Comparison of Solution to the Fuzzy Transportation Problem using Various Methods

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Abstract - **This paper proposes a new method for ranking of fuzzy numbers. Ranking of fuzzy numbers play a vital role in decision making problems, data analysis, socio economics systems, optimization, forecasting etc. Ranking fuzzy numbers is a necessary step in many mathematical models. Many of the methods proposed so far are non-discriminating. This paper presents a new ranking method which converts the fuzzy transportation problem to a crisp valued transportation problem which then can be solved using MODI Method to find the fuzzy optimal solution. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.**

keywords - **Trapezoidal fuzzy numbers, Ranking function, Fuzzy Transportation Problem.**

1. INTRODUCTION:

Transportation problem is used globally in solving certain concrete world problems. A transportation problem plays a vital role in production industry and logistics and supply chain management for reducing cost and time for better service. The transportation problem is a special case of Linear programming problem, which permit us to regulate the optimum shipping patterns between origins and destinations. The solution of the problem will empower us to determine the number of entities to be transported from a particular origin to a particular destination so that the cost obtained is minimum or the time taken is minimum or the profit obtained is maximum. A fuzzy transportation problem is a transportation problem in which the transportation expenditures, supply and demand quantities are fuzzy quantities. Ranking fuzzy number is a necessary step in many mathematical models. The concepts of fuzzy sets were first introduced by Zadeh [1]. Since its inception several ranking procedure have been developed. There onwards many authors presented various approaches for solving the FTP problems [2], [4], [5], [17], [18]. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani [3]. Ranking normal fuzzy number were first introduced by Jain [7] for decision making in fuzzy situations. Chan stated that in many situations it is not possible to restrict the membership function to the general form and proposed the concept of generalized fuzzy numbers. The development in ordering fuzzy numbers can even be found in [6], [7], [8], [9] .To illustrate this proposed method, an example is discussed. As the proposed ranking method is very direct and simple it is very easy to understand and using which it is easy to find the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations. This paper is organized as follows: Section 2, briefly introduced the basic definition of fuzzy numbers. In section 3, a new ranking procedure is proposed. In section 4, MODI method is adopted to solve Fuzzy transportation problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. PRELIMINARIES: In this section we define some basic definitions which will be used in this paper.

2.1 Definition:

If x is a set of objects denoted generally by X, then a fuzzy set A in X is defined as a set of ordered pairs A= {(x, μA(x)/ x Є X}, where μA(x) is called the membership function for the fuzzy set A. The membership function maps each element of X to a membership value between 0 and 1.

2.2 Definition:

A fuzzy set A is defined on universal set of real numbers is said to be a generalized fuzzy number if its membership function has the following characteristics-

(i) $\mu A(x) : R \to [0, 1]$ is continuous
(ii) $\mu A(x) = 0$ for all x Є A $(-\infty, a] \cup [d, \infty)$
(iii) μA(x) is strictly increasing on [a, b] and strictly decreasing on [c, d]
(iv) $\mu A(x) = \omega$ for all x Є [b, c], where $0 < \omega \leq 1$

2.3 Definition:

A generalized fuzzy number A= (a, b, c, d, ω) is said to be a generalized trapezoidal fuzzy number if its membership function is given by,

$$\mu_A(x) = \begin{cases} \frac{w(x-a)}{b-a} & a \le x \le b \\ \omega & b \le x \le c \\ \frac{w(x-a)}{d-c} & c \le x \le d \\ o & \text{otherwise} \end{cases}$$

If ω =1, then A = (a, b, c, d; 1) is a normalized trapezoidal fuzzy number and A is a generalized or non-normal trapezoidal fuzzy number if 0 < ω < 1.

As a particular case if b=c, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by A= (a, b, d; 1)

2.4 Definition:

Let A1= (a1, b1, c1, d1; ω1) and A2= (a2, b2, c2, d2; ω2) be generalized trapezoidal fuzzy number defined on real numbers R then,

A1 + A2 = (a1+a2, b1+b2, c1+c2, d1+d2; min (ω1, ω2))

A1 - A2 = (a1-d2, b1-c2, c1-b2, d1-a2; min (ω1, ω2))

2.5 Definition:

Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

The transportation problem applies to situations where a single commodity is to be transported from various sources of supply (origins) to various demands (destinations). Let there be m sources of supply s_1, s_2, …..............s_m having ai (i = 1,2,......m) units of supplies respectively to be transported among n destinations d_1, d_2………d_n with b_j (j = 1,2…..n) units of requirements respectively.

Let c_{ij} be the cost for shipping one unit of the commodity from source i, to destination j for each route. If xij represents the units shipped per route from source i, to destination j, then the problem is to determine the transportation schedule which minimizes the total transportation cost of satisfying supply and demand conditions.

The transportation problem can be stated mathematically as a linear programming problem as below:

Minimize Z $=\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$

$\sum_{i=1}^{m} x_{ij} \le a_i$ for i = 1, 2, . . . , m

$\sum_{j=1}^{n} x_{ij} \ge b_j$ for j = 1, 2, . . . , n (2.4)

xij ≥ 0 for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

This is a linear program with m × n decision variables, m + n functional constraints, and m × n non-negativity constraints.

3. RANKING OF TRAPEZOIDAL FUZZY NUMBERS: In this section, a new approach for ranking of generalized trapezoidal number is proposed using trapezoid as reference point. Ranking methods map fuzzy number directly in to the real line. That is, M: F→ R which associate every fuzzy number with a real number and then use the ordering ≥ on the real line.

Let A= (a, b, c, d; ω) be generalized trapezoidal fuzzy numbers then R(A) is calculated as follows:

$$R(A) = \frac{d(b-a) + a(d-c)}{b-a+d-c}$$

4. STEPS TO SOLVE THE FUZZY TRANSPORTATION PROBLEM:

To obtain the fuzzy optimal solution for a four index fully fuzzy transportation problem, we go through two phases:

I. Determining an initial basic feasible solution.

There are three popular methods used to determine an initial feasible solution to the classical transportation problem. They are

I.I: North West Corner Method-

Step 1: Start with the cell at the upper left (north-west) corner of the transportation table (or matrix) and allocate commodity equal to the minimum of the rim values for the first row and first column, i.e. min (a1, b1).

Step 2: (a) If allocation made in Step 1 is equal to the supply available at first source (a1, in first row), then move vertically down to the cell (2, 1), i.e., second row and first column. Apply Step 1 again, for next allocation.

(b) If allocation made in Step 1 is equal to the demand of the first destination (b1 in first column), then move horizontally to the cell (1, 2), i.e., first row and second column. Apply Step 1 again for next allocation.

(c) If a1 = b1, allocate x11 = a1 or b1 and move diagonally to the cell (2, 2).

Step 3: Continue the procedure step by step till an allocation is made in the south-east corner cell of the transportation table

I.II: Least Cost Method-

Step 1: Select the cell with the lowest unit cost in the entire transportation table and allocate as much as possible to this cell. Then eliminate (line out) that row or column in which either the supply or demand is fulfilled. If a row and a column are both satisfied simultaneously, then crossed off either a row or a column. In case the smallest unit cost cell is not unique, then select the cell where the maximum allocation can be made.

Step 2: After adjusting the supply and demand for all uncrossed rows and columns repeat the procedure to select a cell with the next lowest unit cost among the remaining rows and columns of the transportation table and allocate as much as possible to this cell. Then crossed off that row and column in which either supply or demand is exhausted.

Step 3: Repeat the procedure until the available supply at various sources and demand at various destinations is satisfied. The solution so obtained need not be non-degenerate.

I.III: Vogel's approximation methods-

Step 1: Calculate the penalties for each row (column) by taking the difference between the smallest and next smallest unit transportation cost in the same row (column). This difference indicates the penalty or extra cost that has to be paid if decision-maker fails to allocate to the cell with the minimum unit transportation cost.

Step 2: Select the row or column with the largest penalty and allocate as much as possible in the cell that has the least cost in the selected row or column and satisfies the rim conditions. If there is a tie in the values of penalties, it can be broken by selecting the cell where the maximum allocation can be made.

Step 3: Adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of them is crossed out and the remaining row (column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step 4: Repeat Steps 1 to 3 until the available supply at various sources and demand at various destinations is satisfied.

II. Improving a basic feasible solution.

The solution obtained if not optimal then it can be further improved using MODI METHOD (UV-METHOD)-

The steps to evaluate unoccupied cells are as follows:

Step 1: For an initial basic feasible solution with m + n – 1 occupied cells, calculate ui and vj for rows and columns. The initial solution can be obtained by any of the three methods discussed earlier. To start with, any one of ui s or vj s is assigned the value zero. It is better to assign zero to a particular ui or vj where there are maximum number of allocations in a row or column respectively, as this will reduce the considerably arithmetic work. The value of ui s and vj s for other rows and columns is calculated by using the relationship. cij = ui + vj , for all occupied cells (i, j).

Step 2: For unoccupied cells, calculate the opportunity cost by using the relationship dij = cij – (ui + vj) , for all i and j.

Step 3: Examine sign of each dij (i) If dij > 0, then the current basic feasible solution is optimal. (ii) If dij = 0, then the current basic feasible solution will remain unaffected but an alternative solution exists. (iii) If one or more dij < 0, then an improved solution can be obtained by entering an unoccupied cell (i, j) into the solution mix (basis). An unoccupied cell having the largest negative value of dij is chosen for entering into the solution mix (new transportation schedule).

Step 4: Construct a closed-path (or loop) for the unoccupied cell with largest negative value of dij. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell. Trace a path along the rows (or columns) to an occupied cell, mark the corner with a minus sign (–) and continue down the column (or row) to an occupied cell. Then mark the corner with plus sign (+) and minus sign (–) alternatively. Close the path back to the selected unoccupied cell.

Step 5: Select the smallest quantity amongst the cells marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell, add it to occupied cells marked with plus signs, and subtract it from the occupied cells marked with minus signs.

Step 6: Obtain a new improved solution by allocating units to the unoccupied cell according to Step 5 and calculate the new total transportation cost.

Step 7: Test optimality of the revised solution. The procedure terminates when all dij ≥ 0 for unoccupied cells.

5. NUMERICAL EXAMPLE:

A company has four sources S1, S2, S3and S4 and four destinations D1, D2, D3 and D4;the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is c_{ij} ,where

$$[c_{ij}]=\begin{pmatrix} (1,2,3,4) & (1,3,4,6) & (9,11,12,14) & (5,7,8,11) \\ (0,1,2,4) & (-1,0,1,2) & (5,6,7,8) & (0,1,2,3) \\ (3,5,6,8,) & (5,8,9,12) & (12,15,16,19) & (7,9,10,12) \end{pmatrix}$$

And fuzzy availability of the product at source are ((1,6,7,12), (0,1,2,3), (5,10,12,17),) and the fuzzy demand of the product at destinations are ((5,7,8,10), (1,5,6,10), (1,3,4,6) (1,2,3,4)) respectively. Then the problem becomes,

Table1: Fuzzy Transportation Problem

	FD1	FD2	FD3	FD4	SUPPLY
FS1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
FS2	(0,1,2,4)	(−1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
FS3	(3,5,6,8,)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
DEMAND	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

By using definition, Fuzzy Transportation Problem is balanced i.e. Sum of supply = Sum of demand

Step 1: By using our new approach given in SECTION 3 the fuzzy transportation problem is changed in to a crisp transportation problem as in table 2

Table2: Crisp Transportation Problem

	FD1	FD2	FD3	FD4	SUPPLY
FS1	2.5	3.5	11.5	7.4	6.5
FS2	1.33	0.5	6.5	1.5	1.5

FS3	5.5	8.6	15.5	9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Step 2:
. Using VAM method we obtain the initial solution as –

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{1}$ 2.5	$\boxed{5.5}$ 3.5	11.5	7.4	6.5
FS2	1.33	0.5	6.5	$\boxed{1.5}$ 1.5	1.5
FS3	$\boxed{6.5}$ 5.5	8.6	$\boxed{3.5}$ 15.5	$\boxed{1}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Hence, the IBFS is,
1×2.5+5.5×3.5+1.5×1.5+6.5×5.5+3.5×15.5+1×9.5=123.5
The IBFS for the fuzzy transportation problem obtained above is the same as compared to the IBFS obtained by Shugani Poonam [18], P. Pandian and G. Natarajan [16].
.Using Least Cost Method we obtain the initial solution as –

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	1.33	$\boxed{1.5}$ 0.5	6.5	1.5	1.5
FS3	$\boxed{1.0}$ 5.5	$\boxed{4.0}$ 8.6	$\boxed{3.5}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Hence, the IBFS is,

$$6.5 \times 2.5 + 1.5 \times 0.5 + 1.0 \times 5.5 + 4 \times\ 8.6 + 3.5 \times 15.5 + 2.5 \times 9.5 = 134.9$$

. Using North West Corner Method we obtain the initial solution as –

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	$\boxed{1.0}$ 6.5	$\boxed{0.5}$ 0.5	6.5	1.5	1.5
FS3	5.5	$\boxed{5.0}$ 8.6	$\boxed{3.5}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Hence, the IBFS is,

$$6.5 \times 2.5 + 1.0 \times 6.5 + 0.5 \times 0.5 + 5.0 \times\ 8.6 + 3.5 \times 15.5 + 2.5 \times 9.5 = 159$$

Step 3:

- Hence by using the MODI method for IBFS of VAM method we shall get the optimal solution as

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{1}$ 2.5	$\boxed{5.5}$ 3.5	11.5	7.4	6.5
FS2	1.33	0.5	$\boxed{1.5}$ 6.5	1.5	1.5
FS3	$\boxed{6.5}$ 5.5	8.6	$\boxed{2}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY

FS1	2.5	$\boxed{5.5}$ 3.5	$\boxed{1}$ 11.5	7.4	6.5
FS2	1.33	0.5	$\boxed{1.5}$ 6.5	1.5	1.5
FS3	$\boxed{7.5}$ 5.5	8.6	$\boxed{1}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is an optimal solution .
Hence the optimum cost is -
3.5×5.5+1×11.5+5.5×7.5+15.5×1+1.5×6.5+9.5×2.5=121

- Similarly by using the MODI method for IBFS of North West Corner Method

we shall get the optimal solution as

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	$\boxed{1.0}$ 6.5	$\boxed{0.5}$ 0.5	6.5	1.5	1.5
FS3	5.5	$\boxed{5.0}$ 8.6	$\boxed{3.5}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	6.5	$\boxed{1.5}$ 0.5	6.5	1.5	1.5
FS3	$\boxed{1.0}$ 5.5	$\boxed{4.0}$ 8.6	$\boxed{3.5}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	6.5	0.5	$\boxed{1.5}$ 6.5	1.5	1.5
FS3	$\boxed{1.0}$ 5.5	$\boxed{5.5}$ 8.6	$\boxed{2.0}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{6.5}$ 2.5	3.5	11.5	7.4	6.5
FS2	6.5	0.5	6.5	$\boxed{1.5}$ 1.5	1.5
FS3	$\boxed{1.0}$ 5.5	$\boxed{5.5}$ 8.6	$\boxed{3.5}$ 15.5	$\boxed{1.0}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is an optimal solution .
Hence the optimum cost is -

$$6.5 \times 2.5 + 5.5 \times 1.0 + 1.5 \times 6.5 + 8.6 \times\ 5.5 + 2.0 \times 15.5 + 2.5 \times 9.5 = 133.55$$

- Similarly by using the MODI method for IBFS of Least Cost Method we shall get the optimal solution as

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{2.5}$ 2.5	$\boxed{4.0}$ 3.5	11.5	7.4	6.5
FS2	1.33	$\boxed{1.5}$ 0.5	6.5	1.5	1.5
FS3	$\boxed{5.0}$ 5.5	8.6	$\boxed{3.5}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is also not an optimal solution hence applying one more iteration of MODI METHOD (UV-METHOD)-

	FD1	FD2	FD3	FD4	SUPPLY
FS1	$\boxed{1.0}$ 2.5	$\boxed{5.5}$ 3.5	11.5	7.4	6.5
FS2	1.33	0.5	$\boxed{1.5}$ 6.5	1.5	1.5
FS3	$\boxed{6.5}$ 5.5	8.6	$\boxed{2.0}$ 15.5	$\boxed{2.5}$ 9.5	11
DEMAND	7.5	5.5	3.5	2.5	

Which is an optimal solution .
Hence the optimum cost is -

$$1.0 \times 2.5 + 5.5 \times 3.5 + 1.5 \times 6.5 + 6.5 \times\ 5.5 + 2.0 \times 15.5 + 2.5 \times 9.5 = 122$$

CONCLUSION

Using North West corner method gives solution in less iterations but yields a bad solution because it is very far from optimal solution. Vogel's approximation method and Least cost method is used to obtain the shortest road. Advantage of Vogel's approximation method and Least cost method yields the best initial basic feasible solution as these methods gives initial solution very near to optimal solution but the solution of Vogel's approximation methods is slow because computations take long time. The cost of transportation with Vogel's approximation method and Least cost method is less than North-West corner method. The result in three methods are different.

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers hence it is not possible to mention which fuzzy ranking method is the best. This paper proposed a new ranking method which is simple and efficient. Most of the time choosing a method rather than another is a matter of preference. The ranking of trapezoidal fuzzy numbers done by the Shugani Poonam [18], P. Pandian and G. Natarajan[16] gives the same values for symmetric fuzzy numbers while it changes slightly for non-symmetric fuzzy numbers but maintains the order of the fuzzy numbers.

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