r_2}}\frac{\partial \mathbf{H}}{\partial \mathrm{z}}\hat{\mathrm{e}}_\mathrm{z} \quad \dots(20)$$

$$\frac{\partial\varphi}{\partial t} + w = \frac{1}{L_e}\,\nabla^2\,\varphi + \frac{N_A}{L_e}\nabla^2\,T \quad \dots(21)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2\mathrm{T} + \frac{\mathrm{N_B}}{L_e}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{z}} - \frac{\partial\varphi}{\partial \mathrm{z}}\right) - 2\,\frac{N_A N_B}{L_e}\,\frac{\partial \mathrm{T}}{\partial \mathrm{z}} \quad \dots(22)$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = \frac{\partial w}{\partial z}\hat{\mathrm{e}}_\mathrm{z} + \frac{p_{r_1}}{p_{r_2}}\,\nabla^2\boldsymbol{H} \quad \dots(23)$$

$$\nabla \mathbf{H} = 0 \quad \dots(24)$$

The dashes (´´) have been dropped for simplicity.

Boundary conditions are:

$$w = 0,\, T = 0,\; \varphi = 0\;\; at\;\; z = 0 \text{ and } w = 0,\, T = 0,\; \varphi = 0\;\; at\;\; z = 1 \quad \dots(25)$$

Since R_m is just a measure of basic static pressure gradient so it is not involved in these and subsequent equations. Now by operating Eq. (20) with $\hat{\mathrm{e}}_\mathrm{z}$.curl curl, we get:

$$\frac{1}{p_{r_1}}\frac{\partial}{\partial t}\nabla^2 w - (1-\mathrm{nF})\nabla^4 w = \mathrm{R_a}\nabla_\mathrm{H}^2\mathrm{T} - \mathrm{R_n}\nabla_\mathrm{H}^2\varphi - \mathrm{Q}\,\frac{\partial^2 w}{\partial z^2} \quad \dots(26)$$

where $\nabla_\mathrm{H}^2 = \frac{\partial^2}{\partial \mathrm{x}^2} + \frac{\partial^2}{\partial \mathrm{y}^2}$ is the two dimensional Laplacian operator on horizontal plane.

III. NORMAL MODES ANALYSIS

The disturbances analyzing in to normal modes and assuming that the perturbed quantities are of the form:

$$[w\,,\,\mathrm{T}\,,\,\varphi\,] = [W(z), T(z), \varphi(z)]\,\exp\,(\,ik_x x + ik_y y + nt) \quad \dots(27)$$

Where k_x and k_y are wave numbers in x and y directions respectively, while n is growth rate of disturbances.

Using eq. (27), eq.(21),(22), and (26) become:

$$W - \frac{N_A}{L_e}(\,D^2 - a^2)T - \left[\frac{1}{L_e}(\,D^2 - a^2) - n\right]\varphi = 0 \quad \dots(28)$$

$$W + \left[(D^2 - a^2) - n + \frac{N_B}{L_e} D - \frac{2N_A N_B}{L_e} D\right] T - \frac{N_B}{L_e} D\varphi = 0 \qquad \ldots(29)$$

$$\left[(D^2 - \ a^2) \frac{n}{p_{r_1}} - (1 - nF)(D^2 - a^2)^2 + QD^2\right] W + a^2 R_a\, T - a^2 R_n \varphi = 0 \qquad \ldots(30)$$

Where $D = \frac{d}{dz}$ and $a = \sqrt{{k_x}^2 + {k_y}^2}$ is the dimensionless the resultant wave number. The boundary conditions of the problem in view of normal mode are written as

$$W = 0, D^2W = 0,\ T = 0, \varphi = 0 \text{ at } z = 0 \text{ and } W = 0, D^2W = 0,\ T = 0,\ \varphi = 0 \text{ at } z = 1 \qquad \ldots(31)$$

IV. METHOD OF SOLUTION

An approximate solution of the system of equations (28)-(30) with the boundary conditions given in eq. (31) is obtained by the Galerkin weighted residuals method. In this method the test functions are the same as the base functions. Accordingly W, T, φ are taken as:

$$W = \sum_{p=1}^{N} A_p\, W_p\ ,\ \ T = \sum_{p=1}^{N} B_p\, T_{p,}\ \varphi = \sum_{p=1}^{N} c_p\, \varphi_p \qquad \ldots(32)$$

where $A_{p,}B_p$ and C_p are unknown coefficients, $p = 1,2,3, \ldots N$ and the base functions W_p ,T_p, and φ_p satisfying the boundary conditions given in Eq.(31). Using expression for W, T and φ in equations (28)-(30) and multiplying the first equation by W_p the second equation by T_p and third equation by φ_p and then integrating in the limits from 0 to 1, we obtain a set of 3N unknown A_p , B_p and C_p; $p = 1,2,3 \ldots. N$.For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number R_a.

V. LINEAR STABILITY ANALYSIS

The function corresponding to the eigen value problem considering the solution

$$W = W_0 \sin \pi z\ ,\ T = T_0 \sin \pi z\ ,\ \varphi = \varphi_0 \sin \pi z \qquad \ldots(33)$$

satisfying boundary conditions given in eq.(31). Substituting solution given in equation (33) in equations (28)-(30), we obtain the Eigen equations as:

$$R_a = \frac{1}{a^2}\left[\left\{(1 - nF)J + \frac{n}{p_{r_1}}\right\} J + Q(J - a^2)\right](J + n) - \frac{\left\{(J\ +\ n) + \frac{N_a}{L_e}\ J\right\}}{\frac{1}{L_e}\ J + n} R_n \qquad \ldots(34)$$

where $J = \pi^2 +\ a^2$

For neutral stability, the real part of n is zero. Hence, on putting $n = i\ \omega$,(ω is the real and dimensionless frequency of oscillation) in eq.(34), we get:

$$R_a =\ \Delta_1 + i\ \omega\ \Delta_2 \qquad \ldots(35)$$

where

$$\Delta_1 = \frac{J}{a^2}\left[J^2 + Q(J - a^2) - \frac{\omega^2}{p_{r_1}} + \omega^2 FJ\right] - \frac{1}{\left\{\left(\frac{J}{L_e}\right)^2 + \omega^2\right\}}\left[\frac{J^2}{{L_e}^2}(L_e + N_a) + \omega^2\right] R_n \qquad \ldots(36)$$

and imaginary part

$$\Delta_2 = \ \frac{1}{a^2}\left[\left\{1 - JF + \frac{1}{p_{r_1}}\right\} J^2 + Q(J - a^2)\right] - \frac{\left[\frac{J}{L_e} - J\left(1 + \frac{N_A}{L_e}\right)\right]}{\left\{\left(\frac{J}{L_e}\right)^2 + \omega^2\right\}} R_n \qquad \ldots(37)$$

R_a will be real since it is a physical quantity Hence, it follow from Eq.(35) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset).

5.1 Stationary Convection

When the stability sets in as stationary convection, the marginal state will be characterized by $\omega = 0$. the Eq.(35) reduces as:

$$(R_a)_s = \frac{(\pi^2 + a^2)}{a^2}\ [(\pi^2 +\ a^2) + \pi^2 Q\ ] - (L_e +\ N_A)R_n \qquad \ldots(38)$$

Here, it is worthwhile mentoning that the expression for R_a is independent of both the prandtl numbers, and the parameters containing the Brownian effects and the thermophoretic effects and presented in the thermal energy equation and the conversation equation for nano particles.

Take $x = \frac{a^2}{\pi^2}$ in Eq. (38), then we have

$$R_a = \pi^2\left[\frac{(1+x)^2}{x} + \frac{Q(1+x)}{x}\right] - (L_e +\ N_A)R_n \qquad \ldots(39)$$

Now

$$\frac{dR_a}{dx} = \pi^2\left[\frac{(-1 + x^2)}{x^2} - \frac{Q}{x^2}\right]$$

The Thermal Rayleigh number R_a given by Eq. (39) takes its minimum value when $x^2 = Q + 1$.

Therefore the critical wave number x shows a substantial increase when the Chandrasekher number Q increases and it is independent of nano particles.

From eq. (39), it is cleared that for the stationary convection, the kinematic visco-elastic parameter F vanishes with n and hence, visco-elastic fluid behaves like an ordinary Newtonian fluid.

From eq. (39), it is cleared that stationary Rayleigh number R_a depends upon dimensionless wave number a , Lewis number, modified diffusivity ratio N_A, and nano particles Rayleigh number R_n , but it is independent of modified particle density increment N_B , Prandtl number P_r and density Rayleigh number R_m.

To study the effects of Lewis number L_e, modified diffusivity ratio N_A, and nano particles Rayleigh number R_n , and magnetic field on stationary convection. We examine the nature of

$\frac{\partial R_a}{\partial L_e}, \frac{\partial R_a}{\partial N_A}, \frac{\partial R_a}{\partial R_n}, \frac{\partial R_a}{\partial Q}$ analytically.

From eq. (39)

$\frac{\partial R_a}{\partial L_e} < 0$, $\frac{\partial R_a}{\partial N_A} < 0$, $\frac{\partial R_a}{\partial R_n} < 0$ and $\frac{\partial R_a}{\partial Q} > 0$

It implies that for stationary convection Lewis number, modified diffusivity ratio, and nano particle Rayleigh number have destabilizing effect whenever magnetic field has stabilizing effect on the fluid layer.

5.2 Oscillatory Convection

For oscillatory convection ($\omega \neq 0$), we must have $\Delta_2 = 0$, Eq.(37) gives

$$\omega^2 = \frac{a^2(\pi^2+a^2)\ \left[\frac{1}{L_e} - \left(1+\frac{N_A}{L_e}\right)\right]}{\left[\left\{1-(\pi^2+a^2)F+\frac{1}{p_{r_1}}\right\}(\pi^2+a^2)^2+\pi^2 Q\right]} R_n - \frac{(\pi^2+a^2)^2}{{L_e}^2} \qquad \ldots(40)$$

Eq.(40) gives the frequency of oscillatory mode, for the value of parameters considered in the range of $10^2 \leq R_a \leq 10^5$, $R_n > 0$, $10^2 \leq R_n \leq 10^6$. We get negative value of ω^2.Thus oscillatory convection is not possible.

VI. SOME IMPORTANT THEOREM

From Eq. (34), we have a cubic equation in n, such that

$$n^3\left\{\frac{JL_e}{a^2}\left(\frac{1}{p_{r_1}} - JF\right)\right\} + n^2\left[L_e\{J^2+Q(J-a^2)\} + \frac{J^2}{a^2}(L_e+1)\left(\frac{1}{p_{r_1}} - JF\right)\right] + n\left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\} - (R_n + L_eR_a)\right] + \left[J^2\{J^2+Q(J-a^2)\} + J\left\{R_n\left(\frac{N_A}{L_e} - 1\right) + R_a\right\}\right] = 0 \qquad \ldots(41)$$

Theorem-1- The system is stable under the condition $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_eR_a) < \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$.

Proof- If $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_eR_a) < \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$, then equation (41) has not any change in sign and so does not allow any positive root. Then the system is stable.

Theorem-2- The system is unstable under the condition $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_eR_a) > \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$.

Proof- If $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_eR_a) > \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$

Then the coefficient of n in the equation (41) is negative, therefore, allows one change of sign and so has at most one positive root. The occurrence of a positive root implies that the system is unstable.

Theorem-3- The system is stable under the effect of magnetic field.

Proof- There is no any change in the sign of equation (41) due to the magnetic field since the term $\{Q(J-a^2)+J^2\} > 0$. So the equation (41) does not allow any positive root, then the system is stable.

Theorem-4- The sufficient conditions for non-existence oscillatory

convection are $R_n < 0$, $1 > (N_A + L_e)$ and $JF < \left(1 + \frac{1}{p_{r_1}}\right)$

Proof- from equation (40), ω^2 is given as:

$$\omega^2 = \frac{a^2 J\ \left[\frac{1}{L_e} - \left(1+\frac{N_A}{L_e}\right)\right]}{\left[\left\{1-JF+\frac{1}{p_{r_1}}\right\}J^2+\pi^2 Q\right]} R_n - \frac{J^2}{{L_e}^2}$$

For $R_n < 0$, $1 > (N_A + L_e)$ and $JF < \left(1 + \frac{1}{p_{r_1}}\right)$; ω^2 is negative, whenever for the existence of oscillatory convection ω^2 must be positive. Then the above conditions are sufficient for the non-existence of oscillatory convection.

VII. RESULTS AND DISCUSSION

Hydromagnetic Instability of visco-elastic Walter's (modal B´) nanofluid layer heated from below is investigated under realistic boundary conditions.

Figure 1 represents the variation of stationary Rayleigh number with Lewis number L_e for different values of R_n . The stationary Rayleigh number R_a is plotted against Lewis number for fixed values of $N_A = 5$, $Q = 100$ and $R_n = 10, 20, 30$. The Rayleigh number decreases with increases in Lewis number, which shows that Lewis number has destabilizing effect on the stationary convection.

Figure 2 represents the variation of stationary Rayleigh number with Lewis number L_e for different values of R_n. The stationary Rayleigh number R_a is plotted against Lewis number for fixed values of $R_n = 10$, $Q = 100$, and $N_A = 5, 10, 15$. The Rayleigh number decreases with increases in Lewis number, which shows that Lewis number has destabilizing effect on the stationary convection.

Figure 3 represents the variation of stationary Rayleigh number with modified diffusivity ratio N_A for different values of nano particle Rayleigh number R_n. The stationary Rayleigh number R_a is plotted against N_A for fixed values of $L_e = 5$, $Q = 100$ and $R_n = 10, 20, 30$. The Rayleigh number decreases with increases in modified diffusivity ratio N_A, which shows that modified diffusivity ratio N_A has destabilizing effect on the stationary convection.

Figure 4 represents the variation of stationary Rayleigh number with modified diffusivity ratio N_A for different values of Lewis number. The stationary Rayleigh number R_a is plotted against N_A for fixed values of R_n = 10, $Q = 100$ and $L_e = 5, 10, 15$. The Rayleigh number decreases with increases in modified diffusivity ratio N_A, which shows that modified diffusivity ratio N_A has destabilizing effect on the stationary convection.
Figure 5 represents the variation of stationary Rayleigh number with nano particle Rayleigh number R_n for different values of modified diffusivity ratio N_A. The stationary Rayleigh number R_a is plotted against R_n for fixed values of $L_e = 5$, $Q = 100$ and $N_A = 10, 20, 30$. The Rayleigh number decreases with increases in nano particle Rayleigh number R_n, which shows that nano particle Rayleigh number R_n has destabilizing effect on the stationary convection.
Figure 6 represents the variation of stationary Rayleigh number with nano particle Rayleigh number R_n for different values of Lewis number. The stationary Rayleigh number R_a is plotted against R_n for fixed values of $N_A = 10$, $Q = 100$ and $L_e = 5, 10, 15$. The Rayleigh number decreases with increases in nano particle Rayleigh number R_n, which shows that nano particle Rayleigh number R_n has destabilizing effect on the stationary convection.
Figure 7 represents the variation of stationary Rayleigh number with magnetic field Q for different values of modified diffusivity ratio N_A . The stationary Rayleigh number R_a is plotted against Q for fixed values of $R_n = 1$, $L_e = 10$ and $N_A = 5, 10, 15$. The Rayleigh number increases with increases in Q which shows that magnetic field has stabilizing effect on the stationary convection.
Figure 8 represents the variation of stationary Rayleigh number with magnetic field Q for different values of nano particle Rayleigh number R_n . The stationary Rayleigh number R_a is plotted against Q for fixed values of $N_A = 5$, $L_e = 10$ and $R_n = 1, 5, 10$. The Rayleigh number increases with increases in Q which shows that magnetic field has stabilizing effect on the stationary convection.

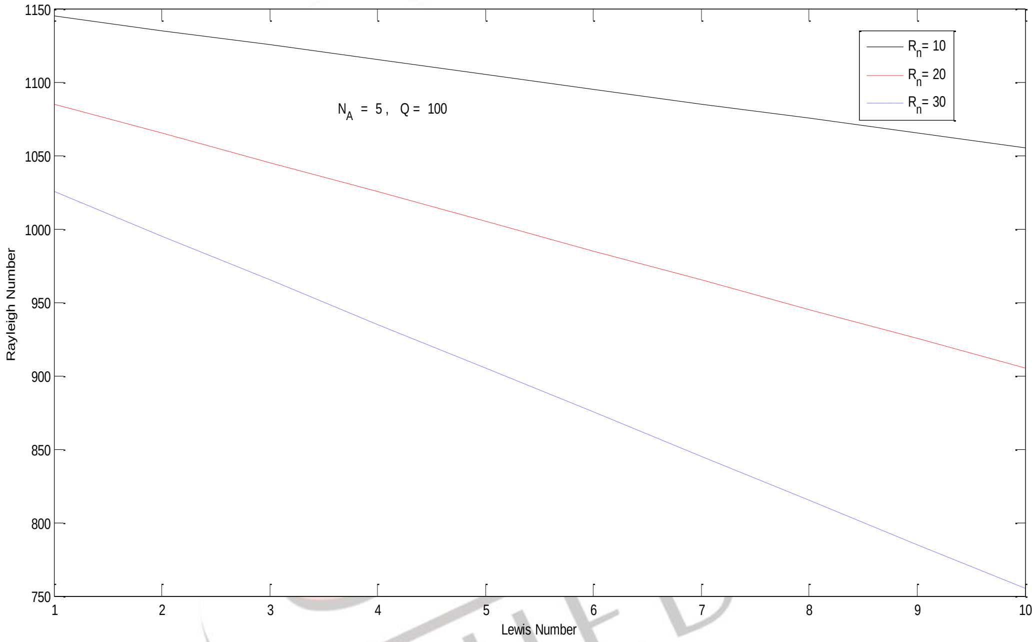

Fig.1: Variations of stationary Rayleigh number with Lewis number

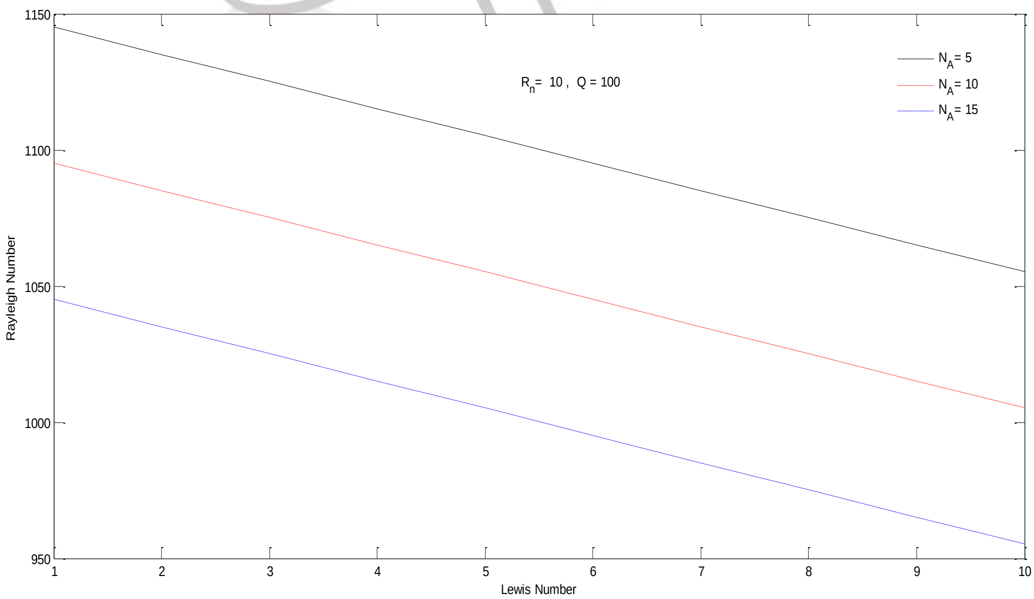

Fig.2: Variations of stationary Rayleigh number with Lewis number

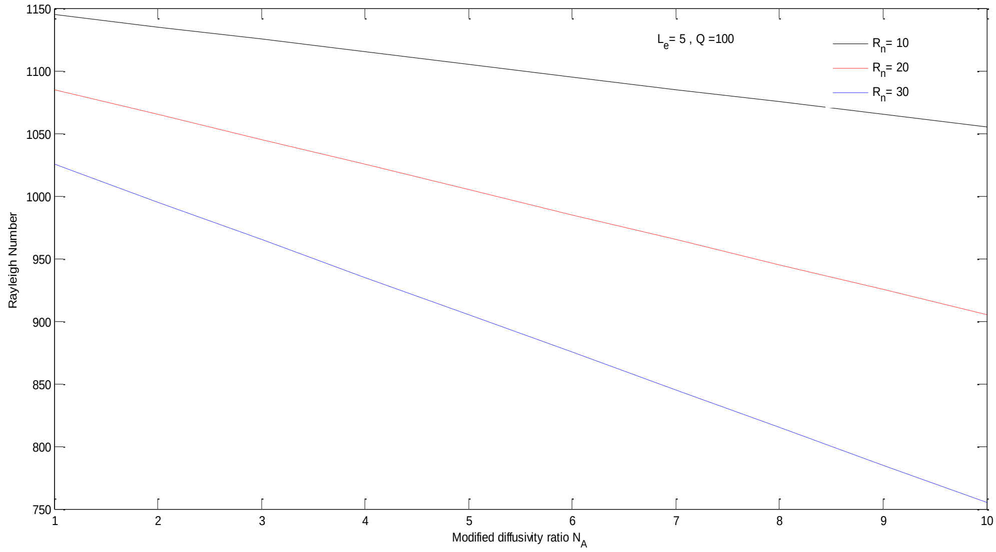

Fig.3:Variations of stationary Rayleigh number with Modified diffusivity ratio N_A

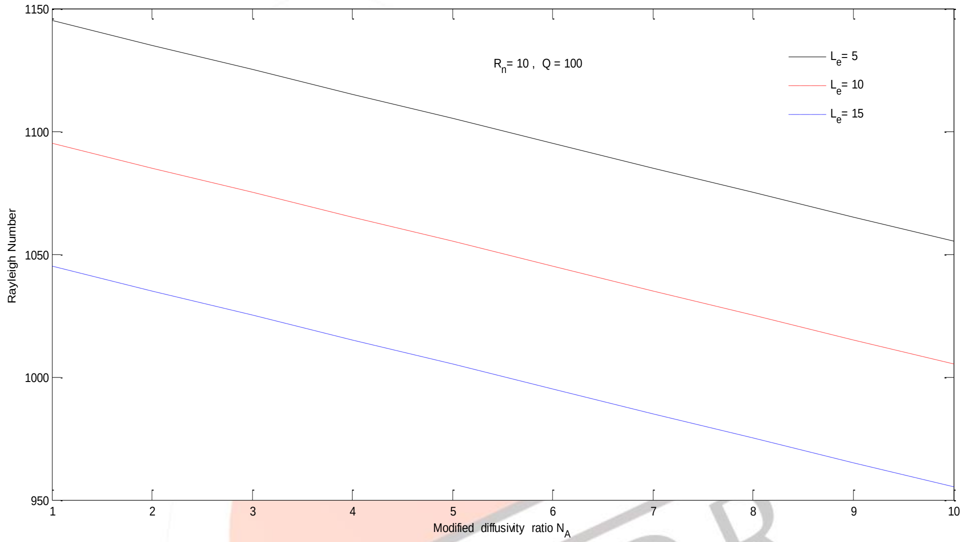

Fig.4:Variations of stationary Rayleigh number with Modified diffusivity ratio N_A

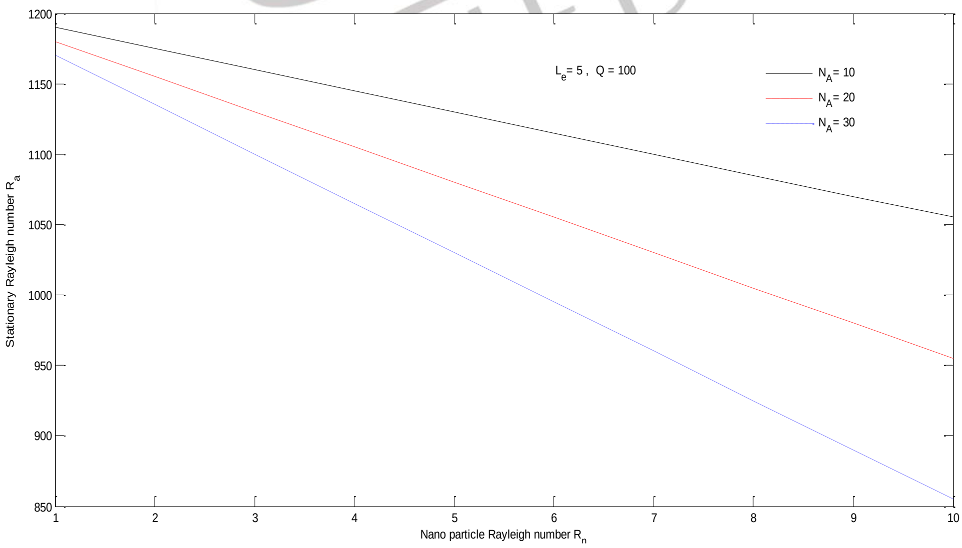

Fig.5: Variations of stationary Rayleigh number with Nano particle Rayleigh number R_n

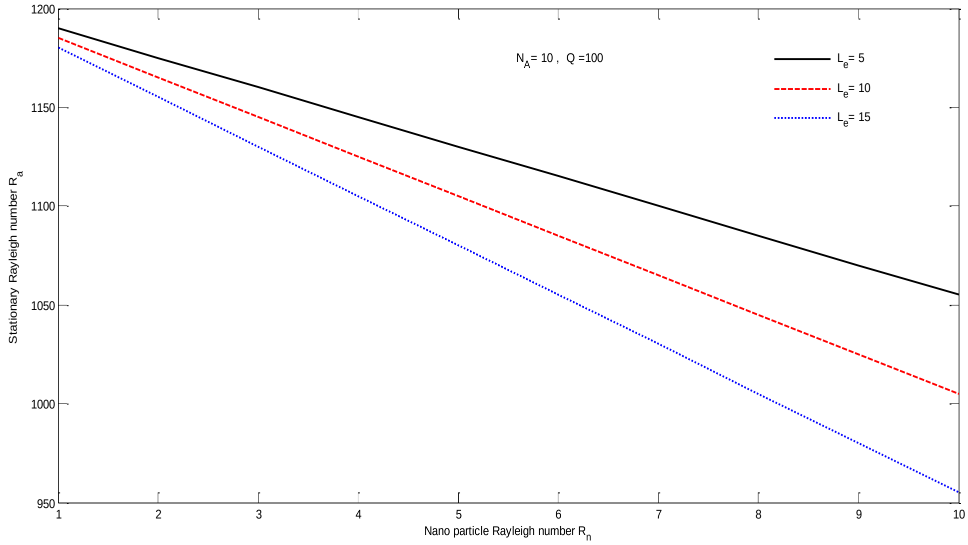

Fig.6: Variations of stationary Rayleigh number with Nano particle Rayleigh number R_n

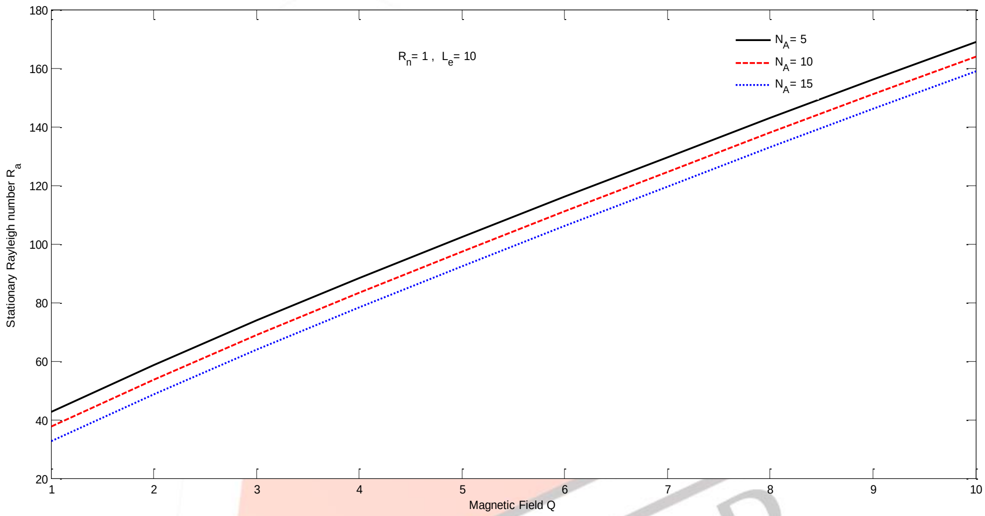

Fig.7: Variations of stationary Rayleigh number with Magnetic field Q

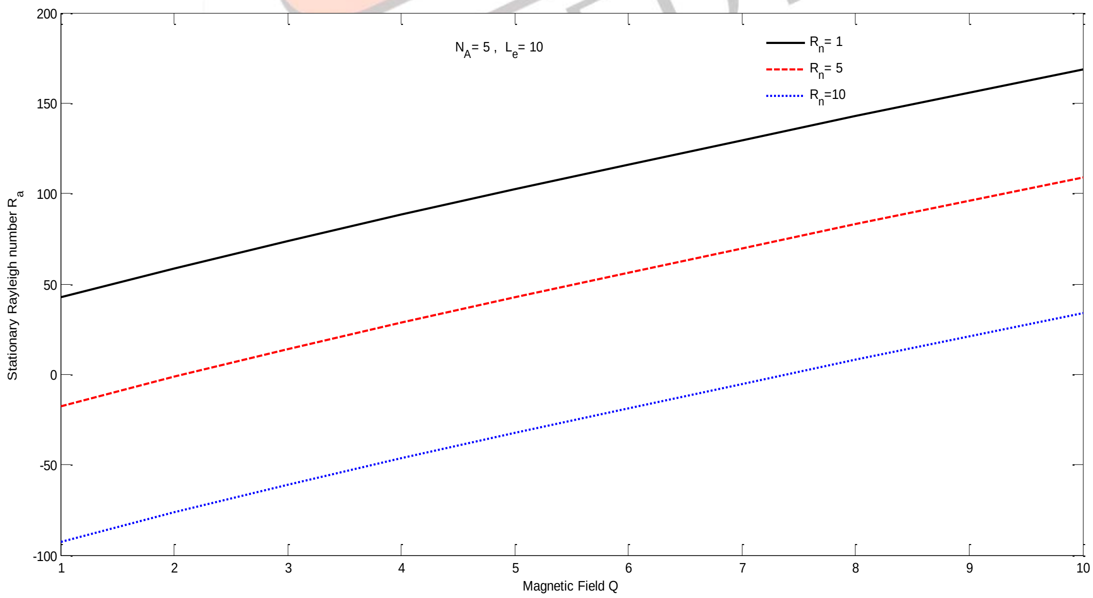

Fig.8:Variations of stationary Rayleigh number with Magnetic field Q

CONCLUSIONS

Hydromagnetic Instability of visco-elastic Walter's (modal B´) nanofluid layer heated from below is investigated by using linear instability analysis. The main conclusions from the analysis of this paper are as follows:

(1) For the stationary convection magnetic field has stabilizing effect on the system.
(2) Lewis number, modified diffusivity ratio and nano particle Rayleigh number have destabilizing effect on the stationary convection.
(3) For the stationary convection, the visco-elastic nanofluid behaves like an ordinary fluid.
(4) The system is stable for the condition $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_e R_a) < \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$.
(5) The system is unstable for the condition $\frac{1}{p_{r_1}} > JF$ and $(R_n + L_e R_a) > \left[\frac{J^3}{a^2}\left(\frac{1}{p_{r_1}} - JF\right) + \frac{J(L_e+1)}{a^2}\{Q(J-a^2)+J^2\}\right]$ and $\frac{N_A}{L_e} > 1$.
(6) The sufficient conditions for the non-existence oscillatory convection are $R_n < 0, 1 > (N_A + L_e)$ and $JF < \left(1 + \frac{1}{p_{r_1}}\right)$
(7) The system is stable under the effect of magnetic field if $\{Q(J-a^2)+J^2\} > 0$.

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