

Deterministic and Probabilistic models in Inventory Control

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Abstract - Inventory is essential to provide flexibility in operating a system or organization. An inventory can be classified into raw material inventory, work in process inventory and finished goods inventory. The raw material inventory removes dependency between various machines of a product line. The finished goods inventory removes dependency between plants and its customers or markets. The main functions of an inventory can be classified into smoothing out irregularities in supply, minimizing the production cost and allowing organizations to cope up with perishable materials. In an industry it is always necessary to keep the inventory optimal by minimizing the cost of ordering and handling. This can be done by adopting some inventory control models. The applications of them vary from industry to industry.

Index Terms - Purchase cost, Ordering Cost, Carrying cost, Shortage cost, Order size, Annual Demand, Cycle time, Economic Order Quantity, Economic Batch Quantity, Stock out, Single period

I. INTRODUCTION

Inventory can be defined as stock of goods kept in a warehouse for future sale or using it in common day to day activities they may include raw materials, work in process goods, finished goods, packing material and general supplies. In order to meet the time, companies must keep on hand a stock of goods that is awaiting sale. The purpose of inventory theory is to determine rules that management can use to minimize the cost associated with maintaining inventory and meeting customer demand. Inventory control is the supervision of supply, storage and accessibility of items in order to ensure an adequate supply without excessive over supply. It can be referred to as internal control. It is always good to maintain an optimal inventory. There are two basic inventory questions generally taken by managers those are when to replenish the inventory of an item? And how much of an item to order when the inventory of that item is to be replenished? If one places frequent orders, the cost of ordering will be more, but the inventory carrying cost will be less. On the other hand if one places less frequent orders the ordering cost will be less but the carrying cost will be more.

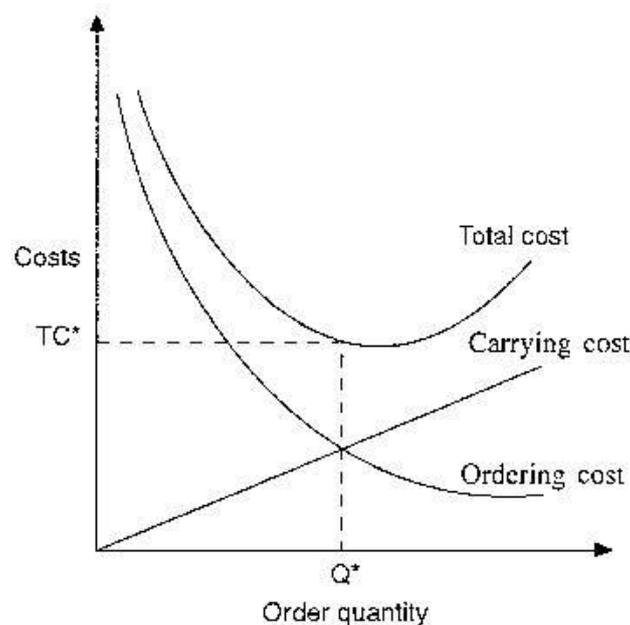


Figure 1

In Figure 1 for an increase in order size (Q), the carrying cost increases and the ordering cost decreases. The total cost curve represents the sum of ordering cost and carrying cost for each order size. The order size, at which the total cost is minimum, is called economic order quantity (EOQ) or optimal order size.

II. DIFFERENT MODELS FOR INVENTORY CONTROL

The classic inventory model is generally used either to forecast optimum inventory or to evaluate two or more inventory systems. Two fundamental techniques are generally employed by industries to develop inventory reserve estimates and they are the deterministic and probabilistic methods. The deterministic method concedes a single best estimation of inventory reserves grounded on recognized engineering, geological, and economic information. The probabilistic method employs the known economic, geological and engineering data to produce a collection of approximate stock reserve quantities and their related probabilities. Each inventory reserve categorization gives a signal of the prospect of revival. The advantage of a probabilistic approach lies in the fact that by using values lying within a bandwidth and modelled by a defined distribution density, the reality can be modelled better than by using deterministic figures.

Deterministic models of inventory control are used to determine the optimal inventory of a single item when demand is mostly largely obscure. Under this model inventory is built up at a constant rate to meet a determined, or accepted, demand. For instance a contract is received in January for 100 model trains and the delivery to be completed by November/holiday shopping. Since the deadline is 10 months so the trains can be produced at a rate of ten per month.

III. DETERMINISTIC MODELS

Method based on the assumption that all parameters and variable associated with an inventory are known or can be computed with certainty, and that the replenishment lead time is constant and independent of the demand. The various deterministic models taken into account are:

1. Purchase model with instantaneous replenishment and without shortages
2. Manufacturing model with shortages
3. Purchasing model with instantaneous replenishment and with shortages
4. Purchase model with shortages

Firstly Let us assume some variables as follows:

r = Annual demand in units

k = Production rate of the item

C_o = Cost per set up

C_c = Carrying cost per unit per year

C_s = Shortage cost

Q = Order size

Q_1 = Maximum inventory

Q_2 = Maximum stock out

P = Cost of production per unit

t = total cycle time

t_1 = Period of production as well as consumption

t_2 = Period of consumption only

t_3 = Period of shortage

t_4 = Period of production as well as consumption of the item satisfying back order

Purchase model with instantaneous replenishment and without shortages

In, the case of purchase model with instantaneous replenishment and without shortages the orders of equal sizes are placed at periodical intervals. The items against an order are replenished instantaneously and the items are consumed at constant rate. The purchase price per unit is same irrespective of the order size. The purchase model can be represented as shown in the figure 2

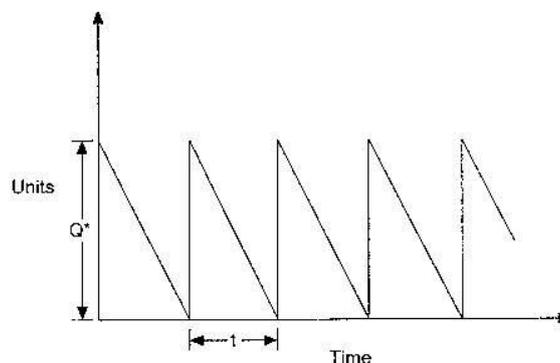


Figure 2

From Figure 2 the following equations can be inferred:

- The number of orders per year = Annual Demand/ Order Size
- Average inventory = Order size/2
- Cost of ordering per year = Annual demand in units/ Order size x Ordering cost
- Cost of carrying per year = Order size/ 2 x Carrying cost per unit per year

With the help of the above equations the economic order quantity (EOQ) can also be inferred as:

$$EOQ = \sqrt{2CoD}/Cc \quad (1)$$

Manufacturing model without shortages

Suppose we take into account a company which manufactures an item which is required for its main product, then the corresponding model of inventory is termed as manufacturing model. In this model, shortages are not permitted. The rate of consumption of the item is assumed to be uniform throughout the year. The item is produced and consumed simultaneously for a portion of cycle time. During the remaining cycle time, only the consumption of the item takes place and the cost of production per unit is same irrespective of production lot size. The operation of the manufacturing model without shortages is shown in figure 3

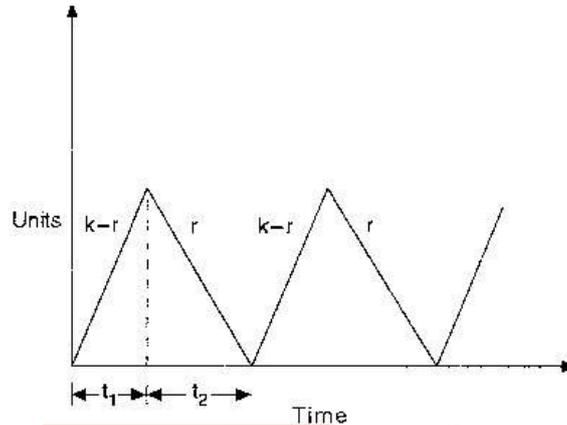


Figure 3

During the period t1, the item is produced at the rate of k units per period and simultaneously it is consumed at the rate of r units per period. During this period, the inventory is built at the rate of k-r units per period. During the period t2, the production of the item is discontinued but the consumption of the same item is continued. Hence, the inventory is decreased at the rate of r units per period during the time t2. Therefore the Economic batch quantity can be given as

$$EBQ = \sqrt{2Co r - Cc [1 - (\frac{r}{k})]} \quad (2)$$

Purchase model with Instantaneous replenishment and with shortages

While, looking at the purchase models with instantaneous replenishment and with shortages, an item on order will be received instantaneously and it is consumed at a constant rate. The purchase price per unit is same irrespective of order size. If there is no stock at the time of receiving a request for the item, it is assumed that it will be satisfied at a later date with a penalty known as backordering. The model is shown in Figure 4



Figure 4

The economic batch quantity for this model can be determined as

$$EBQ = \sqrt{(2CoD)/Cc Cs + Cc/Cs} \quad (3)$$

Manufacturing model with Shortages

In a manufacturing model with shortages an item is produced and consumed simultaneously for a portion of cycle time. During the remaining cycle time, only the consumption of the item only takes place. The cost of production per unit is the same irrespective of the production lot. Stock out is permitted in this model and it is assumed that the stock out units which will be produced at a later date. The operation of the model is shown in Figure 5

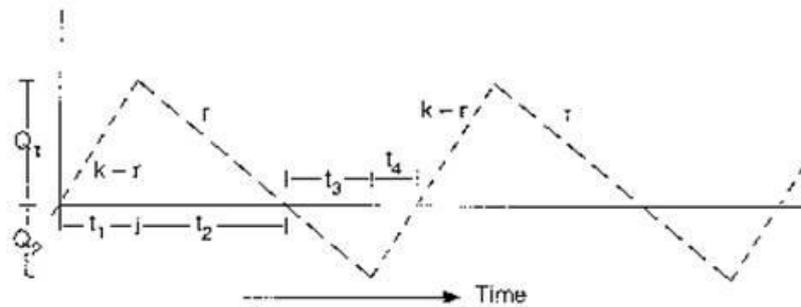


Figure 5

The Economic batch quantity for this model can be given as:

$$EBQ = (\sqrt{2Co/Cc}) * (kr)/k - r * (Cc + Cs)/Cs \quad (4)$$

IV. PROBABILISTIC MODELS

In general it is impossible to determine the demand beforehand so it is difficult to maintain the inventory according to the deterministic model. In general cases, the demand is not constant and deterministic, but probabilistic instead. This type of demand is best described by the probability distribution. The types of models which come under this section can be grouped into 4 types:

1. Single period inventory model with probabilistic demand
2. An order quantity with probabilistic demand
3. A periodic review model with probabilistic demand

Single-period inventory model with probabilistic demand

In a single period inventory model with probabilistic demand, it is first and foremost necessary clarify the term single period. This term refers to the situation where the inventory will only be demanded in one time duration, and cannot be transferred to the next time duration. Newspaper selling and fashion are such examples. Increment analysis is a method that can be used to determine the optimal order quantity for a single-period inventory model. The increment analysis addresses the quantity to be ordered by comparing the cost or loss of ordering a additional unit with the cost or loss of not ordering another additional unit. Let

C_o = cost per unit of overestimating demand
 C_u = cost per unit of underestimating demand

Suppose that the probability of the demand of the inventory items being more than a certain level y is $P(D > z)$, and that the probability of the demand of the inventory items being less than or equal to this level z is $P(D \leq z)$. Then, the expected loss (EL) will be either of the following:

$$\text{For overestimation: } EL(y+1) = C_o P(D \leq z) \quad (5)$$

$$\text{For underestimation: } EL(y) = C_u P(D > z) \quad (6)$$

The optimal value of the demand level, z , being the optimal ordering quantity as well, can be found when

$$EL(z+1) = EL(z) \quad (7)$$

The above expression provides the general condition for the optimal order quantity z in the single-period inventory model. The determination of z depends on the probability distribution. In an order-quantity, reorder-point inventory model with probabilistic demand.

An order-quantity, reorder-point inventory model with probabilistic demand

Multi-period model have the following characteristics:

1. The inventory system operates continuously with many repeating periods or cycles;
2. Inventory can be carried from one period to the next;
3. An order is placed whenever the inventory position reaches the reorder point;
4. Since demand is probabilistic, the following cannot be determined in advance:
 - The time the reorder point will be reached;
 - The time between orders;
 - The lead time.

The inventory pattern can be described by the Figure 6.

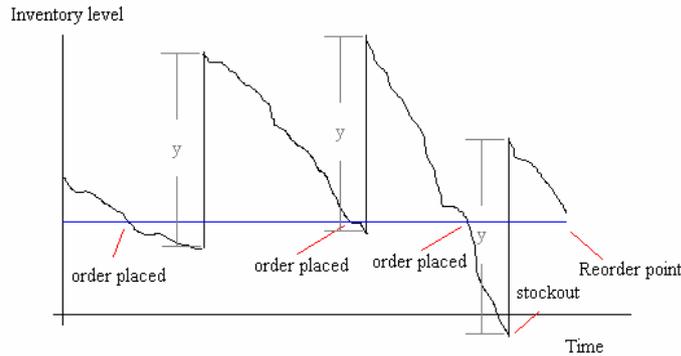


Figure 6

Although we are in a probabilistic demand situation, we can apply the EOQ model as an approximation of the best order quantity. That is

$$y = \sqrt{2KD/h} \tag{8}$$

Where, in this case, D is the expected annual demand

If the expected (or mean, average) demand is μ per unit time, and the standard deviation is σ , then the reorder point r can be expressed as

$$r = \mu + z\sigma \tag{9}$$

Where z is the number of standard deviation necessary to obtain the stock out probability, and it can be find from the standard normal probability distribution table according to the tolerance of stock out probability.

A periodic-review model with probabilistic demand

In a periodic-review model with probabilistic demand the inventory model discussed in 4.2 is a continuous-review model system, where the inventory position is monitored continuously so that an order can be placed whenever the reorder point is reached. We can use a computerized system to perform this task. However, if a company handles multiple products, continuous-review on each of the products may mean heavy work-load and probably low efficiency. In such cases, an alternative inventory model, the periodic review model, is preferred, because this model enables the orders for several items to be placed at the same preset periodic-review time. In this model, we assume that for any single product, the lead time is less than the length of the review period. Then the the question of ordering the quantity at any review period is determined using the following:

$$y = M - H \tag{10}$$

- Where, y = the order quantity;
- M = the replenishment (or the maximum) level;
- H = the inventory on hand at the review period.

Since the demand is probabilistic, the inventory on hand, H, will vary. Thus, the order quantity that must be sufficient to bring the inventory position back to its maximum or replenishment level M can be expected to vary each period. Under the periodic-review model, enough units are ordered each review period to bring the inventory position back to the replenishment level. A typical inventory pattern for a periodic-review system with probabilistic demand is illustrated in Figure 7. Note that the time between periodic reviews is predetermined and fixed.

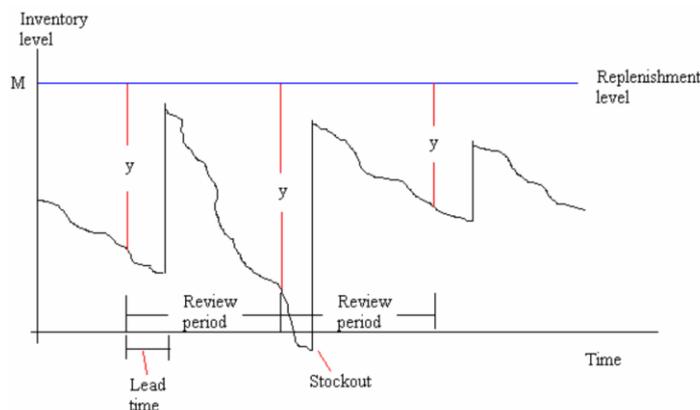


Figure 7

The decision variable in the periodic-review model is the replenishment level M . To determine M , we could begin by developing a total-cost model, including holding, ordering, and stock out (shortage) costs. Instead, we will describe an approach that is often used in practice. In this approach, the objective is to determine a replenishment level that will meet a desired performance level, such as a reasonably low probability of stock out or a reasonably low number of stock outs per year. In the periodic-review model, the order quantity at each review period must be sufficient to cover demand for the review period plus the demand for the following lead time. If during the review period plus the lead-time period the demand can be expressed by the normal probability distribution, then the general expression for M is

$$M = \mu + z\sigma \quad (11)$$

where μ = the mean demand during the review period plus the lead-time period;
 σ = the standard deviation of demand;
 z = the number of standard deviations necessary to obtain the acceptable stock out probability.

V. CONCLUSIONS

The main aim of the paper is to showcase the differences between a deterministic model and a probabilistic model and to suggest an optimum way to minimize the overall holding costs. A deterministic situation is one in which the systems parameters can be ascertained precisely. This is also known as a situation of, as things are sure to occur in the same way. So, deterministic models pre assume the state of affairs to be deterministic. Since it conceives the system to be deterministic, it automatically means that one has full information of the system. Where as a probabilistic situation is a situation of uncertainty and more realistic. Thus, we can conclude that the best inventory plan in most cases would be to minimize the cost of holding of raw materials or finished products. It completely depends on an industry and its operations manager to decide what kind of method they would implement in their industry. Both, of the methods explained in the paper are proven and work efficiently.

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