Mathematical Analysis on the Physiological Effects of Water Pollution

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Abstract - **Mathematical modeling on water pollution with a reference to industrial effluents at various levels has been studied in this paper. The quantification of discharge parameters due to various pollutants and various time intervals is approximated using concentration rate equation with respect to the oxygen transfer at saturation level and concentration level. Study consists of the proposed compositions in the form of visible and invisible chemical and biological industrial discharges. Analytical formulations are established for concentration levels present in the effluents (C_(L-1),C_(L-2),C_(L-2)). Formulations are assumed to be simultaneous linear differential equations for three levels of concentrations of pollutants due to effluents. These differential equations are solved using numerical method to explore the details of three types of compositions of concentration of water pollutants. Results indicate the presence of levels of pollutants which are unsafe for vegetation, agricultural purpose and drinking water requirements. The physiological effects due to this unsafe usage will include causing cholera, hemolysis, impairment in the central nervous system, diarrhea, loose teeth sponge gums.**

keywords - **water, oxygen, effluents, pollutants, industrial.**

I. INTRODUCTION

Water is a natural resource which is very much needed for a multiplicity of purposes. Water is used for drinking and other domestic uses, cooling agent in industries, power generation, irrigation and waste disposal. In the chemical industry water is used as a medium of reaction and a heat transfer agent. Water is a main resource of life for man, plants and other forms of life it cannot be replaced. In this chapter we discussed the origin of waste water, the classification of water pollutants and their effects. Rain is the main source of India. However, the time- distribution of precipitation is quite uneven. The annual growth from rainfall is found to be about 350 million hectare metres (MHM) and out of this, 60 MHM of water evaporates, 100 MHM runs off into surface water bodies and the remaining enters into soil. Based on annual rain fall records and evaporate coefficients, the total annual flow through rivers in India is found to be around 140 MHM. In practice, most of this water flows back to the sea or is through evaporation. A part percolates into the ground. 50 MHM of water is used for irrigation and power generation. From ground water resource data, water obtained from precipitation and stored in aquifers after percolating through intake areas, is limited than those on the surface water resources. It is found that 67 MHM is the net annual recharge in India and in this 67 MHM only 35 MHM may be obtained to be available for utilization. Combining all the above data with surface water, it gives the total utilizable potential of clean water to be around 100 MHM. There are two potential sources of water for human use: (i) desalinated sea- water and (ii) re use waste water.

Discharge of domestic and industrial waste into aquatic systems is a major problem of water pollution. Nearly 60 to 70% population is exposed to polluted drinking water. The rivers and lakes near city centers emit bad odors and fish are being killed along sea coasts. The meat of these dead fishes is tainted and not safe to eat because of maximum of mercury and other chemicals in their bodies. Mercury (Hg) toxicity and its salts are industrial hazards. Excess of Hg (> 100 mg) may leads headache, stomach pain, diarrhea, difficulty in breathing, problem in central nervous system, memory loss, loose teeth spongy gums, troubles in kidney. Pollutants of Water can be classified as oxygen demanding wastes, plant nutrients, disease - causing agents and radioactive substances, thermal discharges and oil.

In the past few years, analytical and experimental studies have been carried out to analyze the concentration of oxygen in polluted water. Masataka Watanabe et. al. [1] compared between numerical observed results for a laboratory test model are presented. W. C. Walton [2] illustrated a model for aquifer test analysis and simple aquifer system evaluation. Jacques C. J. Nihoul [3] proposed a model taking into account man-controlled modifications. Christine Doughty et. al. [4] studied a practical problem on underground heat and fluid flow.D. Adair [5] performed calculations by using a Reynolds-averaged Navier-Stokes code.C. P. Maule et. al. [6] determined the seasonal contributions of rain and snow to soil water. M. Gascoyneet. al. [7] estimated and map the relative hydraulic conductivities.P. Nithiarasu [8] formulated a leaking heat source problem and solved it for different Darcy numbers. Weimei Jiang et. al. [9] employed to simulate sea-land breeze circulation. T. Metzger et. al. [10] estimated thermal dispersion coefficients from minimally intrusive measurements.Apurba Kumar Santraet. al. [11] studied the Effect of copper–water nanofluid as a cooling medium.ATM. M. Rahmanet. al. [12] studied the effects of thermophoresis on an unsteady.S. Bruschiet. al.[13] deal with the testing and modeling of the different phenomena.Estebanezet. al.[14] enabled the analysis of some of the main parameters involved in the design of these devices, thus helping in their optimization. H Chen et al [15] proposed novel kernel is applicable for the quantitative determination of water pollution and is a prospective solution to other problemsin the field of water resource management.

In the present study, the concentration of stagnant polluted water and flowing polluted water have been carried out using analytical and numerical methods.

II. FORMULATION

Consider the difference between the dissolved oxygen(DO) level of saturation(C_S) and the actual concentration (C_L) present in the water. Oxygen is less soluble in water. The mass transfer rate of oxygen can be expressed as the 'mass flux' across unit area in unit time, M_F, so that,

$$M_F = K_{LP}(C_S - C_L) \tag{1}$$

Where K_{LP} is the liquid- phase mass transfer coefficient.

The oxygen transfer per rate of unit volume of water is given by,

$$\frac{dC_L}{dt} = \frac{NA}{V} = K_{LP}\frac{A}{V}(C_S - C_L) \tag{2}$$

Here C_L is referred to as the various possible levels of actual concentration of oxygen with reference to dissolved oxygen then we have to study minimum of two levels as stagnant polluted water $C_{L(SPW)}$ and flowing polluted water $C_{L(FPW)}$ discharged from industries, then

$$\frac{dC_{L(SPW)}}{dt} = K_1\left(C_{s(SPW)} - C_{L(SPW)}\right) \tag{3}$$

$$\frac{dC_{L(FPW)}}{dt} = K_1\left(C_{s(FPW)} - C_{L(FPW)}\right) \tag{4}$$

$$\text{Where } K_1 = K_{LP}\frac{A}{V} \tag{5}$$

On solving equation (3),

$$C_{L(SPW)}(t) = C_{s(SPW)} + c.\,e^{K_1 t} \tag{6}$$

Where c is constant of integration to be obtained by taking the initial values of $C_{L(SPW)}$, $C_{s(SPW)}$,K_1at initial time.

Similarly we get the solution for equation (4) as,

$$C_{L(FPW)}(t) = C_{s(FPW)} + c.\,e^{K_1 t} \tag{7}$$

Where $\frac{dC_L}{dt}$ is the oxygen transfer rate, $r = K_{LP}\frac{A}{V}$ is known as the reaeration rate constant or reoxygenationrate constant.

$$\frac{dC_L}{dt} = r(C_S - C_L) \tag{8}$$

In equation (2), $(C_S - C_L)$ is the difference of the saturation concentration of dissolved oxygen(C_S) and the concentration actually present in water (C_L). Referred to as the oxygen deficit (D).

$$D = (C_S - C_L) \tag{9}$$

If an organic matter is introduced into water and its decomposition is observed, it is observed that the rate of oxidation of organic matter (rate of declination of BOD) can be approximated as a first order chemical reaction whose kinetics may be expressed as,

$$\frac{dL}{dt} = -K_1 L \tag{10}$$

$$\frac{dL}{L} = -K_1 dt \tag{11}$$

Solving equation (11) as,

$$\log(L) = -K_1 t + C_1 \tag{12}$$

taking $t = 0$, and $L = L_u$

$$L = L_u\left(10^{-K'_1 t}\right) \tag{13}$$

If Y represents the amount of BOD utilized at any time, t, then

$$L_u = L + Y \tag{14}$$

$$Y = L_u\left(1 - 10^{-K'_1 t}\right) \tag{15}$$

III. ANALYSIS

Using exponential series for $10^{-k'_1 t}$ we get,

$$Y = L_u\left[\frac{k'_1 t}{1} - \frac{(k'_1 t)^2}{2} + \frac{(k'_1 t)^3}{6} - \frac{(k'_1 t)^4}{24} + - + \; - \ldots\right] \tag{16}$$

Where Y amount of BOD at any time t is, L_u is ultimate BOD, K'_1 is temperature dependent deoxygenation constant, k_1 is deoxygenation constant.

The value of K'_1 changes significantly with the type of waste and the temperature.

Consider,

$$\left(1 - 10^{-K'_1 t}\right) = 2.3K'_1 t\left[1 + \left(\frac{2.3}{6}\right)K'_1 t\right]^{-3} \tag{17}$$

The series expansion gives,

$$\left(1 - 10^{-K'_1 t}\right) = 2.3K'_1 t\left[1 - \frac{1}{2}(2.3K'_1 t) + \frac{1}{6}(2.3K'_1 t)^2 - \frac{1}{24}(2.3K'_1 t)^3 + \ldots\right] \tag{18}$$

$$2.3K_1^{'}t\left[1+\left(\frac{2.3}{6}\right)K_1^{'}t\right]^{-3} = 2.3K_1^{'}t\left[1-\frac{1}{2}(2.3K_1^{'}t)+\frac{1}{6}(2.3K_1^{'}t)^2-\frac{1}{21.6}(2.3K_1^{'}t)^3+\dots\right] \qquad (19)$$

Hence equation (15) can be written as,

$$Y = L_u 2.3K_1^{'}t\left[1+\left(\frac{2.3}{6}\right)K_1^{'}t\right]^{-3} \qquad (20)$$

$K_1^{'}$ and L_u are obtained by,

$$K_1^{'} = 2.61\ \frac{b}{a} \qquad (21)$$

$$L_u = \frac{1}{2.3K_1^{'}a^3} \qquad (22)$$

Several observations of Y as a function of time are required for a meaningful determination of $K_1^{'}$ and L_u. The observation data are, however, limited to the first 10 days because of probable interference from nitrogeneous matter.

IV. RESULTS AND DISCUSSION

In the present study, concentration of stagnant polluted water goes on decreasing with the increase in time also the concentration of flowing polluted water decreases as the time period increases. This shows that water gets polluted when the oxygen present in water reacts with the chemicals in pollutants. Therefore the concentration goes on decreasing as the flow continues. This causes a serious damage on health of human beings.

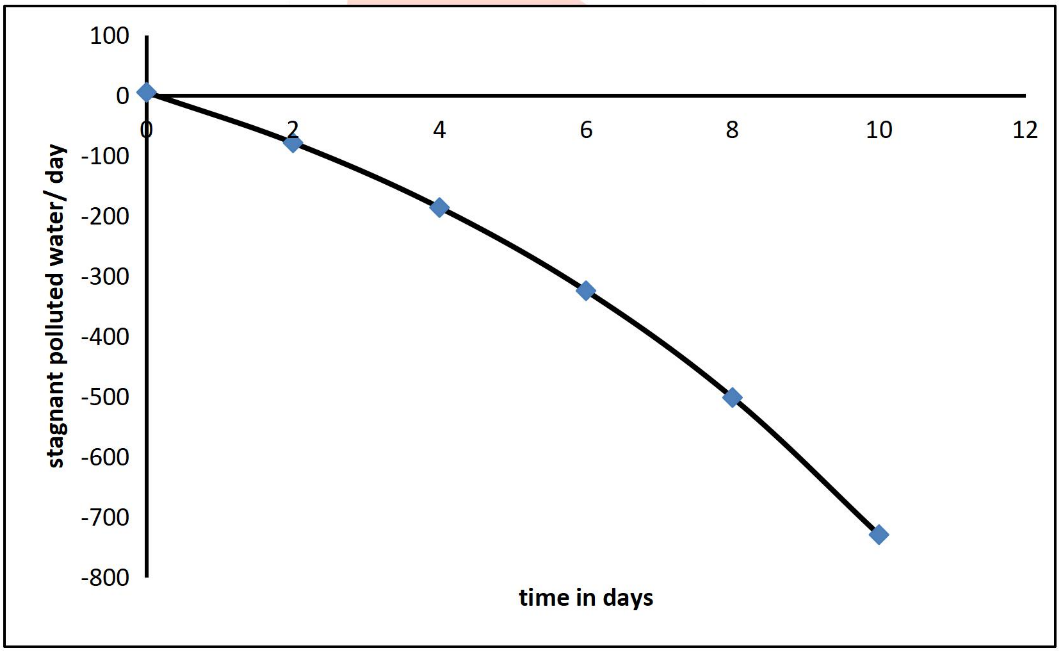

Fig 1: stagnant polluted water v/s time

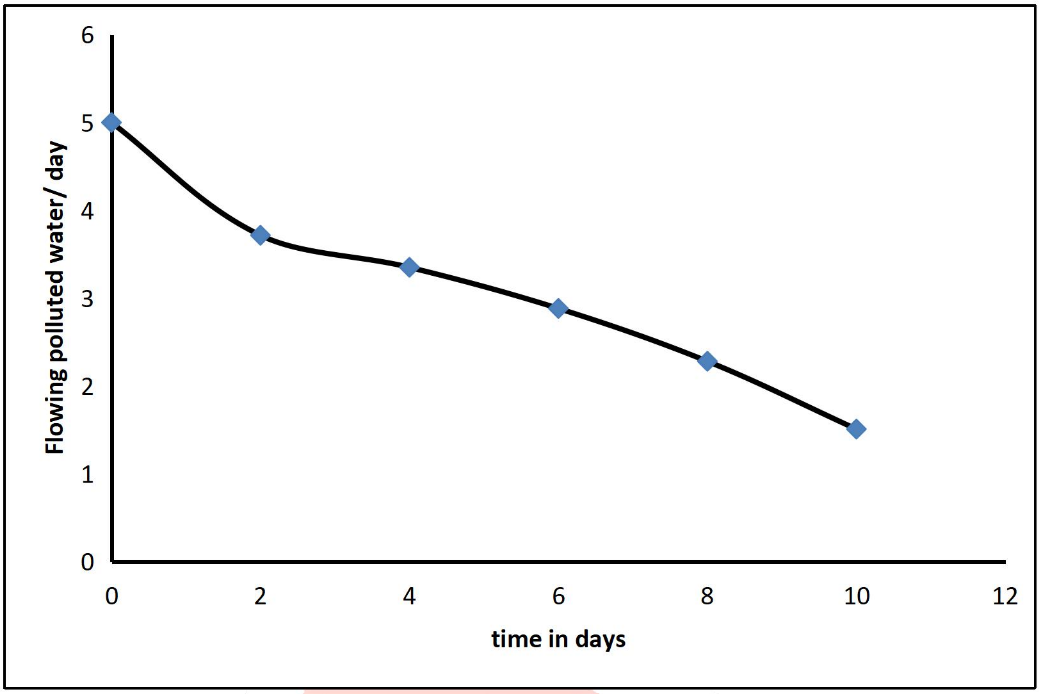

Fig 2: Flowing polluted water v/s time

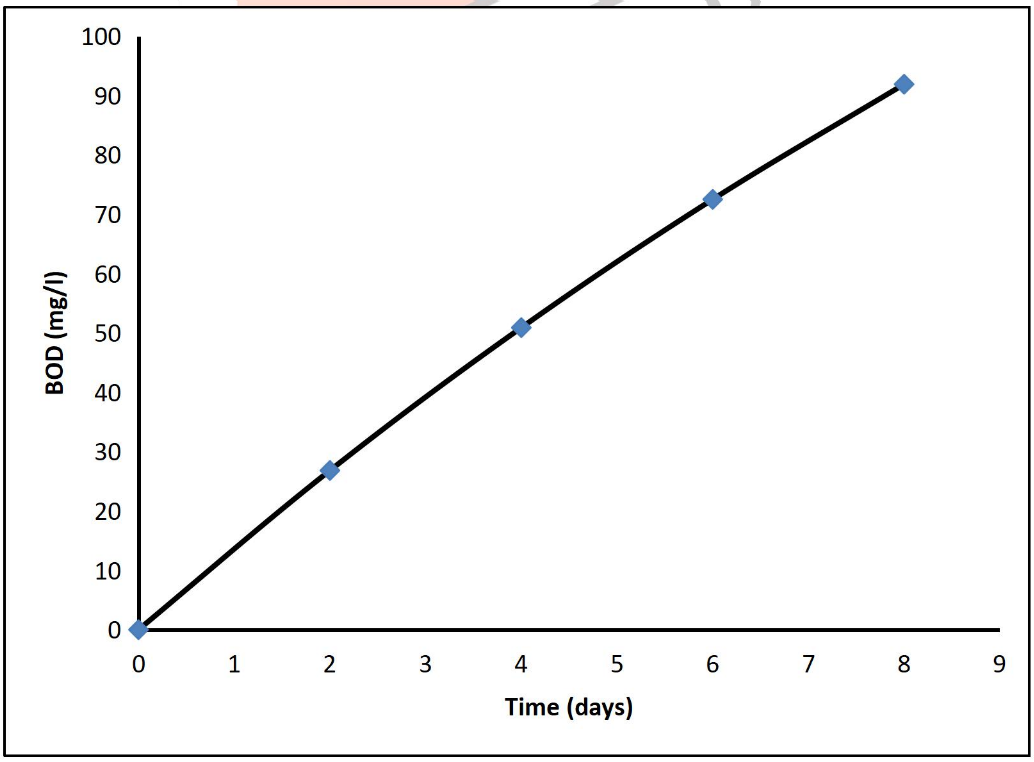

Fig 3: Biochemical Oxygen Demand v/s time

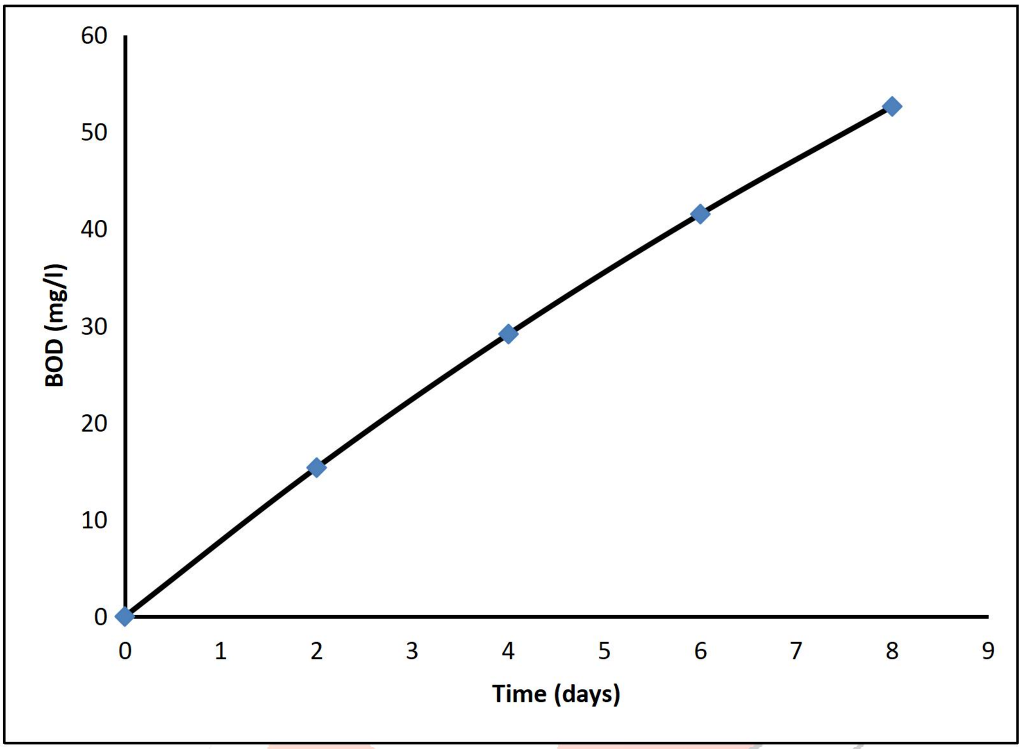

Fig 4: Biochemical Oxygen Demand v/s time

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