The Piston motion in a Free-Piston driver for Shock Tubes & Tunnels

1M Pramod, 2M Bindu Bhargavi, 3Dr. Maruthu Pandiyan
1Student, 2Student, 3Associate Professor
Institute of aeronautical engineering

Abstract - **Hypersonic heating and Viscous heating are the major problems observed in the hypersonic vehicles such as re-entry vehicles or hypersonic missiles. This heating is due to the viscous effect of fluid that is the friction between the air and surface of the vehicle, the surface temperature of the vehicle may exceed 11000K and at those temperatures the survival of material and the avionics inside is not possible, the heat transfer rates or the heat flux is above 6MW/m2. So the thermal protection system is very mandatory for the re-entry vehicles and the hypersonic missiles in order to operate in safe conditions. These TPS system reduces the heat flux and results in lower temperature on the main frame of the vehicle. In order to design a TPS system, we should have the surrounding conditions, such as Density, Pressure, Viscosity etc., So to create those atmospheric conditions in the lab we use few devices such as Hypersonic wind tunnels and Shock Tubes etc., There are many types in HWT based upon speeds and enthalpy, few of them are Blow-down Hypersonic Wind Tunnel, Combustion Driven Shock Tube, Free-Piston Driven Shock Tunnel etc., These hypersonic devices can produce the exact atmospheric conditions can reach up to the speeds of 46 km/s and temperature can go beyond 25000K. The Free Piston shock tunnel is a hypersonic wind tunnel, in which a piston is used to accelerate the air in the driver section and then allowed to ruptures the diaphragm, the sudden expansion of air produces shockwave with a velocity higher than Mach 15 with high enthalpy. The current project deals with the mathematical simulation of the piston motion from its acceleration to brakes, the equations obtained are the coupled nonlinear differential equations. Computational solvers like MATLAB or Python is used to find the solutions of the equations and corresponding plots.**

keywords - **Shock tube, heat flux, Thermal Protection System, Piston Motion.**

I. Introduction

The supersonic vehicles are the future of commercial transportation and for the national security, the first flight vehicle to break the sound barrier was the Bell XS-1 on October 14, 1947, this supersonic journey directed the humanity to go beyond Mach 6.7 (North American X-15) and the Re-entry vehicles made us to go beyond Mach 36. So in aerospace we give more important to the safety either the safety of the crew or the safety of the Air Vehicle, and we never take risks in the air. Here comes the need of the simulating the real atmospheric conditions, in order to get the behavior thermodynamic variables such as the temperature, pressure, density, enthalpy etc. When the air vehicle is at lower speeds, the material behavior with respect to the fluid viscosity and the dense medium is favorable i.e. no need to have thick and heavy metals, but when the speed is going beyond supersonic regimes, the material properties become very worse and lead to the structural failures. So to avoid the failures and other accidental hazards we do simulate the atmospheric conditions with respect to the speed of the vehicle and look for the appropriate material selection for the safety.

The material selection is completely based upon the heat flux, stress distribution and few other loading factors. The stress distribution factor can be resolved by selecting high strength metals with appropriate thickness to avoid fatigue and loading losses, but to protect the metal from the continuous attack of viscosity that leads to form a high temperature gas layer at the surface, these high temperature viscous layers heat the metal and reduces its strength and other properties. So to protect the main structural frame, a new high temperature material was introduced for the first time for Apollo spacecraft in 1962 developed by Lockheed that was a 32-inch-diameter radome and later it came into use for all the controlled and uncontrolled re-entry vehicles. Heat flux is not only observed in re-entry vehicles, but also even in supersonic ballistic missiles, as every nation is concerned in the Defence and tends to have the fastest and very impactful missiles for the security and to become powerful in hypersonic attacks.

To visualize and create the exact atmospheric conditions according to the flight speed and altitude, we invented many devices and named them as wind tunnels, we are having wide range of wind tunnels and categorized them according to the speeds, temperature and enthalpy. Here we are mainly focusing on the supersonic – Hypersonic wind tunnels. The tunnels help us to recreate the Physical conditions around the scaled down air vehicle. The overall run time of the wind tunnel is inversely proportional to the speed of the flow and enthalpy content. The maximum available run time of hypersonic wind tunnels are order to micro to few milli seconds. There are many supersonic and hypersonic wind tunnels and those are categorized based upon flow speed, enthalpy and run time.

Hypersonic wind tunnel	Hypersonic impulse	Shock Tunnel and its	Other hypersonic test

	facilities	variants	facilities
1.Hypersonic Test Facilities 2.Continuous Hypersonic Wind Tunnel 3.Heaters 4.Hypersonic Nozzles 5.Hypersonic Diffuser 6.Blow-down Hypersonic Wind Tunnel 7.Nitrogen Wind Tunnel 8.Continuous Tunnel or Arc Jet Wind Tunnels	1.Impulse Test Facilities 2.Shock Tube 3.Diaphragm-less Shock Tube 4.Combustion Driven Shock Tube	1.Shock Tunnel 2.Free-Piston Driven Shock Tunnel 3.Gun Tunnel 4.Expansion Tube	1.Hot shot Tunnel 2.Launcher or Flight Test Facility

Table.1 – Types of Hypersonic Wind tunnels

II. LITERATURE SURVEY

For many space exploration missions and hypersonic air vehicles, high flow speed, plasma radiation effect and high enthalpy chemical kinetics are important to study the aerothermodynamics of hypersonic flows. Depending upon the type of process and phenomena to be analyzed, these conditions may be simulated using the shock tunnels, shock tubes or expansion tubes that must generate non-equilibrium flows. The first shock tube was invented by Vielle, while studying combustion and detonation in tubes. When the pressure inside the driver section raises, it ruptures diaphragm that results in generating a shock wave. The produced shock wave accelerates the test gas and its other properties such as temperature, pressure, density etc., changes abruptly. Hence, the physical and chemical processes can evolve to their equilibrium state. Shock tubes have great advantages of creating reliable post shock conditions of planetary entries. There are many shock tubes in use by many organizations around the globe. The following surveys main focus is to investigate radiation absorption, chemical kinetics that are seen during atmospheric re-entry at super orbital speeds. Few facilities are capable of producing orbital & suborbital speeds are also be mentioned. All available hypersonic facilities are shown in Figure 1.

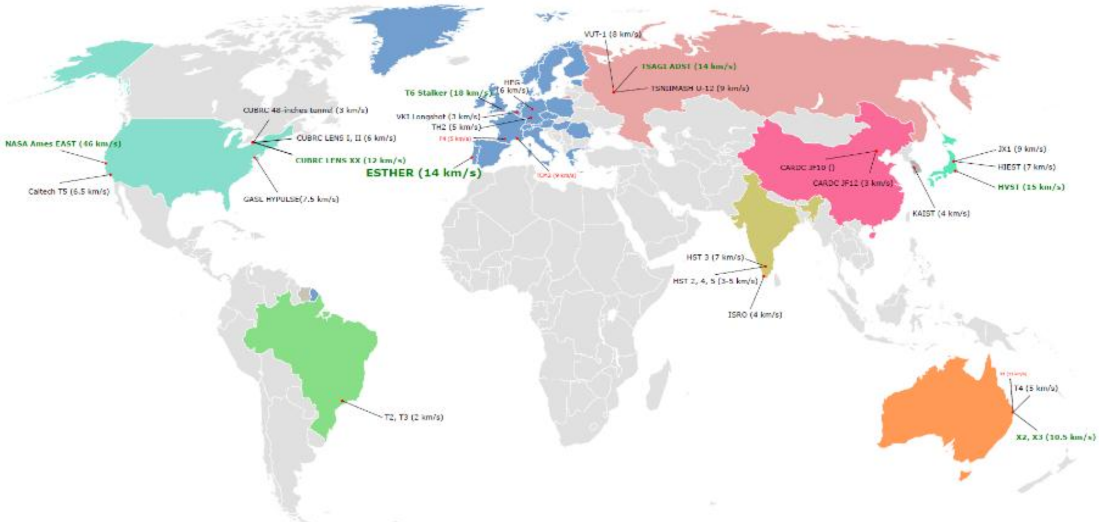

Fig.1- World outlook of Hypersonic Facilities

The flow speed which exceeds 10km/s are known as Superorbital velocities, The only facility that can reach these velocities are Ames EAST shock tube (USA), X2 & X3 expansion tubes (Australia), HVST shock tube (Japan), TsAGI ADST shock tube (Russia) and CUBRC LENS XX expansion tube (USA). There few more shock tubes that are capable of exceeding hypervelocities are under development and construction in Europe i.e. ESTHER shock tube (Portugal) & T6 Stalker tunnel (UK).

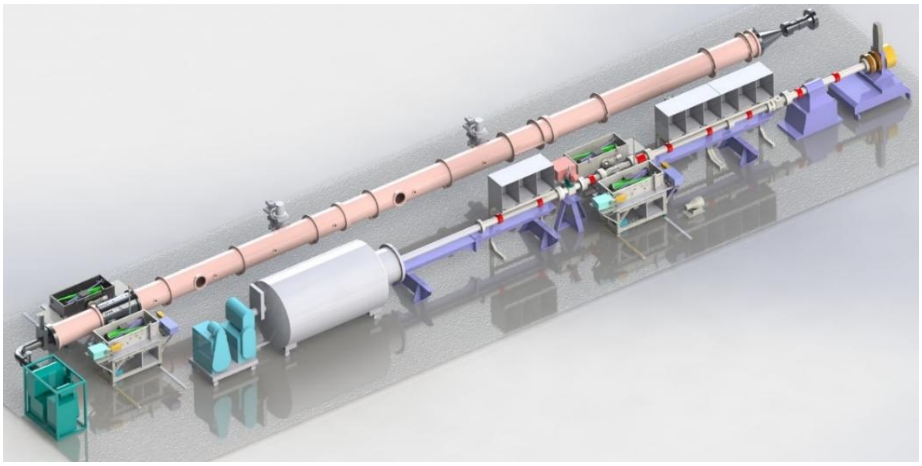

Fig.2- NASA Ames EAST

Ames Electric Arc Shock Tube (EAST) was built by NASA in 1965 is only shock tube in US capable of simulating super-heated gas environments at very high enthalpies i.e. reservoir enthalpy up to 29MJ/Kg.

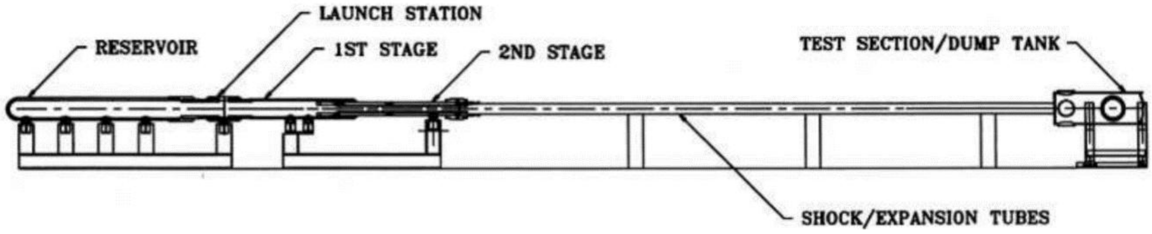

Fig.3- X2 Free Piston shock tube

The X2 free piston driven expansion tube (Fig.3) was built in 1995. It allows experiments in larger models as it is a 23 m long and its driven tube has 85 mm diameter. This facility is mainly used to study radiative heating during atmospheric re-entry, X2 capable of reaching 10.3 km/s and having a test time of about 50 ms.

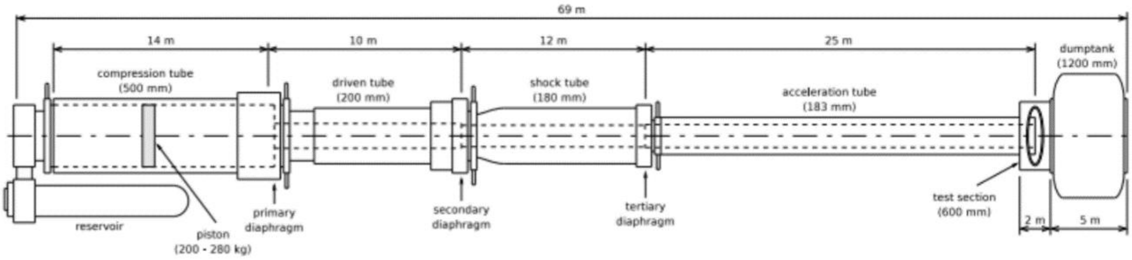

Fig.4- X3 Free Piston Shock Tube

The X3 facility (Fig.4) was commissioned in January 2001 and is a large superorbital free piston driven expansion tube. The length of the facility is 69 m, having a 14 m compression tube, 10 m driven tube, 12 m secondary driver, 25 m acceleration tube and a 2 m test section. The exit diameter of the acceleration tube is 182.6 mm. The dump tank has a length of 5 m and 1.2 m diameter. This facility has a running time of 1 ms and the flow speed can reach 10 km/s in air, with stagnation enthalpies up to 100 MJ/kg.

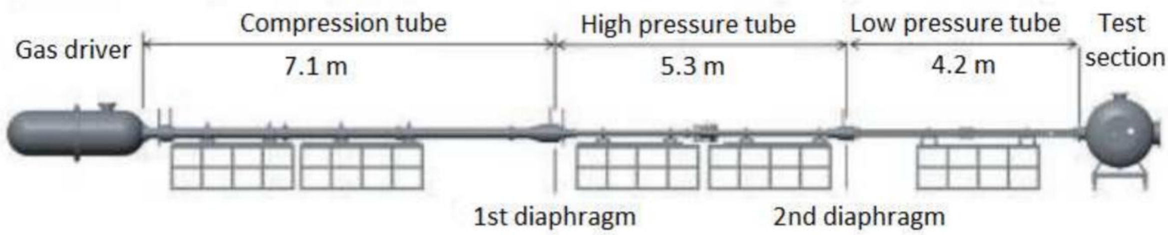

Fig.5- HyperVelocity Shock Tube (HVST)

The Japan Aerospace Exploration Agency (JAXA) currently uses the HyperVelocity Shock Tube (HVST), a free piston double diaphragm shock tube, investigating thermochemical non-equilibrium phenomena and particularly radiation emitted from the highly heated region, behind the shock wave. This shock tube is composed of five sections: air reservoir; free piston compression tube; high and a low pressure compression tubes; and a vacuum tank. The reservoir can be filled with air up to 0.98 MPa.

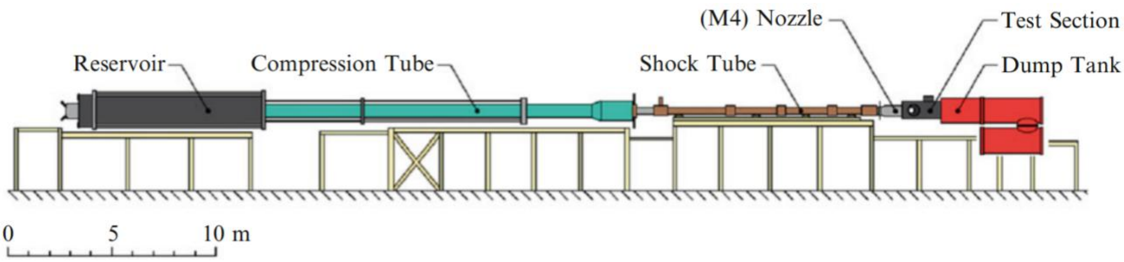

Fig.6- T4 Stalker Tube

The T4 Stalker tube (Fig.6) was commissioned at the University of Queensland in 1987. The reservoir is 7 m long and is designed for pressures up to 14 MPa. The compression tube is 26 m long and has an internal diameter of 229 mm. Pistons of masses 92 and 36 kg are used. Unscored primary diaphragms of up to 6 mm of mild steel plate are used. The shock tube is 10 m long and has an internal diameter of 76 mm. Primary-shock speeds are determined from traces from piezoelectric pressure transducers located at 2 m spacing along the tube and from two transducers located 65 mm from the downstream end of the shock tube that measure the nozzle-supply pressure.

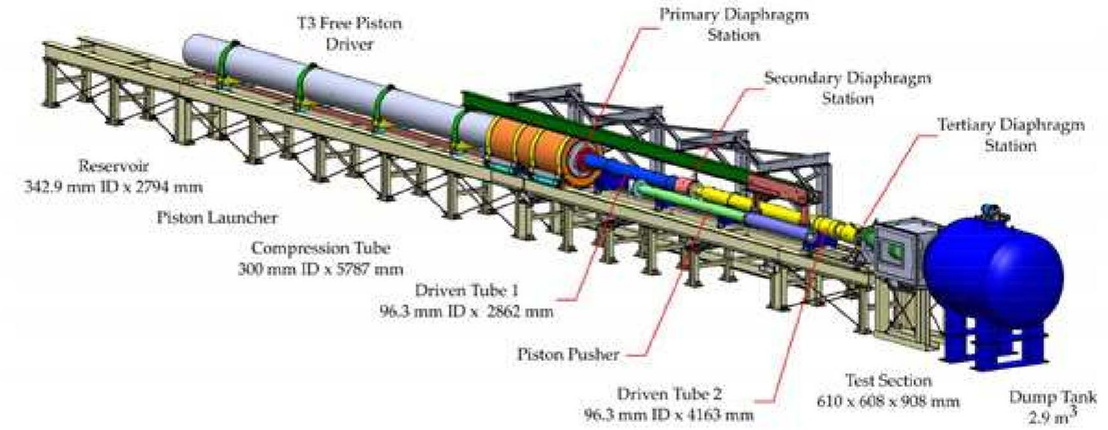

Fig.7- T6 Stalker Tunnel

The T6 Stalker tunnel (Fig .7) has been a joint collaboration between the University of Oxford and the University of Queensland. The tunnel is formed by coupling the decommissioned T3 free piston driver to the barrels, nozzle and test section of the Oxford Gun Tunnel. The result is a facility that allows operations either as a reflected shock tunnel, an expansion tunnel or as a shock tube, for shock layer radiation studies.

III. Methodology

The motion analysis of the piston such as its initialization, acceleration and braking is important as many of the parameters such as enthalpy, temperature, speed of the shock wave etc., are depend upon the piston motion. The analysis is to obtain the numerical solution for the equations by introducing non-dimensional variables Xi, Phi, Tao, etc. by varying key parameters as piston mass, diameter and the pressure ratios.

Model Equations

Let the initial pressure in the air and the helium be p_{A0} and p_{H0}, the piston speed be u (positive to the right), the initial distance of the piston from the diaphragm be L and its variable values being x. The air pressure p_A at the left side of the piston is given by

$$p_A = p_{A0}(1-((\gamma_A-1)/2)(u/a_0))^{(2\gamma_A/\gamma_A-1)} \quad (1)$$

where a_0 is initial speed of sound in air and γ_A is ratio of specific heats in air (7/5) which is considered as perfect gas.

The helium pressure on the right side of piston is given by

$$p_H = p_{H0}(x/L)^{-\gamma} \quad (2)$$

where γ is the ratio of specific heats of helium (5/3). Hence the equation of motion of piston is

$$-M(d^2x/dt^2) = (\pi D^2/4)\{p_{A0}(1-((\gamma_A-1)/2)(u/a_0))^{(2\gamma_A/\gamma_A-1)} - p_H = p_{H0}(x/L)^{-\gamma}\} \quad (3)$$

where M is piston mass, D is compression tube diameter and t is time measured from piston release.

We assume that the reference frame is inertial , which it is not if the compression tube is free recoil and assume that compression tube is so heavy that recoil motion may be neglected.

Let introduce the dimensionless variables

$\xi(\tau) = x/D$, $\tau = t\ a0/D$, $\phi(\tau) = u/a0$ (4)

rewriting equation (3)

$$\xi = -\phi$$
$$\dot{\phi} = b1[(1-0.2\phi)7 - b2\xi - 5/3] \quad (5)$$
$$\xi(0) = L/D$$
$$\phi(0) = 0$$

where dot represents differentiation with respect to τ and parameters b_1 and b_2 are given by

$$b1 = \pi/4(p_{A0}D^3 / Ma_0{}^2) \quad (6)$$
$$b2 = (p_{H0}/p_{A0})(L/D)^{5/3}$$

Piston motion after diaphragm rupture and conditions behind piston analysis can be carried out by obtaining solution of equation (5), the plots obtained shows that the piston acceleration just before diaphragm rupture is greater than its initial acceleration. Let the distance *s* behind the piston from the shock wave is formed due to the steeping of compression waves and it is given by

$$s = 2a_r{}^2/((\gamma_A+1)A) \quad (7)$$

where a_r, A is speed of sound in air at diaphragm rupture, piston acceleration respectively. The pressure on the back face of piston is given by pressure after shock that is generated if piston is stopped instantaneously at rupture of diaphragm. The Mach number M_r is given by

$$u_r/a_r = (2/\gamma_a+1)(M_r-1/M_r) \quad (8)$$

where a_r is speed of sound in air at piston face before shock is generated and it is given by

$$a_r/a_0 = 1+((\gamma-1)/2)(u_r/a_0) \quad (9)$$

The mach number of shock wave is obtained from a solution of quadratic equation formed with the combination of equation (8) & (9) the formed equation is

$$M_r{}^2 - (((\gamma_A+1)(u_r/a_0))/2+(\gamma_A-1)(u_r/a_0))M_r - 1=0 \quad (10)$$

Substitute $\gamma_A = 7/5$

$$M_r = 0.6\ u_r/a_0/1+0.2\ u_r/a_0 + \{(0.6\ u_r/a_0/1+0.2\ u_r/a_0)^2+1\}^{1/2} \quad (11)$$

Therefore the pressure at piston face after shock has been generated is

$$p_{Ar} = p_{A0}(1-0.2u_r/a_0)^7.((7M_r^2-1)/6) \quad (12)$$

IV. Results

The equations were nonlinear in nature (5) and cannot be solved in closed form but are easily solved numerically. The solution examples are showed in Figure 1 and Figure 2 by taking L/D = 100 as phase planes ξ- ϕ plots as time-distance (τ- ξ) plots respectively by varying the values of b1 and b2. The plots clearly provides the information about the motion of piston throughout the operation, the mass, diameter and the initial position of the piston is depend upon the design factors such as the maximum and minimum speed of flow, enthalpy and overall run time of the shock tunnel. The velocity of piston at rupture of diaphragm is very sensitive to the values of b1 and b2. The solution for equation (12) is obtained by using shock jump conditions, Figure 4 provides the variation of p_{Ar}/ p_{A0} vs ur/a0 and it's assumed that pressure behind piston remains constant p_{Ar} after the rupture of diaphragm.

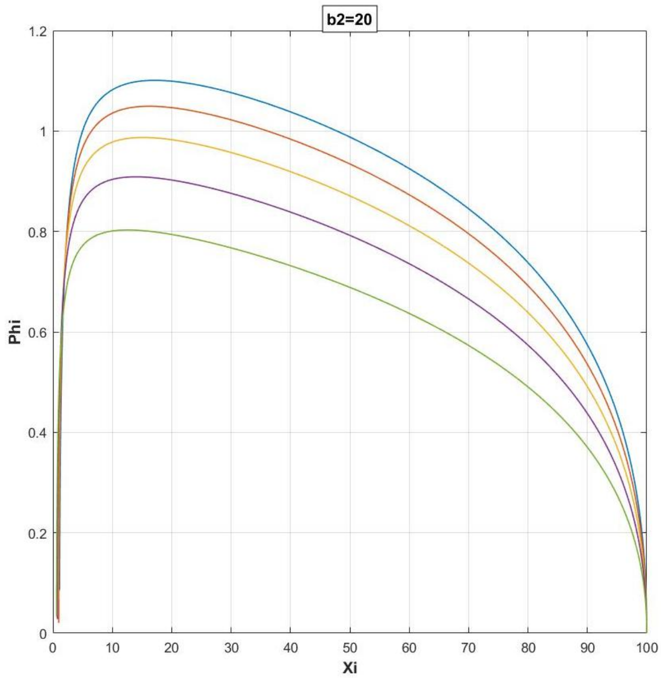

Fig.8- ξ vs ϕ Early part of piston motion. Solution for equation (5) for L/D=100, b2=20

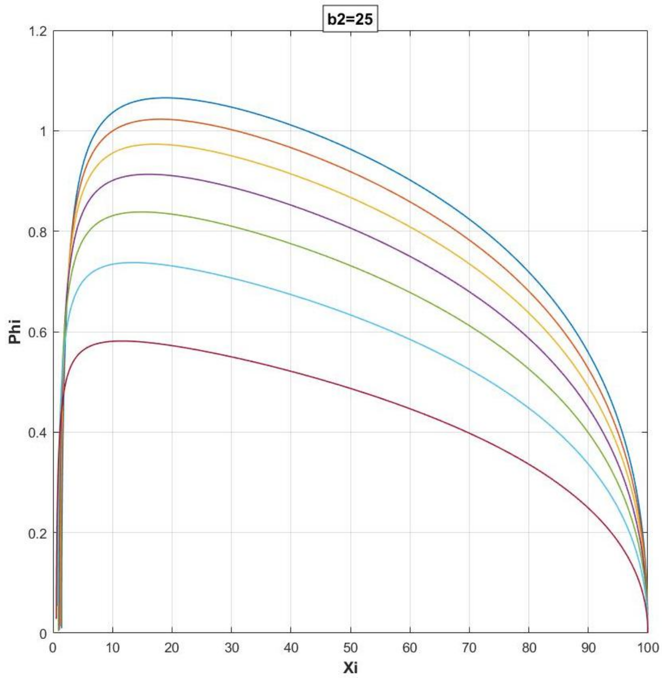

Fig.9- ξ vs ϕ Early part of piston motion. Solution for equation (5) for L/D=100, b2=25

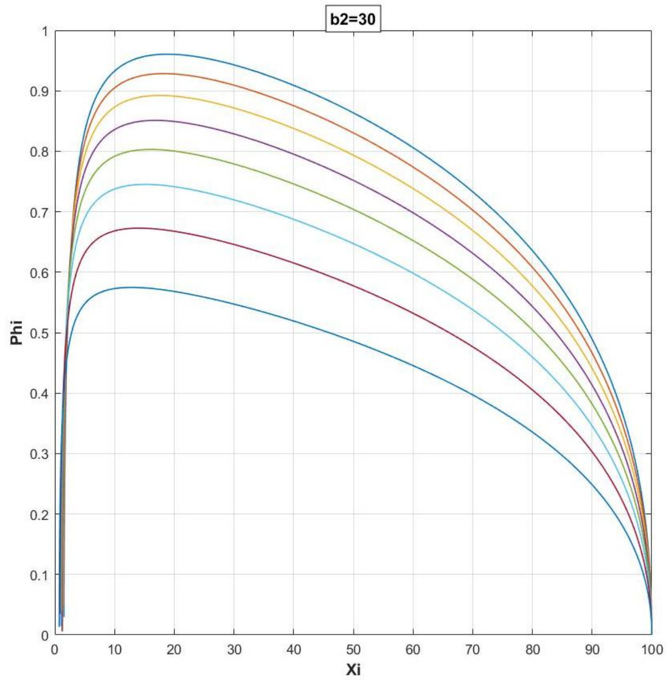

Fig.10- ξ vs ϕ Early part of piston motion. Solution for equation (5) for L/D=100, b2=30

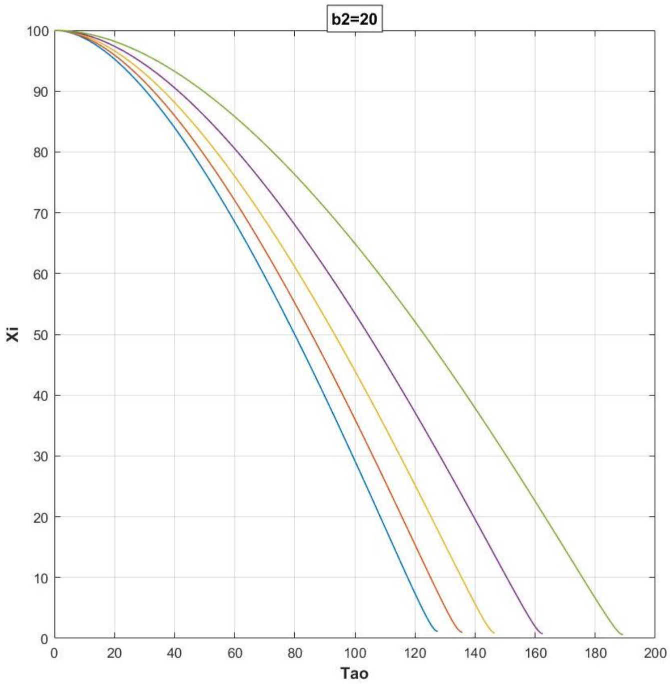

Fig.11- τ vs ξ Early part of piston motion. Solution for equation (5) for L/D=100, b2=20

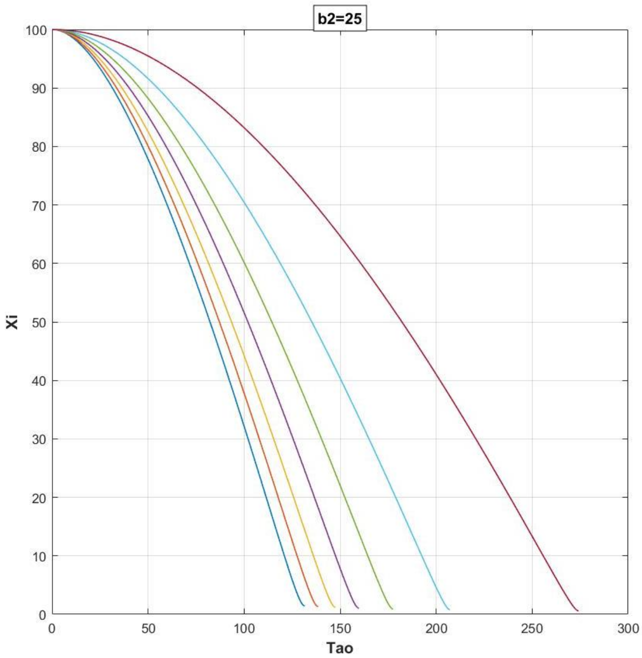

Fig.12- τ vs ξ Early part of piston motion. Solution for equation (5) for L/D=100, b2=25

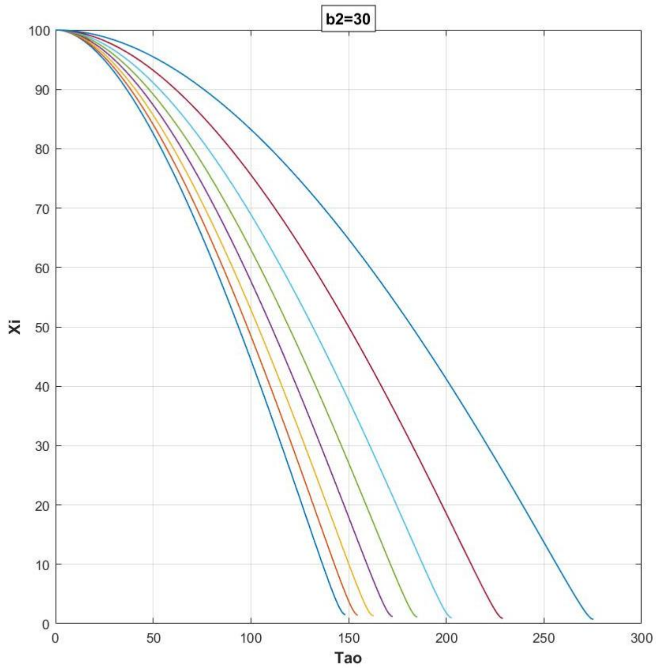

Fig.13- τ vs ξ Early part of piston motion. Solution for equation (5) for L/D=100, b2=30

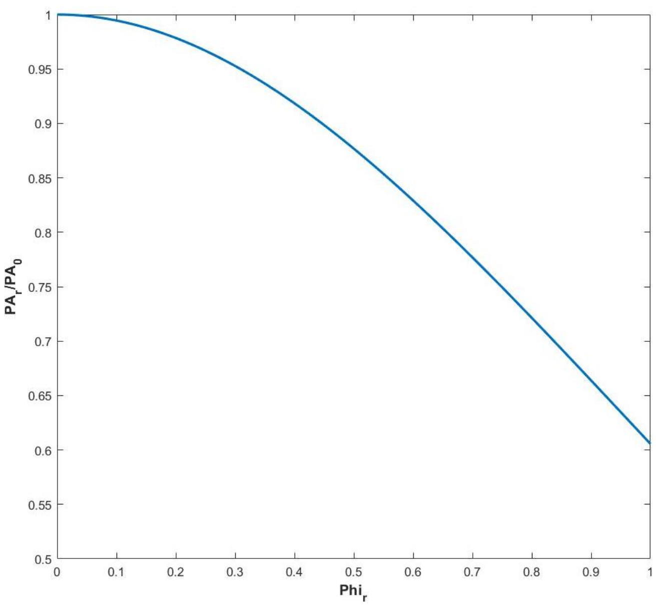

Fig.14- ϕr vs p_{Ar}/p_{A0} Pressure on the rear face of piston stopped instantaneously from a speed ur to which it has been accelerated from rest adjacent to a gas at pressure p_{A0}, speed of sound a_0, γA=7/5 (see equation 12)

V. Future directions

The further analysis of the piston motion and its kinematics might help to produce high enthalpy and high Mach numbers, this study can help to develop new generations of thermal protection system with large areas other than tiles that reduces the heat flux with lightweight materials. The cost to make Free piston shock tunnels can be reduced by using lightweight materials if the 2D heat transfer inside the shock tube is known. The development of hypersonic vehicles, missiles and re-entry vehicles with advanced features can be done with the new generation hypersonic testing facilities.

References

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APPENDIX-A

```
function[f]=fun1(Tao_temp,Xi_temp,Phi_temp)
format longeng;
f=-Phi_temp;
clear Tao_temp Xi_temp Phi_temp;
return
```

A.1 – Defining equation (5).1 as function 1

```
function[g]=fun2(Tao_temp,Xi_temp,Phi_temp,b1_temp,b2_temp)
format longeng;
g = b1_temp*(((1-0.2*Phi_temp)^7)-b2_temp*1.667^(-5/3));
clear Tao_temp Xi_temp Phi_temp b1_temp b2_temp;
return
```

A.2 – Defining equation (5).2 as function 2

```
function [x,y_rk4,z_rk4,b1_rk4]=rk4_3d(f,g,x_bound,y0,z0,b1,b2,h)
format longeng
m=length(b2);
n=ceil((x_bound(1,2)-x_bound(1,1))/h)+1;

x=null(n,m); y_rk4=null(n,m);z_rk4=null(n,m);b1_rk4=null(n,m);
f_rk4=null(n,m); g_rk4=null(n,m);

for j=1:m
    x(1,j)=x_bound(1,1);
    y_rk4(1,j)=y0;
    z_rk4(1,j)=z0;
    b1_rk4(1,j)=b1;
    f_rk4(1,j)=feval(f,x(1,j),y_rk4(1,j),z_rk4(1,j));
    g_rk4(1,j)=feval(g,x(1,j),y_rk4(1,j),z_rk4(1,j),b1_rk4(1,j),b2(1,j));
    for i=1:n-1
        x(i+1,j)=x(i,j)+h;
        b1_rk4(i+1,j)=b1_rk4(i,j)+h;
        k11=h*f_rk4(i,j);
        k12=h*g_rk4(i,j);
        k21=h*feval(f,x(i,j)+h/2.0,y_rk4(i,j)+k11/2.0,z_rk4(i,j)+k12/2.0);
        k22=h*feval(g,x(i,j)+h/2.0,y_rk4(i,j)+k11/2.0,z_rk4(i,j)+k12/2.0,b1_rk4(i,j),b2(1,j));
        k31=h*feval(f,x(i,j)+h/2.0,y_rk4(i,j)+k21/2.0,z_rk4(i,j)+k22/2.0);
        k32=h*feval(g,x(i,j)+h/2.0,y_rk4(i,j)+k21/2.0,z_rk4(i,j)+k22/2.0,b1_rk4(i,j),b2(1,j));
        k41=h*feval(f,x(i,j)+h,y_rk4(i,j)+k31,z_rk4(i,j)+k32);
        k42=h*feval(g,x(i,j)+h,y_rk4(i,j)+k31,z_rk4(i,j)+k32,b1_rk4(i,j),b2(1,j));

        y_rk4(i+1,j)=y_rk4(i,j)+(k11+2.0*k21+2.0*k31+k41)/6.0;
        z_rk4(i+1,j)=z_rk4(i,j)+(k12+2.0*k22+2.0*k32+k42)/6.0;
        f_rk4(i+1,j)=feval(f,x(i+1,j),y_rk4(i+1,j),z_rk4(i+1,j));
        g_rk4(i+1,j)=feval(g,x(i+1,j),y_rk4(i+1,j),z_rk4(i+1,j),b1_rk4(i+1,j),b2(1,j));
        if y_rk4(i+1,j)<=0 || z_rk4(i+1,j)<=0
            x(i+1:n,j)=nan;
            b1_rk4(i+1:n,j)=nan;
            y_rk4(i+1:n,j)=nan;
            z_rk4(i+1:n,j)=nan;
            break;
        end
    end
end
clear i n y0 z0,
return
```

A.3 – Runge – Kutta solver

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube
% piston motion variable before diaphragm rupture
%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= 1e-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=20;
b1=linspace (0.03,0.01,5);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot (Xi, Phi,'-','LineWidth',1);
grid
xlabel ('Xi','FontSize',12,'FontWeight','bold');
ylabel ('Phi','FontSize',12,'FontWeight','bold');
title ('b2=20','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.4 - MATLAB code for solution for equation (5) for L/D=100, b2=20 (ξ vs ϕ)

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube

%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= 1e-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=25;
b1=linspace(0.028,0.004,7);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot (Xi, Phi,'-','LineWidth',1);
grid
xlabel ('Xi','FontSize',12,'FontWeight','bold');
ylabel ('Phi','FontSize',12,'FontWeight','bold');
title ('b2=25','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.5 - MATLAB code for solution for equation (5) for L/D=100, b2=25 (ξ vs ϕ)

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube

%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= 1e-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=30;
b1=linspace (0.02,0.004,8);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot (Xi, Phi,'-','LineWidth',1);
grid
xlabel ('Xi','FontSize',12,'FontWeight','bold');
ylabel ('Phi','FontSize',12,'FontWeight','bold');
title ('b2=30','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.6 - MATLAB code for solution for equation (5) for L/D=100, b2=30 (ξ vs ϕ)

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube

%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= 1e-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=20;
b1=linspace (0.03,0.01,5);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot(Tao, Xi,'-','LineWidth', 1)
grid
xlabel('Tao','FontSize',12,'FontWeight','bold');
ylabel('Xi','FontSize',12,'FontWeight','bold');
title ('b2=20','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.7 - MATLAB code for solution for equation (5) for L/D=100, b2=20 (τ vs ξ)

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube

%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= le-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=25;
b1=linspace (0.028,0.004,7);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot(Tao, Xi,'-','LineWidth', 1)
grid
xlabel('Tao','FontSize',12,'FontWeight','bold');
ylabel('Xi','FontSize',12,'FontWeight','bold');
title ('b2=25','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.8 - MATLAB code for solution for equation (5) for L/D=100, b2=20 (τ vs ξ)

```
% The piston motion in a free piston driver for shock tubes and tunnels-
% Hornung
% Solving of non-linear differential equation defining piston motion
clc;
close all;
format longeng
% INPUT constants
L_D_ratio=100;
% L: Length of cpmpression tube
% D: Diameter of compression tube

%defining function
f= @fun1; %linear function for d_Xi/d_Tao
g= @fun2; %linear function for d_Phi/d_Tao
% Step size
Tao_bound= [0 500];
h=0.2; %step size
n=ceil((Tao_bound(1,2)-Tao_bound(1,1))/h)+1; %number of interval
%tol= le-5;
% initial solution at t=0
Tao(1,1)=0; Xi(1,1)=L_D_ratio; Phi(1,1) =0;
b2=30;
b1=linspace (0.02,0.004,8);
%part 1; SOLUTION BY CLASICAL FOURTH ORDER RUNGE KUTTA METHOD
[Tao,Xi,Phi]=rk4_3d (f,g,Tao_bound,Xi(1,1),Phi(1,1),b1,b2,h);
% PLOTTING THE SOLUTION
figure
plot(Tao, Xi,'-','LineWidth', 1)
grid
xlabel('Tao','FontSize',12,'FontWeight','bold');
ylabel('Xi','FontSize',12,'FontWeight','bold');
title ('b2=30','BackgroundColor',[1 1 1],'EdgeColor',[0 0 0],'FontName','Arial','FontSize',12,'FontWeight','bold');
```

A.9 - MATLAB code for solution for equation (5) for L/D=100, b2=20 (τ vs ξ)

```
clc
Phi=null(1001,1);
Mr=null(1001,1);
PA_r_by_PA_0=null(1001,1);
Mr(1,1)=1;
Phi(1,1)=0;
PA_r_by_PA_0(1,1)=(1-0.2*Phi(1,1))^7*((7*Mr(1,1)^2-1)/6);

for i=2:1001
    Phi(i,1)=Phi(i-1,1)+0.001;
    Mr(i,1)=(0.6*Phi(i,1))/(1+0.2*Phi(i,1))+(((0.6*Phi(i,1))/(1+0.2*Phi(i,1)))^2+1)^(1/2);
    PA_r_by_PA_0(i,1)=(1-0.2*Phi(i,1))^7*((7*Mr(i,1)^2-1)/6);
end
plot(Phi,PA_r_by_PA_0,'-','LineWidth',2);
xlim([0 1])
ylim([0.5 1.0])
xlabel ('Phi_r','FontSize',12,'FontWeight','bold');
ylabel ('PA_r/PA_0','FontSize',12,'FontWeight','bold');
```

A.10 - MATLAB code for solution for equation (12) (ϕr vs p_{Ar}/p_{A0})