# Effects of Thermal Nonequilibrium and Vertical Throughflow on the Onset of Forchheimer-Bénard Convection

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***Abstract* - The influence of vertical throughflow on thermal convection in a horizontal porous layer using a regime of local thermal nonequilibrium (LTNE) is investigated. The flow in the porous medium is governed by the Forchheimer equation and two energy balance equations, one for the solid phase and another for the fluid phase are used. The eigenvalue problem is solved numerically using shooting method combined with Runge-Kuatta-Fehlberg technique. It is noted that throughflow has a stabilizing effect on the stability of the system have the effect of delaying the onset of convection and reducing the size of convection cells. In addition, the influence of parameters representing the thermal non-equilibrium effects on convective instability is discussed in detail.**

***keywords* - Throughflow, porous layer, thermal non-equilibrium model, shooting method**

**Nomenclature**

| | |
|---|---|
| $a=\sqrt{l^2+m^2}$ | The overall horizontal wave number |
| $c$ | Specific heat |
| $c_b$ | Dimensionless Forchheimer coefficient |
| $D=d/dz$ | The differential operator |
| $Da=k/d^2$ | Darcy number |
| $G=c_b\varepsilon^2/Pr\sqrt{Da^{-1}}$ | Inertia parameter |
| $\vec{g}$ | Gravitational acceleration |
| $H=hd^2/\varepsilon k_f$ | Interphase heat transfer coefficient |
| $h$ | Inter-phase heat transfer coefficient |
| $k$ | Permeability of the porous medium |
| $k_f$ | Thermal conductivities of fluid phase |
| $k_s$ | Thermal conductivities of solid phase |
| $l, m$ | The wave numbers in the $x$ and $y$- directions |
| $p$ | pressure |

| | |
|---|---|
| $Pr = \nu\varepsilon / \kappa_f$ | Prandtl number |
| $Pr_D = Pr\varepsilon / Da$ | Darcy-Prandtl number |
| $Q = w_0 d / \kappa_f$ | Throughflow-dependent Peclet number, |
| $\vec{q} = (u, v, w)$ | Velocity vector |
| $R_D = \beta_t g d k \left(T_l - T_u\right) / \varepsilon\nu\kappa_f$ | Darcy-Rayleigh number |
| $T_f$ | Temperature of the fluid |
| $T_s$ | Temperature of the solid |
| $W$ | vertical component of perturbed velocity |

***Greek symbols***

| | |
|---|---|
| $\alpha = \kappa_f / \kappa_s$ | Ratio of diffusivities |
| $\beta$ | Coefficient of thermal expansion |
| $\varepsilon$ | Porosity of the medium |
| $\gamma = \varepsilon k_f / \left(1-\varepsilon\right) k_s$ | Porosity modified conductivity ratio |
| $\kappa = k_f / \left(\rho_0 c\right)_f$ | Effective thermal diffusivity of the fluid |
| $\omega$ | the growth term |
| $\rho_0$ | Reference density |
| $\rho_f$ | Fluid density |
| $\mu_f$ | Fluid viscosity |
| $\because$ | Vertical component of perturbed fluid temperature |
| $\Phi$ | Vertical component of perturbed solid temperature |

## 1. Introduction

Most of the results on thermal convection in porous media are mainly based on the assumption that the fluid and solid phases are everywhere in local thermal equilibrium (LTE) state exists between the solid and liquid phases that remove most practical circumstances. In such circumstances, it is pertinent to take account of the local thermal non-equilibrium (LTNE) effects by considering a two-field model for energy equation each representing the fluid and solid phases separately. It is accepted that the LTNE theory is very important in everyday technology specifically, in drying, freezing of foods, microwave heating, rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes. Many literatures are existing on the problems of LTNE model in the study of thermal convection [1-19]. Recently, Lagziri and Bezzazi [20] have discussed the effects of LTNE at the beginning of the thermal convection of the porous layer for Robin boundaries.

In the current heat transfer research, it is relevant to look for mechanisms to control (suppress or augment) convection. Control of convection by the adjustment of vertical throughflow is found to be more efficient. The effect of vertical throughflow on thermal convective instability in both fluid and porous layers has been studied by many authors extensively [21-26]. However, Shivakumara and Khalili [27] and Shivakumara and Nanjundappa [28] have shown that, irrespective of the nature of boundaries, a small amount of throughflow in either of its direction destabilizes the system; a result which is in contrast to the single component system. Nield and Kuznetsov [29] have studied the effect of throughflow on the onset of thermal convection in a saturated nanofluid porous layer. Kuznetsov and Nield [30] have discussed the onset of convection in a porous medium by allowing the LTNE model with throughflow.

The main aim of the present paper is to investigate the combined effect of LTNE and throughflow on the onset of thermal convection in a horizontal layer of fluid saturated Darcy porous medium. In the discussion, a two field model that represents the fluid and solid phase temperature fields separately is used for energy equation. The Forchheimer-extended Darcy model is used to describe the flow in the porous medium. The boundary conditions are assumed to be impermeable perfect conductors. The resulting eigenvalue problem is solved numerically using shooting method. The results are illustrated graphically.

## 2. Formulation of the problem

We consider an incompressible fluid-saturated horizontal porous layer of thickness $d$ with constant vertical throughflow $w_0$ which is either gravity aligned or otherwise in its direction. The lower surface of the porous layer is held at constant temperature $T_l$, while the upper surface is at $T_u$ $(<T_l)$. A Cartesian coordinate system $(x, y, z)$ is chosen such that the origin is at the bottom of the porous layer and the z-axis vertically upward. The solid and fluid phases of the porous medium are considered to be in LTNE and a two-field model for temperatures is used.

The basic equations are:

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_0 c_b}{\sqrt{k}}|\vec{q}|\vec{q} = -\nabla p + \rho_f \vec{g} - \frac{\mu_f}{k}\vec{q} \tag{2}$$

$$\varepsilon(\rho_0 c)_f \frac{\partial T_f}{\partial t} + (\rho_0 c)_f (\vec{q}\cdot\nabla)T_f = \varepsilon k_f \nabla^2 T_f + h(T_s - T_f) \tag{3}$$

$$(1-\varepsilon)(\rho_0 c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s \nabla^2 T_s - h(T_s - T_f) \tag{4}$$

$$\rho = \rho_0\{1 - \beta(T_f - T_l)\} \tag{5}$$

From Eqs.(3) and (4), it may be noted that the energy equations are coupled by means of the terms, which accounts for the heat lost to or gained from the other phase.

The basic state is not quiescent and given by

$$\vec{q} = \vec{q}_b = w_0\hat{k},\ p = p_b(z),\ T_f = T_{fb}(z),\ T_s = T_{sb}(z),\ h = 0 \tag{6}$$

where the subscript $b$ denotes the basic state. In the basic state, the fluid and solid phase temperatures satisfy the following equations

$$(\rho_0 c)_f\, w_0 \frac{\partial T_{fb}}{\partial z} = \varepsilon k_f \frac{\partial^2 T_{fb}}{\partial z^2} \tag{7a}$$

$$\frac{\partial^2 T_{sb}}{\partial z^2} = 0 \tag{7b}$$

with the boundary conditions

$$T_{fb} = T_{sb} = T_l \text{ at } z = 0$$

$$T_{fb} = T_{sb} = T_u \text{ at } z = d \,. \qquad (8)$$

Solving Eqs. 7(a) and (b) subject to the boundary conditions, we get

$$T_{fb} = T_l - \left(T_l - T_u\right)\frac{\left(1-e^{w_0 z/\kappa}\right)}{\left(1-e^{w_0 d/\kappa}\right)} \qquad (9a)$$

$$T_{sb} = -\frac{(T_l - T_u)}{d}z + T_l \qquad (9b)$$

To this end, we superimpose infinitesimal disturbances on the basic state to study its stability as follows:

$$\vec{q} = w_0\hat{k} + \vec{q}\,', \; p = p_b(z) + p', \; T_f = T_{fb}(z) + T_f{}', \; T_s = T_{sb}(z) + T_s{}' \,. \qquad (10)$$

Substituting Eq.(10) into Eqs.(1)-(5), eliminating the pressure term from the momentum equation, linearizing the equations in the usual way and non-dimensionalizing the coordinates, time, velocity and temperature of the fluid as well as solid phase temperatures using the scales $d$ , $d^2/\kappa_f$, $\varepsilon\kappa_f/d$ and $(T_l - T_u)$, respectively, we obtain the following stability equations (after neglecting the primes for simplicity):

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + G|Q| + 1\right)\nabla^2 w = R_D \nabla_h^2 T_f \qquad (11)$$

$$\left(\frac{\partial}{\partial t} + Q\frac{\partial}{\partial z} - \nabla^2\right)T_f + H\left(T_f - T_s\right) = -wf(z) \qquad (12)$$

$$\alpha\frac{\partial T_s}{\partial t} - \nabla^2 T_s - \gamma H\left(T_f - T_s\right) = 0 \qquad (13)$$

Here, $f(z) = -Qe^{Qz}/\left(e^{Q}-1\right)$ is the basic temperature gradient. As $Q \to 0$, $f(z) \to -1$ then the basic temperature gradient becomes constant.

The perturbed quantities are expressed as normal modes of the form

$$\begin{bmatrix} w \\ T_f \\ T_s \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{bmatrix} \exp\left\{i(lx+my)+\omega t\right\} \qquad (14)$$

After substituting Eq.(14) into Eqs.(11)-(13), we obtain

$$\left\{\frac{\omega}{Pr_D} + G|Q| + 1\right\}\left(D^2 - a^2\right)W = -R_D a^2\Theta \qquad (15)$$

$$\omega\Theta + QD\Theta - \left(D^2 - a^2\right)\Theta + H\left(\Theta - \Phi\right) = -Wf(z) \qquad (16)$$

$$\alpha\omega\Phi - \left(D^2 - a^2\right)\Phi - \gamma H\left(\Theta - \Phi\right) = 0 \qquad (17)$$

We note that the principle of exchange of stability holds valid since there is no mechanism to set up oscillatory motions. Hence, we take $\omega = 0$ in the above equations and arrive at the following system of ordinary differential equations:

$$\left[G|Q| + 1\right]\left(D^2 - a^2\right)W = -R_D a^2\Theta \qquad (18)$$

$$\left(D^2 - a^2\right)\because - QD\because + H\left(\Phi - \because\right) = Wf(z) \tag{19}$$

$$\left(D^2 - a^2\right)\Phi + \gamma H\left(\because - \Phi\right) = 0\,. \tag{20}$$

The above equations are to be solved subject to the following boundary conditions:

$$W = \because = \Phi = 0 \quad \text{at} \quad z = 0,\ 1. \tag{21}$$

### 3. Method Of Solution

The eigenvalue problem constituted by Eqs. (18) – (20) together with Eq. (21) is solved numerically using shooting method along with Runge-Kutta-Fehlberg (RKF) and Newton-Raphson methods with $R_D$ as an eigenvalue. Equations (18) – (20) is solved as an initial value problem with the conditions at $z = 0$ as

$$W(0) = 0,\ DW(0) = 1;\ \because(0) = 0,\ D\because(0) = \eta_1;\ \Phi(0) = 0,\ D\,\Phi(0) = \eta_2 \tag{22}$$

and satisfying the conditions at $z = 1$, namely

$$W(1) = 0,\ \because(1) = 0,\ \Phi(1) = 0. \tag{23}$$

Here, the condition $DW(0) = 1$ is a limitation helps to break the scale invariance of the solutions of Eqs. (18) – (21). Further, the parameters $\eta_1$ and $\eta_2$ are unknowns to be determined together with the Darcy-Rayleigh number $R_D$.

The critical Darcy- Rayleigh number $R_{Dc}$ and the corresponding wave number $a_c$ are obtained for various values of physical parameters $\gamma$, $H$, $G$, $Q$ and the corresponding boundary conditions. To endorse the numerical procedure used, the critical Darcy Rayleigh number $R_{Dc}$ and the wave number $a_c$ obtained for different values of $Q$ when $H = 0$, $G = 0$ and $\gamma = 0$ are compared in Table 1 with those of Chen [31]. From this Table, it is seen that our results are in good agreement with the published ones and thus verifies the accuracy of the method used.

### 4. Numerical Results and discussion

The onset of Forchheimer-Bénard convection in a porous layer is investigated using a local thermal non-equilibrium (LTNE) model in the presence of a uniform vertical throughflow. The eigenvalue problem is solved numerically using the shooting method and the results are presented graphically in Figs.1-3.

The variation of critical stability parameters $(R_{Dc}, a_c)$ is shown in Figs. 1-3 as a function of $|Q|$ for various values of $H$ (with $G = 0.1$ and $\gamma = 0.5$), $\gamma$ (with $G = 0.1$ and $H = 20$) and $G$ (with $H = 20$ and $\gamma = 0.5$) respectively. The direction of throughflow is not altering the stability of the system and the effect of increasing $|Q|$ is to increase $R_{Dc}$ and hence it has a stabilizing effect on the system. This is because; the effect of throughflow is to confine significant thermal gradients to a thermal boundary layer at the boundary toward which the throughflow is directed. The effective length scale is thus smaller than the thickness of the porous layer $d$ and so the Rayleigh number, which is proportional to the porous layer thickness, will be much smaller than the actual value of $R_{Dc}$. Therefore higher values of $R_{Dc}$ are needed for the onset of convection with increasing $|Q|$. For different values of $H$ (see Fig 1(a)), it is noted that the curves of $R_{Dc}$ increases with increasing $H$, in general, indicating its effects is to delay the onset of convection. The variation of $R_{Dc}$ as a function of $|Q|$ for different values of $\gamma$ with $G = 0.1$ and $H = 20$ is presented in Fig. 2(a). We note that $R_{Dc}$ decreases with increasing $\gamma$ indicating its effect is to hasten the onset of convection because heat is transported to the system through both by solid and fluid phases. The influence of inertia parameter $G$ on the stability characteristics of the system is illustrated in Fig. 3(a). From this figure it is noted that $R_{Dc}$ increases with increasing $G$. In other words, increase in the inertia effect is to delay the onset of convection.

The corresponding variation of critical wave number $a_c$ is shown in Figs. 1(b)-3(b). We note that increasing $|Q|$ and $H$ is to increase the critical wave number, while opposite is the case with increasing $\gamma$. That is, increase in the value of $|Q|$ and $H$, decrease in $\gamma$ is to reduce the size of convection cells. Moreover, there is no substantial change in the values of $a_c$ with increasing $G$.

### 5. Conclusions

The onset of Darcy-Benard convection in a porous layer is investigated using LTNE model by considering the effect of throughflow in the vertical direction. The shooting method is used to solve the eigenvalue problem for impermeable isothermal boundary conditions. The results of the above-mentioned study can be summarized as follows:

(i) Throughflow has a stabilizing effect on the stability of the system have the effect of delaying the onset of convection.

(ii) The curves of Rayleigh number $R_{Dc}$ increases with increasing the inter-phase heat transfer coefficient $H$.

(iii) The porosity modified conductivity ratio $\gamma$ has destabilizing effect on the stability of the system.

(iv) Increase in the value of non-dimensional parameter $G$ is to delay the onset of convection.

(v) The wave number increases with increasing in $|Q|$ and $H$, while opposite is the case with increasing $\gamma$. Moreover, there is no effect on wave number in increasing the value of $G$.

Table 1: Comparison between the present results for different values of $Q$ when $H = 0$, $G = 0$ and $\gamma = 0$ those of Chen [31].

| $Q$ | Present results | | Chen results [31] | |
|---|---|---|---|---|
| | $R_c$ | $a_c$ | $R_c$ | $a_c$ |
| 0 | 39.4784 | 3.142 | 39.4703 | 3.14 |
| 1 | 41.3011 | 3.1975 | | |
| 2 | 45.2339 | 3.3084 | 45.0682 | 3.29 |
| 3 | 52.0288 | 3.5050 | | |
| 4 | 61.665 | 3.7849 | 61.6487 | 3.79 |
| 5 | 73.423 | 4.1947 | | |
| 6 | 86.6774 | 4.7255 | 86.5861 | 4.73 |
| 7 | 100.565 | 5.3772 | | |
| 8 | 114.846 | 6.0818 | 114.7731 | 6.09 |
| 9 | 129.298 | 6.8066 | | |
| 10 | 143.895 | 7.5362 | 143.4251 | 7.61 |

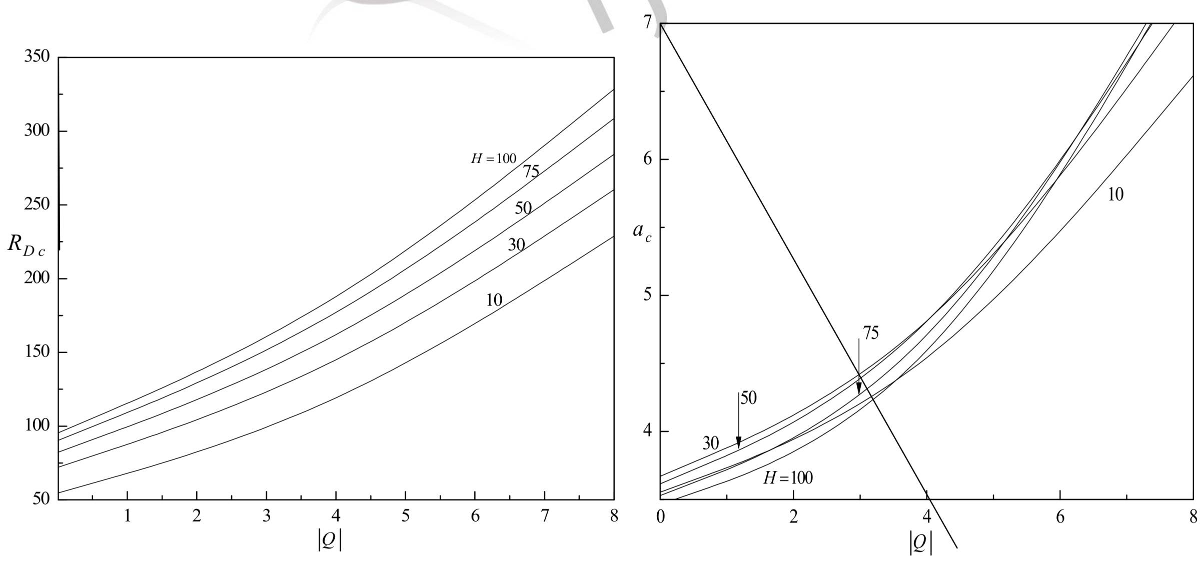


**Fig. 1(a).** Variation of $R_{Dc}$ with $Q$ for different values of $H$ when $G = 0.1$ and $\gamma = 0.5$.

**Fig. 1(b).** Variation of $a_c$ with $Q$ for different values of $H$ when $G = 0.1$ and $\gamma = 0.5$.

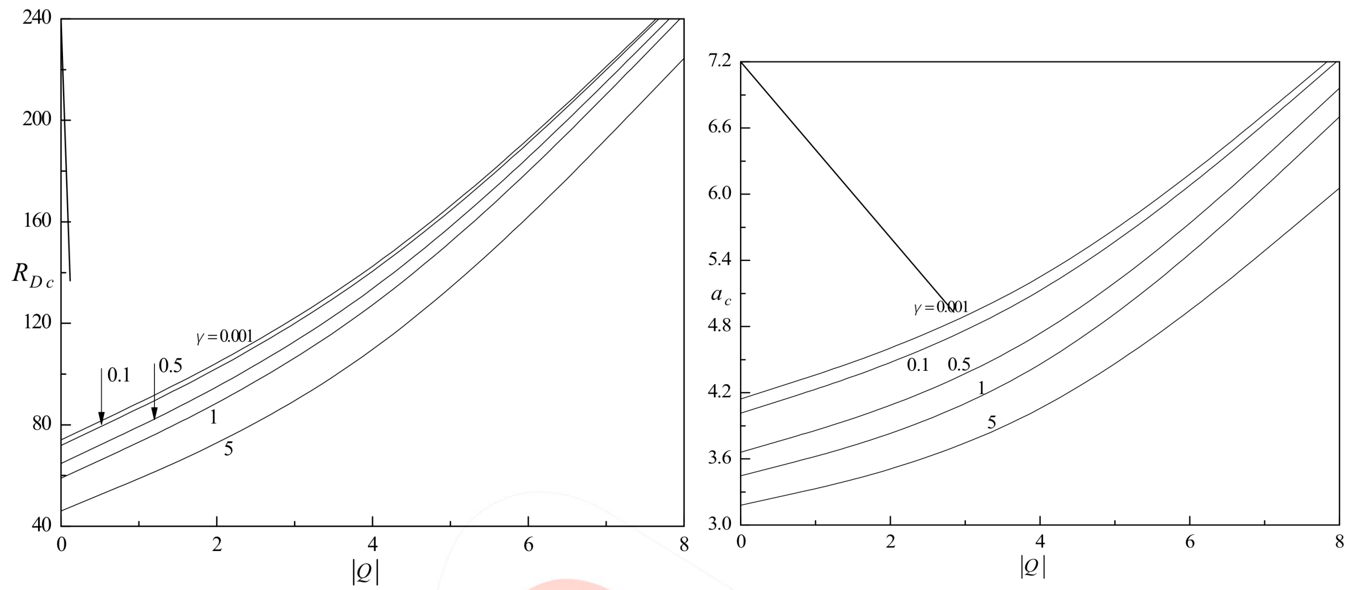


**Fig. 2(a).** Variation of $R_{Dc}$ with $Q$ for different values of $\gamma$ when $G = 0.1$ and $H = 20$.

**Fig. 2(b).** Variation of $a_c$ with $Q$ for different values of $\gamma$ when $G = 0.1$ and $H = 20$.

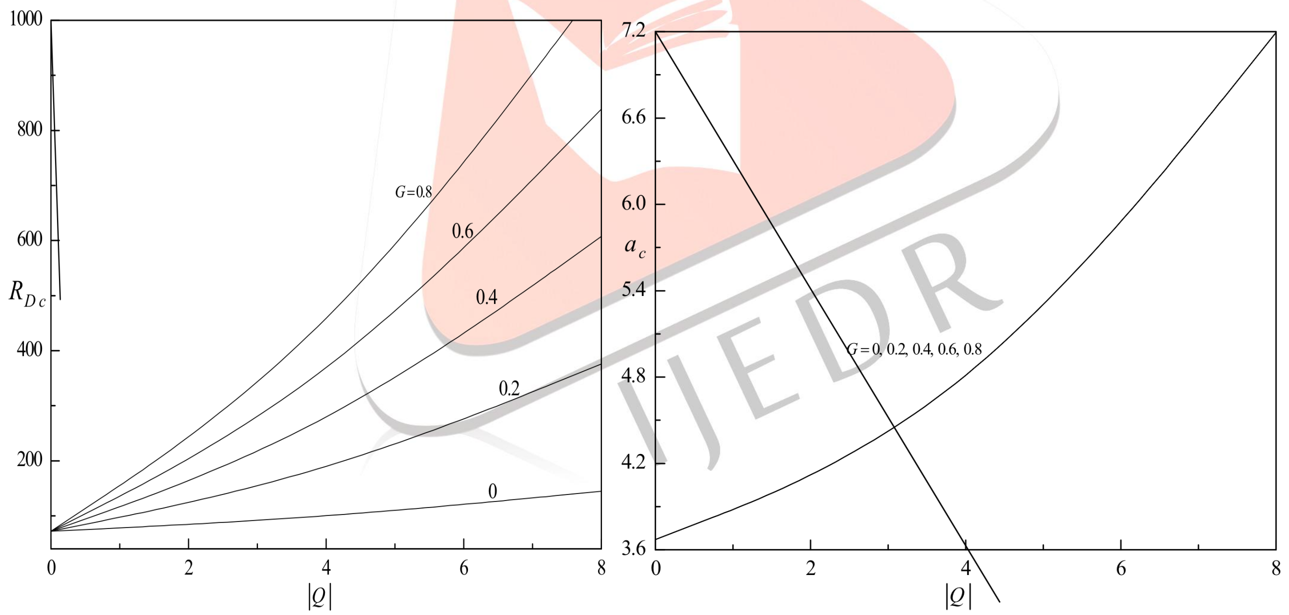


**Fig. 3(a).** Variation of $R_{Dc}$ with $Q$ for different values of $G$ when $\gamma = 0.5$ and $H = 30$.

**Fig. 3(b).** Variation of $a_c$ with $Q$ for different values of $G$ when $\gamma = 0.5$ and $H = 30$.

**References**

[1] N. Banu, D.A.S. Rees, Onset of Darcy–Bénard convection using a thermal non-equilibrium model, Int. J. Heat Mass Transfer 45 (2002), 2221–2228.

[2] M.S. Malashetty, I.S. Shivakumara, S. Kulkarni, The onset of Lapwood–Brinkman convection using a thermal non-equilibrium model, Int. J. Heat Mass Transfer 48 (2005) 1155–1163.

[3] B. Straughan, Global nonlinear stability in porous convection with a thermal nonequilibrium model, Proc. Roy. Soc. A 462 (2006) 409–418.

[4] D. L. Damm and A. G. Fedorov, Local thermal non-equilibrium effects in porous electrodes of the hydrogen-fueled SOFC, J. Power Sources, 159 (2006), 1153–1157. (doi:10.1016/j.jpowsour.2005.12.008).

[5] A. Nouri - Borujerdi, A.R. Noghrehabadi, D.A.S. Rees, Onset of convection in a horizontal porous channel with uniform heat generation using a thermal non-equilibrium model, Trans. Porous Media 69 (2007) 343–357.
[6] L. Virto, M. Carbonell, R. Castilla, P.J. Gamez-Montero, Heating of saturated porous media in practice: several causes of local thermal non-equilibrium, Int. J. Heat Mass Transfer 52 (2009) 5412– 5422.
[7] B. Straughan, non-equilibrium temperatures and with Cattaneo Porous convection with local thermal effects in the solid, Proc. Roy. Soc. A 469 (2013) 20130187.
[8] M.S. Malashetty, Mahantesh Swamy, Effect of rotation on the onset of thermal convection in a sparsely packed porous layer using a thermal non-equilibrium model, Int. J. Heat Mass Transfer 53 (2010) 3088–3101.
[9] I.S. Shivakumara, A.L. Mamatha, M. Ravisha, Effects of variable viscosity and density maximum on the onset of Darcy-Benard convection using a thermal non-equilibrium model, J. Porous Media 13 (2010) 613-622.
[10] J. Lee, I.S. Shivakumara, A.L. Mamatha, Effect of nonuniform temperature gradients on thermogravitational convection in a porous layer using a thermal nonequilibrium model, J. Porous Media 14 (2011) 659–669.
[11] I.S. Shivakumara, Jinho Lee , K. Vajravelu, A.L. Mamatha, Effects of thermal nonequilibrium and non-uniform temperature gradients on the onset of convection in a heterogeneous porous medium, Int. Comm. Heat Mass Transfer 38 (2011) 906–910.
[12] I.S. Shivakumara, Jinho Lee, A.L. Mamatha, M. Ravisha, Boundary and thermal non-equilibrium effects on convective instability in an anisotropic porous layer, J. Mech. Sci. Tech. 25 (4) (2011) 911-921.
[13] I. S. Shivakumara, Jinho Lee, M. Ravisha and R. Gangadhara Reddy, The effects of local thermal nonequilibrium and MFD viscosity on the onset of Brinkman ferroconvection, Meccanica 47(6) (2012), 1359-1378.
[14] A. Barletta and D. A. S. Rees, Local thermal non-equilibrium effects in the Darcy–Bénard instability with isoflux boundary conditions, Int. J. Heat Mass Transf. 55 (2012), 384–394.
[15] M. Celli, A. Barletta and L. Storesletten, Local thermal non-equilibrium effects in the Darcy–Bénard instability of a porous layer heated from below by a uniform flux, Int. J. Heat Mass Transf. 67 (2013), 902–912.
[16] A. Barletta, M. Celli and H. Lagziri, Instability of a horizontal porous layer with local thermal non-equilibrium: Effects of free surface and convective boundary Conditions, Int. J. Heat Mass Transf. 89 (2015), 75–89.
[17] B. Straughan, Convection with Local Thermal Nonequilibrium and Microfluidic Effects. Springer, Heidelberg (2015)
[18] Nield, D.A., Bejan, A.: Convection in Porous Media, 5th edn. Springer, New York (2017)
[19] M. Ravisha, I. S. Shivakumara and A. L. Mamatha, Cattaneo–LTNE porous ferroconvection, Multidiscs. Mod. Mat. Str. 15(4) **(2019),** 779-799.
[20] H. Lagziri and M. Bezzazi, Robin Boundary Effects in the Darcy–Rayleigh Problem with Local Thermal Non-equilibrium Model, Transp. Porous Med. https://doi.org/10.1007/s11242-019-01301-2 (2019)
[21] R. A. Wooding, Rayleigh instability of a thermal boundary layer in a flow through a porous medium at large Reynolds number or Peclet number, J. Fluid Mech. **9** (1960), 183–92.
[22] F. M. Sutton, Onset of convection in a porous channel with net throughflow, Phys. Fluids **13** (1970), 1931–4.
[23] G. M. Homsy and A. E. Sherwood, Convective instabilities in porous media with throughflow, AIChE J. **22** (1976), 168–174.
[24] M. C. Jones and J. M. Persichetti, Convective instability in a packed bed with throughflow, AIChE J. **32** (1986), 1555–7.
[25] D. A. Nield, Convective instabilities in porous media with throughflow, AIChE. J. **33** (1987), 1222–4.
[26] I. S. Shivakumara, Effects of throughflow on convection in porous media, Proc. 7th Asian Congr. Fluid Mechanics, 2 (1997), 557–60.
[27] I. S. Shivakumara and A. Khalili, On the stability of double diffusive convection in a porous layer with throughflow, *Acta Mech.* **152** (2001), 165–75.
[28] I. S. Shivakumara and C. E. Nanjundappa, Effects of quadratic drag and throughflow on double diffusive convection in a porous layer, Int. Commun. Heat Mass Transfer **33** (2006), 357–63.
[29] D.A. Nield, A.V. Kuznetsov, The onset of convection in a layered porous medium with vertical throughflow, Transp. Porous Media 98 (2013) 363-376.
[30] A. V. Kuznetsov and D. A. Nield, Local thermal non-equilibrium effects on the onset of convection in an internally heated layered porous medium with vertical throughflow, Int. J. Thermal Sci. 92 (2015), 97-105.
[31] Falin Chen, Throughflow effects on convective instability in superposed fluid and porous layers, J. Fluid Mech. 231 (1990), 113-133.