Optimal solution of fuzzy linear programming problems for trapezoidal number by using method of matrix inversion

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Abstract - **In this study we are going to discuss about on Fuzzy Linear Programming Problem under fuzzy environment with trapezoidal numbers. To obtain optimum solution of fuzzy linear programming problem we are applying matrix inversion method and discuss the results with numerical example and illustrate this method.**

keywords - **Fuzzy Linear Programming Problem (LPP), Matrix Inversion Method, Trapezoidal Number.**

INTRODUCTION

Many areas of engineering and management can benefit from linear programming (LP). Fuzzy numbers are often used to represent the parameters of LP in these applications due to the complexity of the real-world problems. Due to these reasons, fuzzy linear programming (FLP) has gained much attention from researchers.

Recently fuzzy set theory has been applied in many research regions, since fuzzy set theory is effective to solve the decision-making problems with imprecise data [1–3]. According to Delgado et al. [4], the parameters of constraints are fuzzy numbers, but the parameters of the objective function are crisp. Rommel fanger [5] has also proposed a general model for the FLP problems and the main difference compared with [4] is that here parameters of the objective function are also fuzzy numbers. Considering the different hypotheses. Some methods are based on the concepts of the superiority and inferiority of fuzzy numbers [6], the degrees of feasibility [7], the satisfaction degree of the constraints [9], and the statistical confidence interval [10]. Other kinds of methods are multi-objective optimization method [6], penalty method [11], and semi-infinite programming method [12]. Mahdavi-Amiri and Nasseri [13] develop a new dual algorithm for solving the FLP problem directly. Ganesan and Veeramani [14] propose a method for solving fuzzy linear programming problem (FLPP) without converting them to crisp LPP. Maleki et al. [15] propose a good method for solving an FLP problem, and an auxiliary problem is introduced in their model.

In recent years, several kinds of the fully fuzzy linear programming (FFLP) problems in which all the parameters and variables are represented by fuzzy numbers have appeared in the literature [16–20]. Some authors [16, 17] have discussed FFLP problems with crisp inequality constraints and obviously different methods for solving them have been proposed. In these methods the fuzzy optimal solutions of the FFLP problems are obtained by converting FFLP problem into crisp linear programming (CLP) problem. Lotfi et al. [18] can only obtain the approximate solution of the FFLP problems, but the method proposed by Kumar et al. [19] can find the fuzzy optimal solution which satisfies the constraints exactly. Guo and Shang [20] propose the computing model to the positive fully fuzzy linear matrix equation, and the fuzzy approximate solution is obtained by using pseudoinverse. However, in most of previous literatures, all constraints of FFLP problems have the crisp form.

Reference [21] examined the linear programming limitations and mathematical formulation are also explained. Some practical applications of linear programming are discussed. They provide insight on the latest applications of linear programming problem in various fields like sports, lean manufacturing, financial planning, and radiotherapy. Reference [22] analysis Fuzzy Linear Programming Problem with Triangular and Trapezoidal fuzzy numbers by using ranking method technique with α-Cut to get optimal solution for solving fuzzy linear programming problem with trapezoidal and triangular fuzzy numbers and compare the existing methods. Reference [23] explained the advantage of excel solver technique is easy to implementation and also get the faster and appropriate solution for large amount of data. Reference [24] examined that Linear Programming in an operation research for optimum best solution in real world problem using excel solver.

PRELIMINARIES

In this section some basic definitions and arithmetic operations are reviewed.

BASIC DEFINITIONS

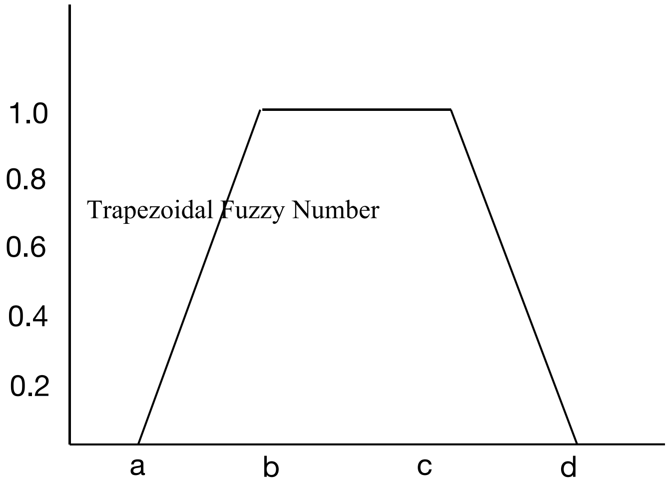
1. Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. A = {(x, μA(x); x ∈ X}, Here μA: X → [0,1] is a mapping called the degree of membership function of the fuzzy set A and μA(x) is called the membership value of xε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2. Trapezoidal Fuzzy Numbers: A fuzzy number $\tilde{A}$ is a trapezoidal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$. where a_1, a_2, a_3, a_4 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{a_4 - a_3} & \text{for } a_3 \le x \le a_4 \\ 0 & \text{for } x > a_4 \end{cases}$$

An alternative concise expression using min and max is:

Trapezoidal (x; a, b, c, d) = Max$\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$



3. Operations of Trapezoidal Fuzzy Numbers

The following are the four operations that can be performed on trapezoidal fuzzy numbers:

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then,

- Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- Multiplication: $\tilde{A} \times \tilde{B} = (t_1, t_2, t_3, t_4)$

Where
$$t_1 = \min(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$$
$$t_2 = \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$$
$$t_3 = \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$$
$$t_4 = \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$$

- Division: $\frac{\tilde{A}}{\tilde{B}} = \left(\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right), \min\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right),\right.$
$\left.\max\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \max\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right)\right)$

4. Ranking for Trapezoidal fuzzy number

If a number $\tilde{A}$ is a trapezoidal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$. where a_1, a_2, a_3, a_4 are real numbers and its membership-function $R(A)$ is given by

$$R(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

Fuzzy Linear Programming Problem

$$\left.\begin{array}{l} \text{Min or Max:} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \cdot x_{ij} \\ \text{Subject to constraint:} \\ \quad \sum_{j=1}^{n} \tilde{a}_{ij}x_j \ (\le\ , =\ , \ \ge)\ \tilde{b}_j \qquad i = 1, 2, \dots\dots\dots m \\ \quad x_j \ \ \ge \ \ 0 \qquad\qquad\qquad j = 1, 2, \dots\dots\dots n \\ \quad \text{Here,} \\ \quad c_j = \text{Objective Values} \\ x_j = \text{Contribution per units} \\ \tilde{a}_{ij} = \text{Input-output coefficient} \\ \tilde{b}_i = \text{Total availability of the ith resource} \end{array}\right\} \qquad (1)$$

Method of Matrix Inversion:

Step 1: Construct of LPP:

Min or Max $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} . x_{ij}$

Subject to Constraints

$$\sum_{j=1}^{n} \tilde{a}_{ij}x_j \ (\leq\ , =\ ,\ \geq)\ b_j\ ,\quad i = 1, 2, \ldots, m$$

$$x_j \quad \geq \quad 0, \quad j = 1, 2, \ldots, n$$

Step 2: Subject to constraints considered as system of linear equations:

$$\tilde{a}_{11}x_1 + \tilde{b}_{12}x_2 + \tilde{c}_{13}x_3 = \tilde{d}_1$$
$$\tilde{a}_{21}x_1 + \tilde{b}_{22}x_2 + \tilde{c}_{23}x_3 = \tilde{d}_2$$
$$\tilde{a}_{31}x_1 + \tilde{b}_{32}x_2 + \tilde{c}_{33}x_3 = \tilde{d}_3$$

Step 3: System to equation can be change into the matrix form

Coefficient Matrix $A = \begin{bmatrix} \tilde{a}_{11} & \tilde{b}_{12} & \tilde{c}_{12} \\ \tilde{a}_{21} & \tilde{b}_{22} & \tilde{c}_{22} \\ \tilde{a}_{31} & \tilde{b}_{32} & \tilde{c}_{33} \end{bmatrix}$ Constant Matrix $B = \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \end{bmatrix}$

Variable Matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Step 4: If Coefficient Matrix A is non-singular (i.e. $|A| \neq 0$) matrix than we proceed to further solution, otherwise stop here.

Step 5: Now coefficient matrix A change into inverse matrix i.e. $A^{-1} = \frac{AdjA}{|A|}$

Step 6: To find the value of basic variables, the system of equation can be written as i.e. $X = A^{-1}B$.

Step 7: To get optimum solution put obtained value of basic variables in objective functions of given numerical problem.

Numerical Problem:

$$\text{Min } Z = (3,6,9,12)x_1 + (30,35,40,45)x_2 + (22,23,24,25)x_3$$

Subject to Constraints

$$(3,5,7,9)x_1 + (4,8,12,16)x_2 + (4,7,9,12)x_3 \leq (54,74,94,114,)$$
$$(3,9,15,21)x_1 + (1,3,5,7)x_2 + (2,6,10,14)x_3 \leq (14,34,54,74)$$
$$(4,8,12,16)x_1 + (12,14,16,18)x_2 + (2,4,6,8)x_3 \leq (22,44,66,88)$$

Solution by method of matrix inversion:

Step 1: Fuzzy Linear Programming Problem, as given in numerical examples

$$\text{Min } Z = (3,6,9,12)x_1 + (30,35,40,45)x_2 + (22,23,24,25)x_3$$

Subject to Constraints

$$(3,5,7,9)x_1 + (4,8,12,16)x_2 + (4,7,9,12)x_3 \leq (3,6,9,12)$$
$$(1,3,5,7)x_1 + (2,4,6,8)x_2 + (2,6,10,14)x_3 \leq (4,8,12,16)$$
$$(1,3,5,7)x_1 + (2,4,9,14)x_2 + (2,4,6,8)x_3 \leq (7,8,9,10)$$

Step 2: Subject to constraints considered as system of linear equations:

$$(3,5,7,9)x_1 + (4,8,12,16)x_2 + (4,7,9,12)x_3 = (3,6,9,12)$$
$$(1,3,5,7)x_1 + (2,4,6,8)x_2 + (2,6,10,14)x_3 = (4,8,12,16)$$
$$(1,3,5,7)x_1 + (2,4,9,14)x_2 + (2,4,6,8)x_3 = (7,8,9,10)$$

Step 3: System to equation can be change into the matrix form

Coefficient Matrix $A = \begin{bmatrix} (3,5,7,9) & (4,8,12,16) & (4,7,9,12) \\ (1,3,5,7) & (2,4,6,8) & (2,6,10,14) \\ (1,3,5,7) & (2,4,9,14) & (2,4,6,8) \end{bmatrix}$

Constant Matrix $B = \begin{bmatrix} (3,6,9,12) \\ (4,8,12,16) \\ (7,8,9,10) \end{bmatrix}$ and Variable Matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Step 4: Now we are going to change coefficient matrix into determinate and we obtained that

$$|A| = \begin{vmatrix} (3,5,7,9) & (4,8,12,16) & (4,7,9,12) \\ (1,3,5,7) & (2,4,6,8) & (2,6,10,14) \\ (1,3,5,7) & (2,4,9,14) & (2,4,6,8) \end{vmatrix} = (-2362, -644, 837, 3228)$$

Here, we found that coefficient matrix A is non-singular matrix because of $|A| \neq 0$.

Step 5: Now we are going to change coefficient matrix in inverse matrix i.e.

$$A^{-1} = \begin{bmatrix} (-0.06, -0.09, 0.11, 0.08) & -(-0.05, -0.07, 0.08, 0.07) & (-0.09, -0.14, 0.11, 0.07) \\ -(-0.04, -0.05, 0.06, 0.04) & (-0.03, -0.03, 0.04, 0.03) & -(-0.05, -0.08, 0.06, 0.04) \\ (-0.04, -0.05, 0.04, 0.03) & -(-0.05, -0.06, 0.06, 0.04) & (-0.03, -0.05, 0.06, 0.04) \end{bmatrix}$$

Step 6: To find the basic variable, the system of equation can be written as i.e.

$$X = A^{-1}B$$

and obtained the basic variable as follows,

$$x_1 = (-2.74,\ -3.03,\ 2.82,\ 2.46); \quad x_2 = (-1.46,\ -1.53,\ 1.56,\ 1.86) \quad \&$$

$$x_3 = (-1.42,\ -1.62,\ 1.62,\ 1.56)$$

To Obtained Optimal Solution, we put the value of basic variable in object function and get $\text{Min } Z = (-134.08,\ -125.19,\ 125.22,\ 159.72)$

CONCLUSIONS

In this study Fuzzy Linear Programming with trapezoidal numbers solved by matrix inversion method. In this investigation we found that method of matrix inversion gives the optimum result. Here we also found that method of matrix inversion easy to understand as comparative to exist methods of LPP. We also observe that FLPP is most effective modal to take decision manager in different type of unpredictable issues.

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