Fuzzy structure of max-product and application

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***Abstract* - In this paper, the max product of potent fuzzy structure , max product of connected fuzzy graph structure, regular fuzzy graph structure, degree and total degree, application of max product and it is extend to fuzzy soft bi partite application.**

***keywords* - Fuzzy structure, max-product, soft bi-partite.**

1.INTRODUCTION

Fuzzy set which is a superset of crisp set is the starting work of Zadeh. Zadeh found many real life and other application in the field of telecommunication, discrete mathematics, networking, chemical industry, decision making, computer science. Rosenfeld initiated fuzzy subgroup by using fuzzy subset concept. A relation between vertices V and edge E is the mathematical representation of a graph. Graph theory represents real life application but sometimes it is failed. This failure leads to Fuzzy graph. Kaffman initiated fuzzy graph using Zadeh's concept. The quickly growing Fuzzy graph has many application in the field like planning and scheduling, communication, image capturing, clustering, networking, data mining and household things.

2.PRILIMINARIES

1. A Structure with non-empty set and relation on non-empty set which are disjoint is irreflexive and symmetric. The Structure is called Fuzzy Structure.

The Structure H=(X, S), p=1,2,…….n
X=non-empty set
S =relation on non-empty set

2. Let a fuzzy set σ be on fuzzy set X and fuzzy set μ , p=1,2, …..n be on fuzzy set S respectively. H is called fuzzy structure H=(σ, μ) if σ(a)∧σ(b)>0 a, b∈ X

3. Let H1 and H2 be fuzzy structures.

H1= (σ1, μ ′), p=1,2,……n
H2= (σ2, μ ″), p=1,2,……n

Consider Crisp structure,

H*1=(X1, S ′), j=1,2,…….n
H*2=(X2, S ″), j=1,2,……n
S = (a1,b1)(a2,b2) a1=a2 then b1,b2∈S ″ if b1=b2 then a1,a2∈S ′

We define, σ=σ1×σ2

σ(a, b)=σ1(a)∨σ2(b), a, b ∈ X, X=X1×X2
μ =μ ′*μ ″

The product, H1*H2= (σ, μ), p=1,2……n is called Max-product of fuzzy structure and the product H*1×H*2=(X, S), where X=X1*X2 is the crisp structure

μ ((a1,b1)(a2,b2))=σ1(a1)∨μ ″(b1b2) a1=a2, b1,b2 ∈S ″
σ2(b1)∨μ ′(a1a2) b1=b2, a1,a2∈S ′

4. Fuzzy structure μ is potent fuzzy structure if μ (b1,b2)=σ(b1)∧σ(b2) b1,b2∈S
p=1,2,…..n H is potent fuzzy structure if H is μ potent fuzzy.

5. The degree of vertex in Max-product H1*H2 of two fuzzy structure

H1=(σ1, μ ′) where p=1.,2,….n
H2=(σ2, μ ″) where p=1,2,….n

Then,

Degree of H1*H2(a , b_i)= $\sum_{a\ a\ ..S\ ',b_i=b}$ μ ′(a a)∨σ2(b_i)+$\sum_{b_ib\ ..S\ '',a\ =a}$ μ ″(b_ib)∨σ1(a)

THEOREM 1:

Max-product of two potent fuzzy structure is a potent fuzzy structure.

PROOF

Let H1 = (σ1,μ ′) where p=1,2,…...n

H2 = (σ2,μ ″) where p=1,2,……n be potent fuzzy structure.
Then μ ′(b1b2)=σ1(b1)∧σ1(b2) where b1,b2∈S ′ p=1,2,….n
μ ″(a1a2)=σ2(a1)∧σ2(a2) where a1,a2∈S ″ p=1,2,….n

By proceeding,
Case 1: a1=a2 where b1,b2∈S ″
μ ((a1,b1)(a2,b2))=σ1(a1)∨μ ″(b1b2)
= σ1(a1)∨[σ2(b1)∧σ2(b2)]
= [σ1(a1)∨σ2(b1)]∧[σ1(a1)∨σ2(b2)]
=[σ(a1,b1)∧σ(a2,b2)]
Case 2: b1=b2 where a1,a2∈S ′
μ ((a1,b1)(a2,b2))=σ2(b1)∨μ ′(a1a2)
=σ2(b1)∨[σ1(a1)∧σ1(a2)]
=[σ1(a1)∨σ2(b1)]∧[σ1(a2)∨σ2(b1)]
=[σ(a1,b1)∧σ(a2,b2)]
Thus, μ ((a1,b1)(a2,b2))=σ(a1,b1)∧σ(a2,b2)
Hence, H=H1*H2 is potent fuzzy structure.

THEOREM 2:

Max-product of two connected fuzzy structure is a connected fuzzy structure.

PROOF

Let H1=(σ1,μ ′) where p=1,2,…n
H2=(σ2,μ ″) where p=1,2,…n be connected
And crisp structure
H*1=(X1,S ′) where p=1,2,…n
H*2=(X2,S ″) where p=1,2,…n
Let X1=(a1,a2,….a) and X2=(b1,b2,…b)
then 0<μ ′∞(a a_i) where a , a_i∈X1
0<μ ″∞(b b_i) where b , b_i∈X2
The Max-product of H1 and H2 is H=(σ, μ).
Consider M subgroups H of vertex set {a , b_i}where i=1,2,….n. Each subgroup H connected. Since a 's are the same and H2 is connected, each b_i is adjacent to X2. Also H1 is connected,
a is adjacent to X2. Therefore, there exist one edge between any pair of above 'M' subgroup.
Thus,
0<μ ∞((a b_i)(a b) where (a b_i)(a b)∈S
Hence, H is connected fuzzy structure.

THEOREM 3:

If H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, σ1<=μ ″ where p=1,2,….n , then degree of any vertex in Max-product H1*H2 is
Degree of H1*H2(a , b_i)=degree H1*(a)σ2(b_i)+degree H2(b_i)

PROOF

Let H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, σ1≤μ ″ then degree of any vertex in Max-product of H1*H2 is
Degree of H1*H2(a , b_i) = $\sum_{a\ a\ ..S\ ',b_i=b}$ μ $'(a\ a\ )\vee \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ )\vee\sigma 1(a\ )$
$=\sum_{a\ a\ ..S\ ',b_i=b} \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ )$
=Degree of H1*(a)σ2(b_i)+Degree of H2(b_i)

THEOREM 4:

If H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, σ1≤μ ″ where p=1,2,….n , and σ2 is constant of value c, then degree of any vertex in Max-product H1*H2 is
Degree of H1*H2(a , b_i)=degree H1*(a)c+ degree H2(b_i)

PROOF

Let H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, σ1≤μ ″ and σ2 is constant then degree of any vertex in Max-product of H1*H2 is

Degree of H1*H2(a , b_i) = $\sum_{a\ a\ ..S\ ',b_i=b}$ μ $'(a\ a\ )\vee \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ )\vee\sigma 1(a\ )$
$=\sum_{a\ a\ ..S\ ',b_i=b} \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ )$
= degree H1*(a)c+ degree H2(b_i)

THEOREM 5:

If H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, μ ″≤σ1 and μ ′≤σ2 where p=1,2,….n , then degree of any vertex in Max-product H1*H2 is

Degree of H1*H2(a , b_i)=degree H1*(a)σ2(b_i)+degree H2*(b_i)σ1(a)

PROOF

Let H1=(σ1, μ ′) and H2=(σ2, μ ″) are two fuzzy structure, μ ″≤σ1 and μ ′≤σ2 where p=1,2,….n , then degree of any vertex in Max-product H1*H2 is

Degree H1*H2(a , b_i)= $\sum_{a\ a\ ..S\ ',b_i=b}$ μ $'(a\ \ a\ \ )\vee\sigma 2(b_i)+\sum_{b_ib\ \ ..S\ '',a\ \ =a}$ μ $''(b_ib\ \ )\vee\sigma 1(a\ \ )$

$=\sum_{a\ a\ ..S\ ',b_i=b}\ \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}\ \sigma 1(a_i)$

=Degree of H1*(a)σ2(b_i)+Degree of H2*(b_i)σ1(a)

THEOREM 6:

If H1 and H2 are fuzzy structure , μ ″≥σ1 where p=1,2,…n then the total degree is given by,

Total Degree of H1*H2(a , b_i)=degree H1*(a)σ2(b_i)+total degree H2(b_i).

PROOF

Let H1 and H2 are fuzzy structure , μ ″≥σ1 then σ ′≤σ2 where p=1,2,…n then the total degree is given by,

Degree of H1*H2(a , b_i)

$=\sum_{a\ a\ ..S\ ',b_i=b}$ μ $'(a\ \ a\ \ )\vee\sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ \ )\vee\sigma 1(a\ \ )$+σ(a , b_i)

$=\sum_{a\ a\ ..S\ ',b_i=b}\ \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ \ )+[\sigma 1(a\ \ )\vee\sigma 2(b\triangleleft)]$

=degree of H1*(a)σ2(b_i)+[degree of H2(b_i)+ $\sigma 2(b_i)$]

= degree of H1*(a)σ2(b_i)+total degree of H2(b_j)

THEOREM 7

If H1 and H2 are fuzzy structure , μ ″≥σ1 where p=1,2,…n and σ2 is constant then the total degree is given by,

Total degree of H1*H2(a , b_i)=total degree H2(b_i)+ degree H1*(a)c

PROOF

Let H1 and H2 are fuzzy structure , μ ″≥σ1 where p=1,2,…n and σ2 is constant and also shows that σ2≥σ1 and μ ′≤σ2 then the total degree is,

Degree of H1*H2(a , b_i)

$=\sum_{a\ a\ ..S\ ',b_i=b}$ μ $'(a\ \ a\ \ )\vee\sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ \ )\vee\sigma 1(a\ \ )$+σ(a , b_i)

$=\sum_{a\ a\ ..S\ ',b_i=b}\ \sigma 2(b_i)+\sum_{b_ib\ ..S\ '',a\ =a}$ μ $''(b_ib\ \ )+[\sigma 1(a\ \ )\vee\sigma 2(b\triangleleft)]$

=degree of H2(b_i)+degree of H1(a)c+ $\sigma 2(b_i)$

= total degree of H2(b_j)+degree of H1*(a)c

APPLICATION

The difference between the employees in an organization: In an organization there may be different employees specialized in different fields. All though they are in the same organization they have problems. There are also good relationship between some employees but not all employees. There are issues like higher posting, experience, salary. There may be many problems but they can be solved in some point expect some issues. We can use fuzzy structure to identify the most issues between the employees. Consider the set G={A,B,C,D}. Let σ be fuzzy set on G

Table 1

Employee	Degree of membership
A	0.9
B	0.7
C	0.8
D	0.6

Now we are going to see the membership values between pair of employees.

Table 2

Issues	(A,B)	(A,D)
Salary	0.6	0.7
Higher posting	0.8	0.3
Experience	0.2	0.5

Table 3

Issues	(B,A)	(B,C)
Salary	0.1	0.5
Higher posting	0.2	0.8
Experience	0.4	0.5

Table 4

Issues	(C,B)	(C,D)
Salary	0.6	0.7
Higher posting	0.3	0.4
Experience	0.2	0.1

Table 5

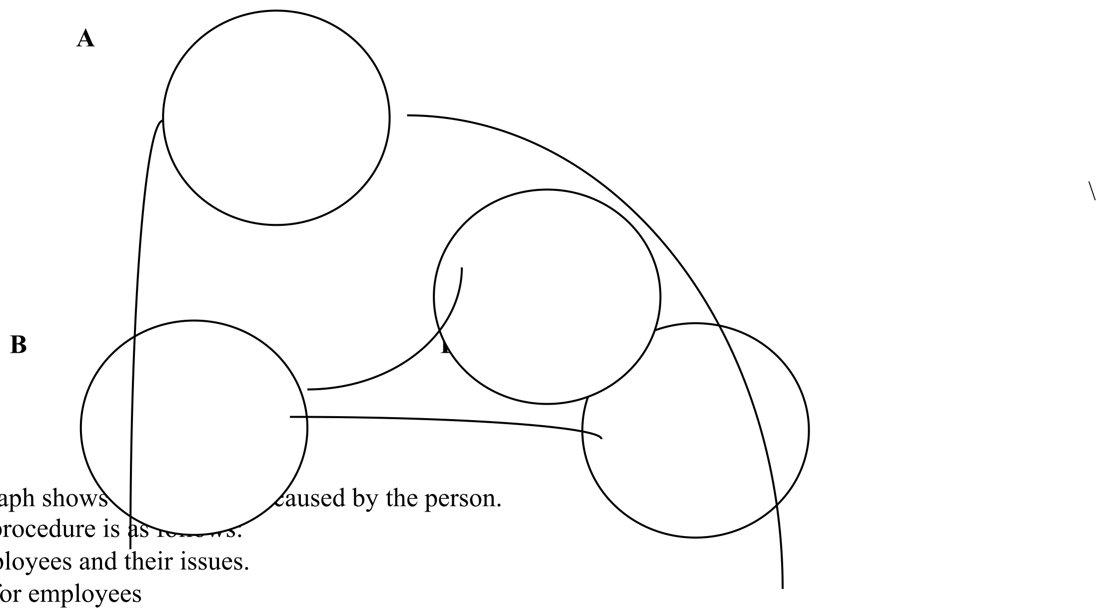
Issues	(D,A)	(D,B)
Salary	0.9	0.2
Higher posting	0.2	0.3
Experience	0.1	0.8

On set G many relations can be defined. Let S1= salary, S2= Higher posting, S3=Experience.
Let

S1={(A,B)(D,A)}
S2={(C,B)(A,D)}
S3={(B,C)(D,B)}

μ1,μ2,μ3 be fuzzy set

μ1={((A,B),0.6), ((D,A),0.9)}
μ2={((C,B),0.3), ((A,D),0.3)}
μ3={((B,C),0.5), ((D,B),0.8)}



The above graph shows caused by the person.
The general procedure is as follows:

- Input the employees and their issues.
- Define a set for employees
- Write the membership values of each employees.
- Find the issues by pairing every employees.
- Construct a fuzzy graph structure.

FUZZY SOFT BI-PARTITE GRAPH
DEFINITION:

A fuzzy soft G (A,V)=((A, ρ)(A, μ)) is said to be soft bi partite. If the vertex V is partitioned into two disjoint vertex $\mu_e(x_i,y_\triangleleft)=\rho_e(x_i)V(y_\triangleleft)$ for all $x_i \in v_i$ and $y_\triangleleft \in v_\triangleleft$.

If fuzzy soft is said to be soft bi partite then the size of fuzzy soft is given by

$S(G(A, v_i \cup v_{\lhd}))=\sum_{e\in A}\sum_{x_i y_{\lhd}\in v_i \cup v_{\lhd}} \mu_e(x_i, y_{\lhd})$

Example:

Consider a fuzzy soft graph G(A,V) where $V=v_i \cup v_j=\{s1,s2,s3,t1,t2,t3\}$ *and* $E=\{e1,e2,e3\}$*. Here G(A,V) described by table and* $\mu_e(s_i,t_{\lhd})=0$ *for all*
$(s_i, t_{\lhd})\in v_i \times v_{\lhd} \setminus \{(s1,t1)(s1,t2)(s2,t1)(s2,t2)(s3,t3)(s3,t2)(s3,t3)$ *for all* $e\in E$

Table represents soft bi-partite graph

ρ	s1	s2	s3	t1	t2	t3
e1	0.2	0.6	0	0.7	0.3	0
e2	0.8	0.9	1.0	0	0.8	0.6
e3	0	0.4	0.9	0.6	0.3	0.8

M	s1,t1	s1,t2	s2,t1	s2,t2	s3,t3	s3.t2	s3,t3
e1	0.8	0.2	0.7	0	0	0	0
e2	0	0.5	0	0.7	0.5	0.4	0
e3	0	0	0.6	0.4	0	0.6	0.2

Fuzzy soft graph size is

$S(e1)=\sum_{e\in A}\sum_{x_i y_{\lhd}\in v_i \cup v_{\lhd}} \mu_e(x_i, y_{\lhd})$
$=0.8+0.2+0.7=1.7$

$S(e2))=\sum_{e\in A}\sum_{x_i y_{\lhd}\in v_i \cup v_{\lhd}} \mu_e(x_i, y_{\lhd})$
$=0.5+0.7+0.5+0.4$
$=2.1$

$S(e3)=\sum_{e\in A}\sum_{x_i y_{\lhd}\in v_i \cup v_{\lhd}} \mu_e(x_i, y_{\lhd})$
$=0.6+0.4+0.6+0.2$
$=1.8$

$S(G(A, v_i \cup v_{\lhd}))=\sum_{e\in A}\sum_{x_i y_{\lhd}\in v_i \cup v_{\lhd}} \mu_e(x_i, y_{\lhd})$
$=1.7+2.1+1.8$
$=5.6$

The degree of vertices (s1)=1.0
The degree of vertices (s2)=1.9
The degree of vertices (s3)=1.9
The degree of vertices (t1)=1.3
The degree of vertices (t2)=1.4
The degree of vertices (t3)=1.4

CONCLUSION

In this we have discussed about the max-product. When the max-product of two fuzzy structure is potent then the max-product is also potent. Application of max-product in an organization is discussed. This is extended up to fuzzy soft structure application. This can be further extended up to fuzzy rough structure, soft fuzzy structure, rough fuzzy structure.

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