Analysis of a Partial Differential Equation and Real World Applications Regarding Water Flow in Baoding, Hebei Province, China.

1Dr. Isaac Devakumar
1AP Calculus Teacher
1Aiton High School

Abstract - **In this research article an "Exact solutions of a nonlinear diffusion-convention equation" a partial differential equation is analyzed. Moreover the characteristics and the evolution of time evolution phenomenon as the speed of the wave solution are adjusted. The equation is a governing equation for the flow of water under gravity through a homogeneous isotropic porous medium.**

keywords - **Partial Differential Equation**

Introduction

The partial differential equation for the given scenario has been expressed, which illustrates the flow of water under gravity through homogeneous isotropic porous medium, has been derived as follows:

$$\partial u/\partial t=\partial(u^n)/\partial x+\partial^2(u^m)/\partial x^2, \quad n\geq m>1$$

By simplifying the above equation let's begin our research. The simplified form of ordinary differential equation is as follows:

$(c + nU^{n-1} - 2\mu m U^{m-1} Y)\, dY/dY + (m-1)U^{m-2}\mu (1-Y^2)(dU/dY)^2 + m$ � �$^{m-1}\mu\, ((1-Y^2)d^2U/dY^2 = 0$

The aim of this research is to analyze the exact solutions for the equation. Eventually, the analysis of the exact solutions leads to a hypothesis involving water flow, which we apply to the real world problem of water flow in Baoding city, Hebei Province, China.

Baoding city water resource:

The Baoding city water resource is one of the primary sources of ground-water in China, and most of China's water systems utilize this ground water system. In total it covers over 500,000 square miles of land, and a total of ten- billion gallons of water is drawn up from it each day. The Baoding water supply system consists of three levels, namely: the upper confining unit, middle semi-confining unit, and lower semi-confining unit.

Water Scarcity: As water is a renewable resource, it needs to be dealt with much care. 97% of the earth contains the salt water whereas 3% only the pure water. Most of the pure water comes from rain. Water scarcity is a global issue, this research project we will be honing in on water scarcity in the Baoding city.

In 2020, the mean air temperature in China was 10.25°C, which was 0.7°C above normal (1981–2010 average), and the annual rainfall was 694.8 mm, which was 10.3% above normal. In general, disasters caused by rainstorms and flooding were more serious than those by drought. In summer, southern China experienced the most severe flooding with extreme heavy rainstorms since 1998. Drought brought slight impacts and losses in China. The seasonal transition from spring to summer was earlier than normal. High temperatures occurred earlier than normal with extreme values, and lasted longer than normal in summer over the south of China. The number of landfalling typhoons was lower than normal. Cold-air processes had a wide influence and brought a substantial decrease in air temperature in local areas. This report explores the characteristics of wave-like solutions of a partial differential equation and analyzes the behavior and time evolution of the phenomenon as the speed of the wave solution is adjusted.

Methods:

When we change the PDE to ODE, the new variable is:

$$Y = \tanh(\mu T)$$

where, $T = x - ct$

So, u(x, t) will becomes U(Ψ). By this change of variables we get ODE.

Power Series Method: The solution obtained by this method is as follows:

$$R_1 \qquad R_2$$

$(x, t) = U(\varphi) = S(Y) = \sum_{k=0} a_k Y^k + \sum_{l=1} b_l Y^{-l}$

Simplified form of the above equation is as follows:

$(x, t) = U(\varphi) = S(Y) = a_0 + a_1 + b_1/Y$

When these two equations are added, we get an approximation of u(x, t)

Now we substitute m = 2 and n = 3:

We get, $(c + 3S^2 - 4\mu Y S) dS/dY + 2(1-Y^2)(dS/dY)^2 + 2\mu(1-Y^2) S d^2S/dY^2 = 0$

Now let's solve for exact solution. This is the only solution that can be used for real applications, since it is bounded on the total real line.

The Problem Statement and the Solution:

Problem:

This research problem is to solve the PDE which models the flow of water under gravity through a homogeneous isotropic porous medium and to observe the physical changes of the traveling wave solutions.

Solution:

There are four solutions to the ODE, and we arrive at them using the tangent hyperbolic method, expressed in terms of tangent x or tangent hyperbolic x. We use the tangent hyperbolic solution, the reason being that it is the only solution of the four which is usable for real applications, to obtain significant physical solutions. The solution is as follows:

$(x, t) = \sqrt{-c}\,.\tanh((1/2)\sqrt{-c}\,.(x - ct)), c < 0$

The real solution is,

$\tanh x = (e^x - e^{-x})/(e^x + e^{-x}) \approx \lim_{x \to \infty} \to 1$

Because the other three solutions contain discontinuities.

Substituting the values for c and t:

Substituting the values of speed:

c = -0.01, -0.02, and -0.03 and

time: t = 0, 500, 1000, and 1500 seconds into solution.

When c = -0.01,

$(x, t) = \sqrt{(0.01)} \tanh(1/2 \sqrt{(0.01)} * (x + 0.01t)), c = -0.01$

$(x, t) = 0.1 \tanh((0.1/2)(x + 0.01t)), c = -0.01$

When c = -0.02,

$(x, t) = \sqrt{0.02} \tanh((1/2)\sqrt{0.02}(x + 0.02t)), c = -0.02$

When c = -0.03,

$(x, t) = \sqrt{0.03} \tanh((1/2)\sqrt{0.03}(x + 0.03t)), c = -0.03$

Conclusion

From the solution of the equation we conclude that, we infer that as the speed of the tangent hyperbolic wave-like solution increases the greater the amplitude becomes. From this research we hypothesize that in the real world problem, pertaining to Baoding water resource system, as the flow of water into the water tanks the amount of water will not decay linearly; rather, it is exponential decay. We suggest further investigation into this problem in the future because as water scarcity becomes an issue the need for sustaining the water level in the aquifer becomes extremely important.

References

[1] Vanaja, V. (2009). Exact solutions of a nonlinear diffusion-convection equation. *Physica Scripta, 80, 7pp.* doi: 10.1088/0031-8949/80/04/045402

[2] McCullough, S. The Floridan Aquifer. Retrieved from http://ww2.valdosta.edu/ tmanning/hon399/san dra.htm.

[3] https://www.sciencedirect.com/science/article/pii/S1674283421000209

[4] (2006). Water For Food -A Matter of Survival. *InfoResources Focus, 3/* . Retrieved fromhttp://www.inforesources.ch/pdf/focus063e.pdf.

[5] Maliet, W., & Hereman, W. (1996). The tanh method:I. exact solutions of nonlinear evolution and wave equations. *Phys. Scr. 54, 5638.*

[6] Analysis of a Partial Differential Equation and and Real World Applications Regarding Water Flow in the State of Florida Katelyn Merwin Embry-Riddle Aeronautical University, article-8.

[7] Sajjadi, S. G., & Smith, T. A. (2008). Applied Partial Differential Equations. Boston, MA: Huffington Mifflin Harcourt Publishing Company.