A Mathematical Model to Determine Sensitivity of Vibration Signals for Localized Defects and to Find Effective Number of Balls in Ball Bearing

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Abstract **- Rolling element bearing faults are among the main causes of breakdown in rotating machines. Present study shows the effect of the different individual working parameters on vibration signals keeping other parameters constant. But it is important to know what happens if we vary all the parameters simultaneously. Due to this, one can understand the significant parameter that affects performance of the machine, so he can avoid the sudden breakdown by altering that particular parameter and can continue the working of same machine up to the time of scheduled breakdown, after detecting the first defect or fault in the working bearing. This paper presents a mathematical model which considers the contacts between the balls and races as non linear springs. Hertzian contact deformation theory is applied to this model. The intensity of the vibration signals varies according to working parameter. In order to find out the significant parameter, which affects the vibration signal i.e. the performance of the machine, Taguchi's Methodology is used. The results, in terms of vibration signals and analysis are presented in the paper, which can give idea about the sensitivity of the vibration signals.**

I. INTRODUCTION

Rolling element bearings are one of the major machine component used in industries like power plants, chemical plants and automotive industries that require precise and efficient performance. Working or performance of rotating machine will hamper, if the bearing fails to work properly. This deterioration in the performance can be detected using the vibration signals which are generated from the bearing due to defect present in the bearing [1]. So, Condition Monitoring of rolling element bearing is the most important thing in case of rotating machineries to detect the status of it and avoid failures. Several condition monitoring techniques are available. Vibration analysis has many advantages in industries as a predictive maintenance technique. The defects can be detected in earlier stage with the help of the vibration signals acquired from the bearing. This model will help to predict the effect of localized defects on the ball bearing vibrations. In this study, vibration response of the rolling bearings with defect on outer race, inner race and the rolling elements will be studied and analyzed.

Every defect excites the system at its characteristic frequency. So the vibration signals during contact of the races or rolling element with the defect may show large amplitude at that frequency i.e. there will be problem in the working of the system when defect comes in contact with races or rolling elements.[2][3] The vibration signal may get affected by the various parameters such as speed of rotor, radial load, axial load, speed of the cage, distributed and localized defects [2]. The vibration signal also varies according to the size of the spalls and the location of the spalls [4], the waviness in the inner and outer race grooves [5]. The defect produced may be on outer race, inner race or rolling element. The significance of these parameters on vibration signals can be determined by using Taguchi method.

Rolling element bearing exists in broad range of applications across almost all industries and they play an effective and important role in all aspect of our modern life, so their good condition is vital for the machine performance. The defects developed into the bearing, produces some noise or vibrations in the system. Unnoticed local defects in their early stage may result in progressively higher noise and vibrations and finally leading to the failure of the system, causing severe economic and personal losses. So it is important to detect the defect and severity of the defect that are generated in the bearing..

The study presents mathematical model for predicting the effect of a localized defect on the ball bearing vibrations. In the analytical formulation, the contacts between the ball and the races are considered as non-linear springs. The contact force is calculated using the Hertzian contact deformation theory [7]. A computer program is developed to evaluate the effect of the defect on the raceways with the results in the form of vibration signals. The model yields the acceleration and the Fast Fourier Transform of the signals. The effect of the defect size and its location and speed of the rotor and radial load has been investigated. Numerical results for 6204 deep groove ball bearing have been obtained and discussed. The results obtained from the analytical model and experiments have also been presented and analyzed. The vibrations produced in the bearing are normally due to following reasons;

Reasons for noise

1. Radial clearance is high.
2. Waviness of races, grooves.
3. Flaw in the bearing.
4. Occurs mostly with grease and only rarely with oils and will be more in winter because of oils.

5. Type of grease and amount of grease/oil.
6. A particular range of speed produces vibration and it can be because of resonance speed.
7. Also in case of defective bearing (spall on inner race, outer race and balls), when two bodies (balls and races) comes in contact exactly at the place of spall, the normal motion will get disturbed and system will start vibrating. It will vibrate with certain amplitude for time span of defect in and defect out.
8. If there is any foreign particle in the bearing system, it may be because of contaminated grease/oil and atmosphere around the bearing system. If it comes in contact with the rolling element i.e. balls, then there will be vibration, as an obstacle is coming in the path of balls in the bearing.
9. Damage due to improper handling and improper fitting of the bearing in its housing.

Solutions to the possible reasons

1. Manufacturing cares can be taken to cancel out the waviness and other problems.
2. We can use quality and clean grease/oil, to cancel the entry of foreign particle in the bearing.
3. The atmosphere closer to the system can be kept very clean to avoid exposure of dust etc to the system.

Whenever we are using rolling element bearings in rotating machines, there will be some sort of vibrations which are listed earlier. Out of those, main reasons are related to manufacturing and cleanliness of the bearing and working conditions of the bearing. So we have to focus on working time difficulties so that we can reduce the losses in future due to sudden breakdown. Industries mainly prefer periodic maintenance breakdowns. So they will obviously check or see for how much time they can continue the working of machine with the same bearing. So, they can avoid reduction in production rate. The vibration signal obtained from the bearing system mainly depends upon working conditions/parameters such as; Rotating speed, Radial Load acting on the Bearing, Defect Size, Defect Position, Quality of the grease/oil using in the bearing, Cleanliness of the bearing, etc. Out of all these factors, the main easily uncontrollable factors are speed. Load, defect size, and location of the defect. So remaining factors we can control easily as compared to this.

We have to look after the effects of one parameter/factor on the vibrations, indirectly the working quality of the bearing system (performance), together with remaining factors. Here we are finding out the effect of the working parameters when a defect with the different size lies at the different location of the bearing. Normally we will consider two defect sizes which are as 1mm, 2mm; and the corresponding area of the defect will be 0.7854mm2, 3.14159 mm2 respectively. We are considering these defect sizes, as we will get the information regarding the vibration signals for defective area, which is acceptable. According to industrial considerations, if the defective area in the ball bearing becomes equal to 6.25 mm2 to 6.50 mm2[9], the bearing is considered as the faulty bearing and it is suggested that it should be replaced as early as possible. One can find the effect of the defect of 3mm size to find the vibrations for non-acceptable region. Also, we are considering SKF 6204 ball bearing for experimentation. The speed levels decided are 1600rpm and 1900rpm. The speed levels considered here are just to find the effect of the speed on the vibration signal.

In case of location of the defect, we considered two possible locations; groove of inner race on outer periphery, groove of outer race on inner periphery. As all other parameters are at two levels, we have considered three levels in case of loads in kilograms; 2kg, 4kg.

II. MODELLING OF ROLLING ELEMENT BEARING

Rolling element bearing is the most critical element in the rotating machines. So the detection of the defects with the help of vibration signals is much important to check the severity of the bearing. So we can apply working conditions to find the vibration signal response of bearing. Also we can vary the working conditions and can see the effect of those conditions on vibration signals so that we will get the idea that, because of which factor the production hampers or vibration amplitude becomes higher. So to study the effects of different parameters such as speed, load, defect size and location of defect on vibration signals and time domain parameters, we are considering these parameters with their levels for the proposed mathematical model.

Table 1 Control Factors and their levels

Levels \ Factors	Defect Location	Defect Size	Speed	Radial Load
1	Inner Race	1mm	1600	2kg
2	Outer Race	2mm	1900	4kg

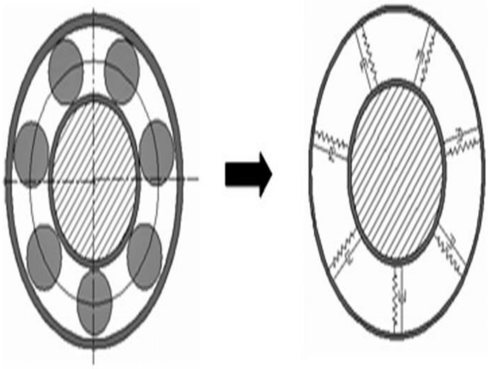


Fig1. Model of the Bearing

Assumptions of Mathematical Modelling

1. Inner race is rotating with the speed of shaft.
2. Outer race is fixed with help of housing.
3. Revolving and rotating balls will act as a spring fig.1 and it
   Will get compressed whenever comes in the loaded region.
4. Balls are equidistant.
5. Temperature of the surrounding is constant.
6. There is no slipping of the balls.
7. The races are rigid and undergo only local deformation due to contact stresses.
8. The deformation takes place according to Hertzian theory of elasticity.

If we want to find out the Hertzian forces acting at the contact point due to contact deformation, then it can be give as,

$$F = K\,\delta^n \qquad (1)$$

Where, F= force due to deformation, K= stiffness of the bearing, δ= contact deformation and n= 3/2 for ball bearing So in order to determine the force applied at the contact due to deformation, we have to find the stiffness of the ball bearing and contact deformation. The stiffness of the ball bearing mainly depends upon geometry of it, Fig.2 [7].

r_{gi} = 4.2mm, r_{go} = 4.7mm, $f_o = (r_{go}/D) = 0.5875$, $f_i = (r_{gi}/D) = 0.525$

For contact in between ball and inner race[7],

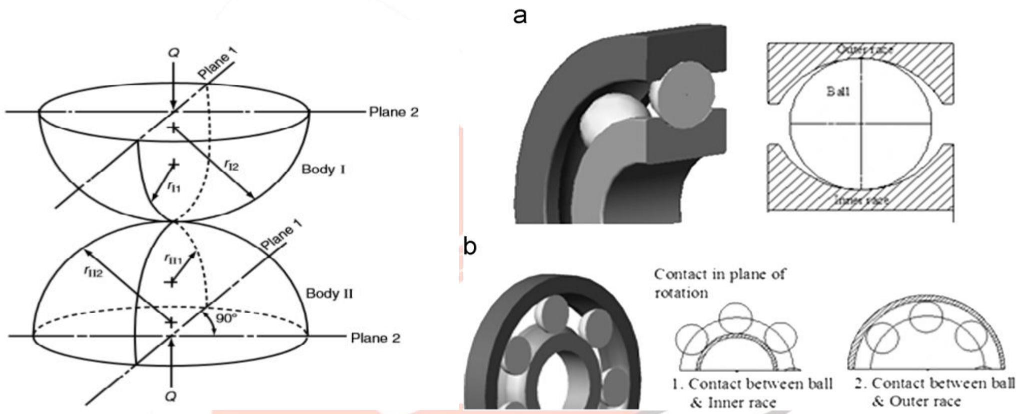


Fig.2 Geometry of the Contacting Bodies [7] and Ball Bearing Model[2]

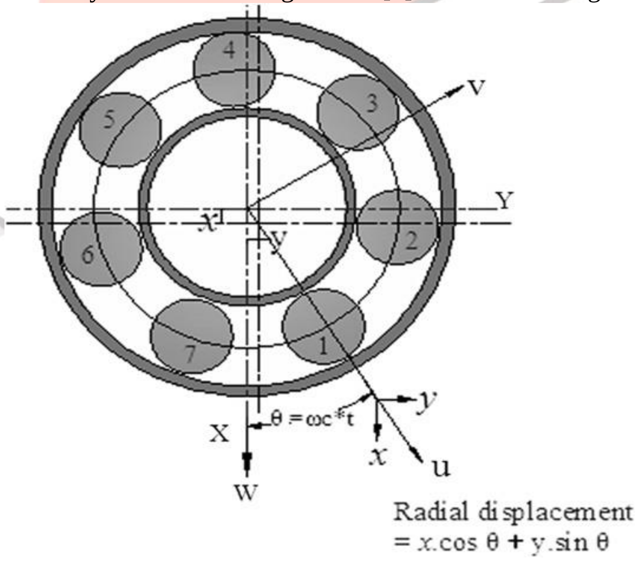


Fig.3 Bearing system subjected to loading[4]

For contact in between ball and outer race,[7]

$r_{I1} = \frac{D}{2}$, $r_{I2} = \frac{D}{2}$, $r_{II1} = \frac{1}{2}d_o$, $r_{II2} = r_{go}$.

Here,

D = Ball diameter, 8mm,

d_i= Inner Race Diameter, 25.2mm,

d_o = Outer Race Diameter, 41.8mm,

d_m= Pitch Diameter, 33.5mm,

r_{gi}= Inner Race Groove Diameter, 4.2mm,

r_{go}= Outer Race Groove Diameter, 4.7mm,

r= Radius of Curvature of body
r_{12}= radius of curvature of body 1 in plane 2.
$\Sigma\rho$= curvature sum= $\{(1/ r_{11})+(1/ r_{12})+(1/ r_{1I1})+(1/ r_{1I2})\}= \rho_{I1}+ \rho_{I2} + \rho_{II1} + \rho_{II2}$ (2)
$F(\rho)$= Curvature difference = $\frac{(\rho_{I1}- \rho_{I2})+ (\rho_{II1}- \rho_{II2})}{\Sigma\rho}$ (3)

So from this we could get the values of δ* i.e. the dimensionless contact deformation. We will get the values of δ* with the help of curvature difference F(ρ) using table of F(ρ) and δ*[7].
Total deflection between two raceways is the sum of the approaches between the rolling elements and each raceway. Here, K_i and K_o are inner and outer raceways to ball contact stiffness;

$$K =\left[\frac{1}{(1/K_i)^{1/n}+(1/K_o)^{1/n}}\right]^n \quad (4)$$

And this are given by $K_p = 2.15 \times 10^5 \sum \rho^{-1/2}(\delta^*)^{-3/2}$, we have values of the curvature sum and contact deformation as;[2]
For inner race, Σρ= 0.34033, F(ρ)=0.930, δ*= 0.62415, so; K_{pi}=747403.0758 N/mm^2
For outer race, Σρ= 0.23901, F(ρ)=0.68913, δ*= 0.86365, so; K_{po}=541928.3328N/mm^2
Hence, for SKF 6204 Ball Bearing, K=7.097611554 × 10^9 N/m$^{3/2}$

Considering the damping constant for steel material ball bearing, C=200Ns/m. So, now considering the initial conditions which mostly are obtained with the help of geometry of the ball bearing,[7]

Fundamental Train Frequency, $F_{FTF}= \frac{N_S}{(2\times60)}\left[1-\frac{D}{d_m}\cos\alpha\right]$, (5)

Inner Race Frequency, $F_{ID}= Z\times\left[\frac{N_S}{60}-F_{FTF}\right]$, Outer Race Frequency, $F_{OD}= Z\times F_{FTF}$, (6)(7)

Ball Spin Frequency, $F_B= \frac{N_S}{(2\times60)}\frac{d_m}{D}\left[1-\left(\frac{D}{d_m}\right)^2\cos\alpha\right]$, Cage Speed, $\omega_C= \left(\frac{2\pi N_S}{60\times2}\right)\left[1-\frac{D}{d_m}\cos\alpha\right]$ (8)(9)

In order to find out the sensitiveness of factors and their levels, we will consider the Taguchi's methodology. Doing this, we will get the information which factor and level are significant. So we calculated the vibration signals and for analysis we used L8 orthogonal array.

If x and y are the deflections along X- and Y-axis and Cr is the internal radial clearance, the radial deflection at the ith ball, at any angle yi is given by,

$$\delta = [(x\cos\theta_i + y\sin\theta_i) - C_r] \quad (10)$$

The mathematical model is done by considering that the balls in the loaded region will give the restoring force as those are considered as the springs. These springs will give the restoring force only when are compressed. The deflection of the bearings is considered as X and Y along X axis and along Y axis. So in order to evaluate restoring force, the clearance in the bearing should be neglected. Therefore the restoring force after contact deformation along X and Y direction can be given as, [4]

$$F_{XX} = [(x\cos\theta_i + y\sin\theta_i) - C_r]^{\frac{3}{2}}\cos\theta_i \quad (11)$$
$$F_{YY} = [(x\cos\theta_i + y\sin\theta_i) - C_r]^{\frac{3}{2}}\sin\theta_i \quad (12)$$

This restoring force will be the force acting by the balls which are in the loading region and for non-defective bearing. But when there is defect in the bearing having height, H_D, and φ, the angle made by the defect with the centre of the bearing, which is given by ratio of defect size to raceway radius, the total deflection occurred into the bearing will consider the clearance as well as the height of the defect, i.e.

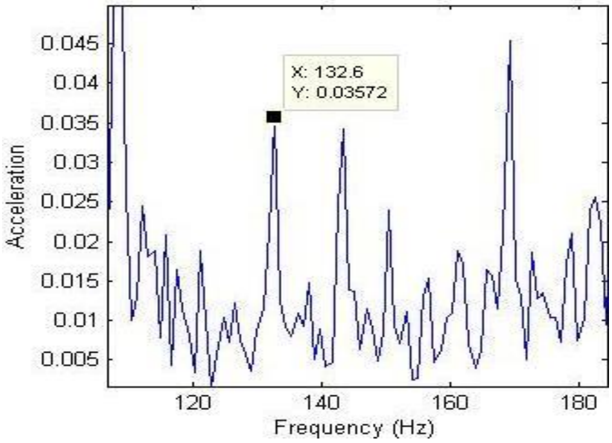
$$F_{XX} = \sum_{i=1}^{Z}\left[(x\cos\theta_i + y\sin\theta_i) - \left(C_r + H_D\sin(\pi(\theta_t-\theta_i)/\varphi)\right)\right]^{\frac{3}{2}}\cos\theta_i \quad (13)$$
$$F_{YY} = \sum_{i=1}^{Z}\left[(x\cos\theta_i + y\sin\theta_i) - \left(C_r + H_D\sin(\pi(\theta_t-\theta_i)/\varphi)\right)\right]^{\frac{3}{2}}\sin\theta_i \quad (14)$$

Since outer race is stationary, $\theta_t = \omega_c t + 2\pi(Z-i)/Z$ and
as inner race is rotating with the shaft, $\theta_t = (\omega_c - \omega)t + 2\pi(Z-i)/Z$
The equation of motion for two degree of freedom in X and Y direction is given by,
In X direction, $M\ddot{x} + C\dot{x} + F_{XX} = W$ and in Y direction $M\ddot{y} + C\dot{y} + F_{YY} = 0$

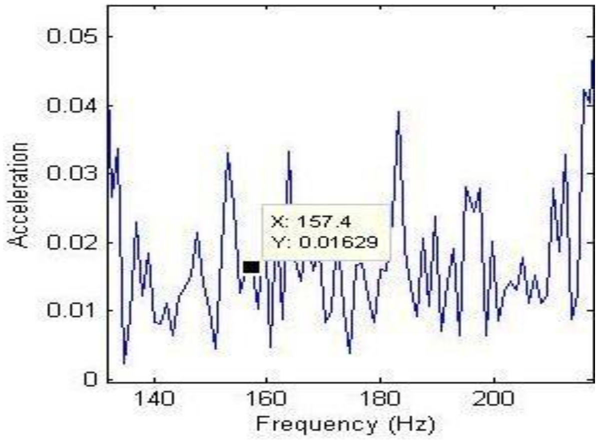
The depth of the defect for finding out the displacement of the centre of the ball when it goes into the spall or defect is evaluated using the relation, Depth= R - (R cosØ) where Ø is the angle inscribed by the defect width at the centre of the ball rotating in the bearing and R is the radius of the ball.

III. RESULTS AND ANALYSIS

These equations of motions are solved using Euler Method. From the Fast Fourier Transform of an acceleration-



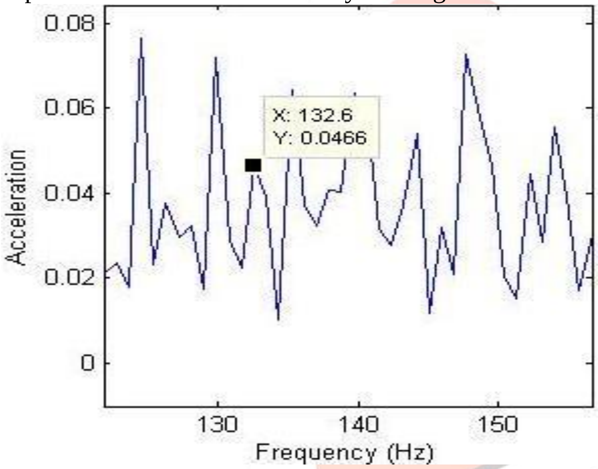
(a)



(b)

Fig 4.a Fast Fourier Transform of the vibration (acceleration) signals calculated from the model.

signal, which are obtained with the help of MATLAB, it is found that the peaks are obtained at the characteristic defect frequencies in all conditions. From this it is clear that, the acceleration varies at that instant due to certain change in the working parameters. The FFT of the acceleration signals shows that, the amplitude of the vibration varies as we vary the different factors. From the readings available, with the help of Taguchi's Methodology we found the effective parameter and their level, which affects the performance of the machine by causing excessive vibrations.



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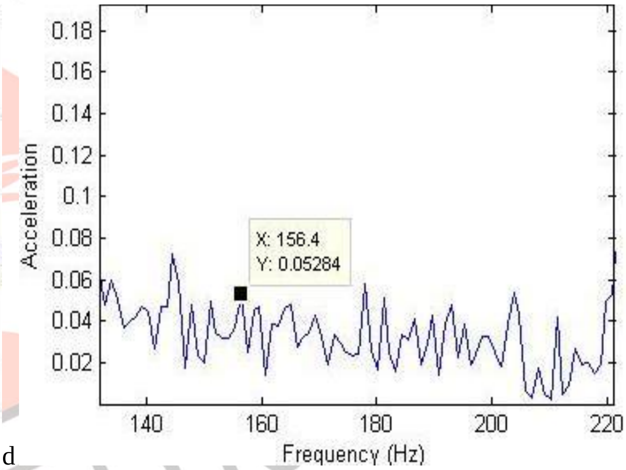

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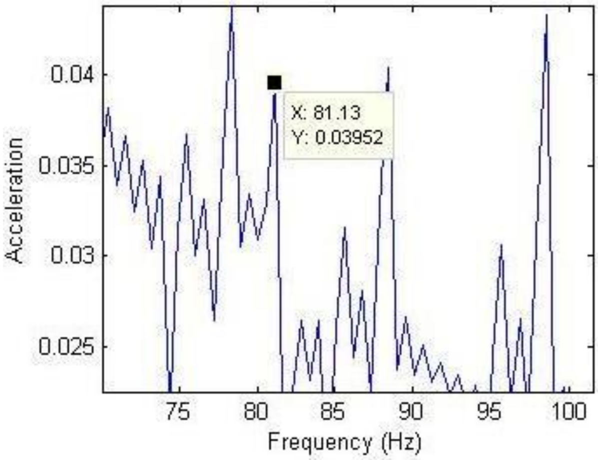

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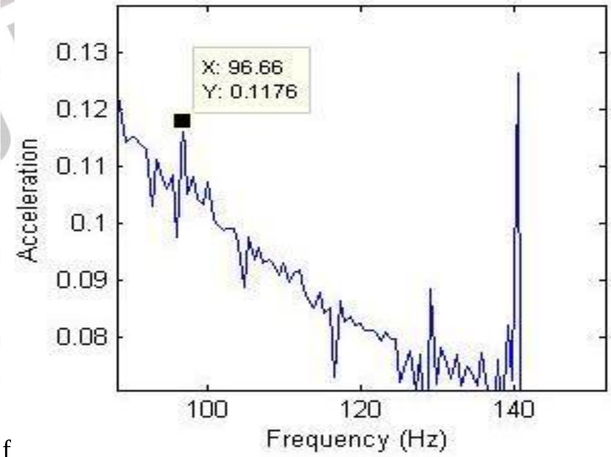

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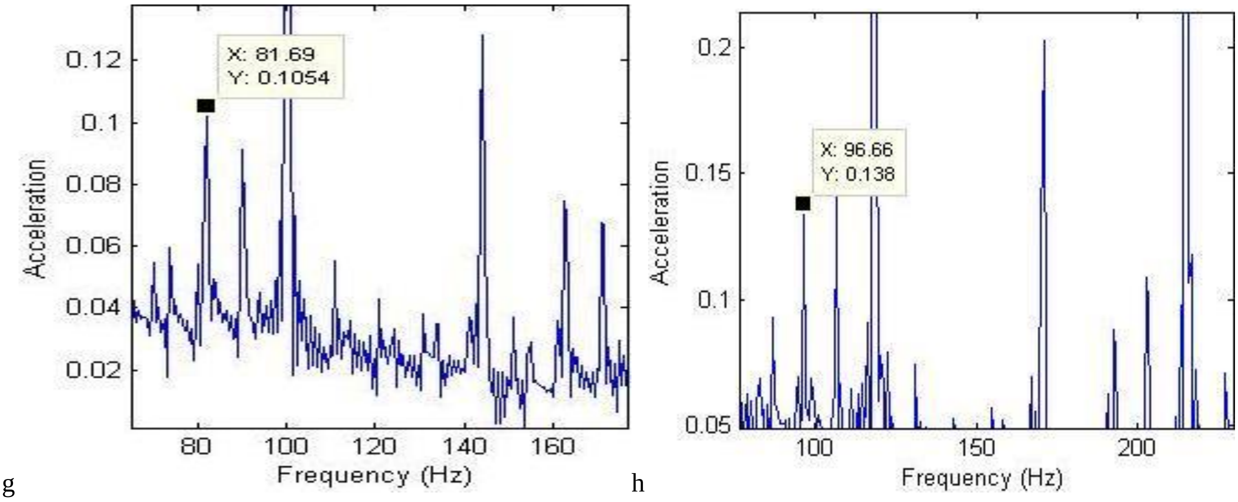

Fig 4.b Fast Fourier Transform of the vibration (acceleration) signals calculated from the model.

When the number of balls in the bearing are 8(Fig.5b), the response of the means, gave race as the most significant factor, which affects the vibrations in the ball bearing and it mainly depends upon location of the defect. When the defect or spall is on the inner race, the vibration amplitude is less as compared to the case when the defect is on the outer race. The amplitude of the vibration increases when the defect is on the inner race. The amplitude of vibration also varies as speed of shaft, size of defect and load acting on the bearing element varies. It is low at lower speed and increases as speed increases. The size of defect is the second significant factor which is affecting the variation in the vibration amplitude. At the start of the development of the defect, vibrations are less. In the same way, rotating speed is also affecting parameter, which causes the vibration to vary. The amplitude of vibration results lesser, when the speed is 1600 rpm than 1800 rpm. Also it can be concluded that when the vertically downward load is acting on the inner race, it hampers the vibrations inside the bearing system when there is defect in the bearing. The results shows that the amplitude of vibration increases at higher loads and it remain less when the load acting on the bearing is less. The amplitude increased at low load 2kg and it is reduced at 4kg. On the same line, the effect of varying the number of balls is observed using the same program and the responses are plotted as shown below. Fig.5(a) represents the response for Nb=7 and Fig.5 (c) represents the response for Nb=9.

The ball bearing available in the market is normally comes with 8 balls. But in order to check what will happen if we

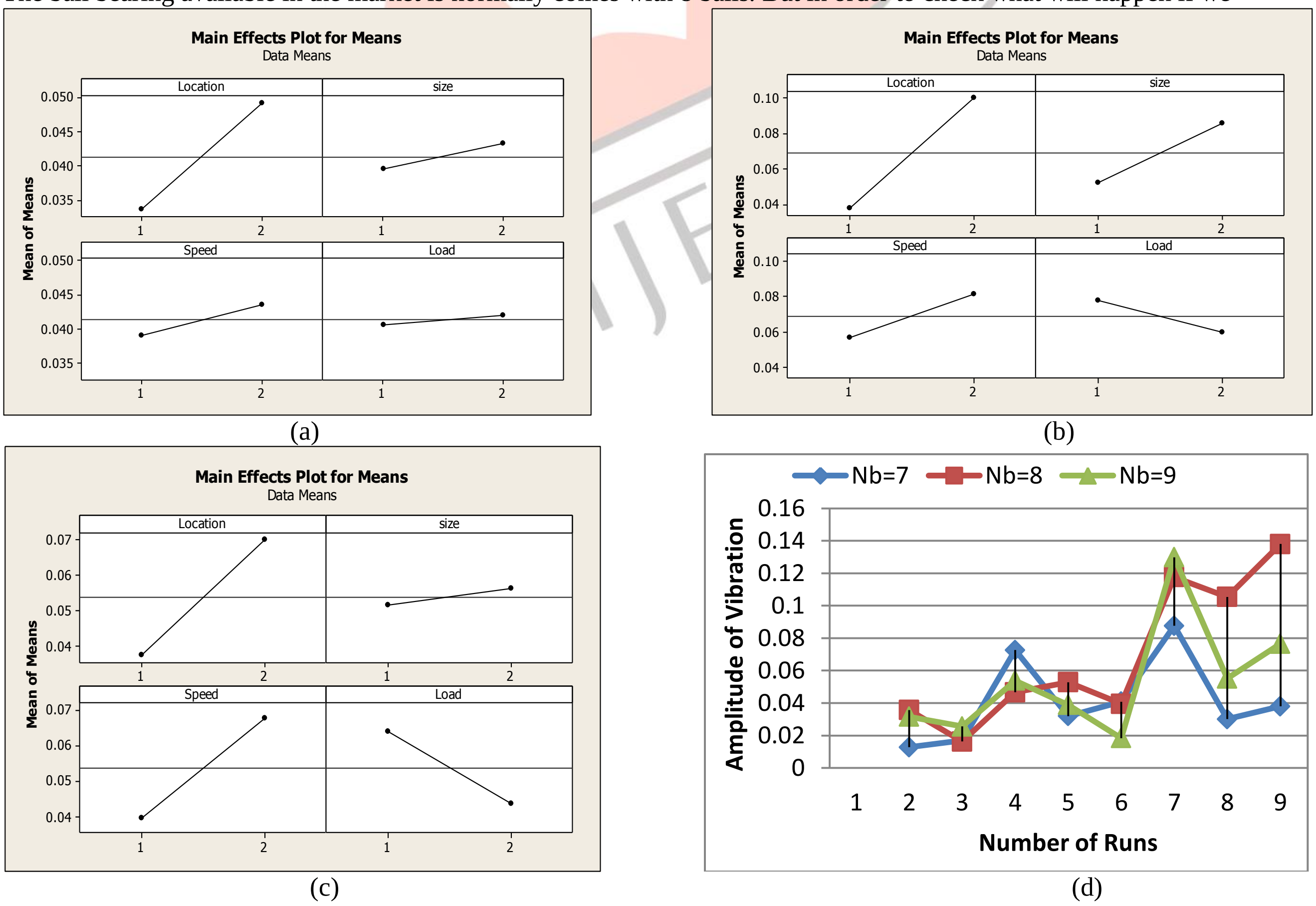

Fig.5 Response of the bearing system to varying working parameters for different number of balls in the system

vary number of balls in the bearing, we performed same experiments with same factors and its levels with different number of balls. First we considered the mathematical model of the ball bearing with 7 balls i.e. 7 non-linear springs. On the same line, we can consider more number of non linear springs i.e. 9 balls supporting inner race. Here it is found that size of the defect does not show noticeable variation in the amplitude of vibration, in case of Nb=7 and nb=9. In these cases also, the amplitude of the vibration is large when the defect is present on the outer race. At low load vibration amplitude increases and at larger load vibration reduces by sufficient value in case of Nb=9, same as Nb=8. In case of Nb=7, the nature of the behavior of the vibration signals with the radial load reverses but the factor becomes less significant.

In the case of Nb=7, only position of the defect is most significant factor, while in case of Nb=9 only size of the defect shows less significance. If we compare remaining factors, they are not much significant or not contributing so much vibration in the total vibration of the ball bearing. But out of all the factors, location of the defect is the vibration enhancing factor. Also, the location and size of defect are not controllable factors in the analysis work, so those cannot be controlled. The only things that we can control are speed and radial load. Though the factor, location of defect is most significant, we have to select a bearing model in which the speed and radial load are least significant and it results in case of Nb=8. Also it can be concluded that, the vibration amplitude is distributed closely and remains lesser in the bearing model of Nb=7. The overall response using vibration amplitude of the bearing system due to various factors and their levels is plotted and shown in the Fig.4 (d).

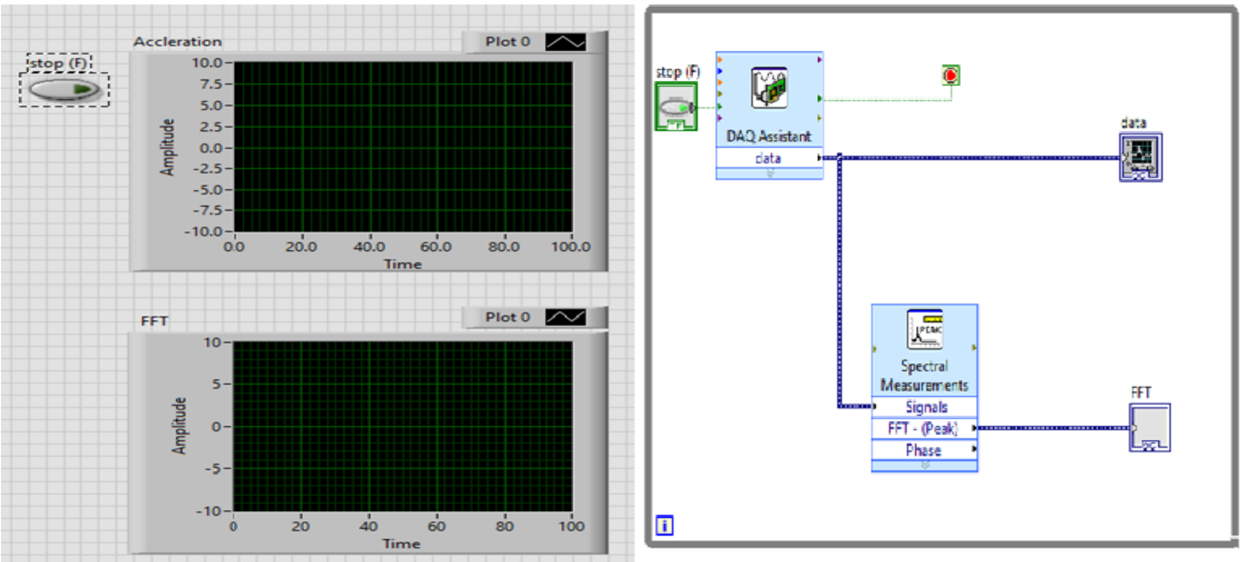

Fig.6 Block diagram and indicator diagram for LabView used for Experimentation

The response obtained with the help of Matlab programs are checked by doing experimentation and the results are almost matching. The data acquisition is achieved using NI Data Acquisition Kit with an Accelerometer sensor. The block diagram and an indicator diagram for the experimentation are as shown in Fig.6. An accelerometer was mounted on the bearing housing using LN-screw and FFT of the acceleration signals is obtained with the help of the Lab-View software. Sound and vibration module card was used for acquiring the desired signals.

IV. CONCLUSION

The most significant factor in this domain is the location of the defect, so one cannot control that. Only finding out the effect of number of balls and significant factor out of the given working parameters is not sufficient. One can find the effect of the defects in terms of vibration amplitude on bearing system when the defects or spalls are on the balls. Doing this, will help to understand the effect of the spalls on the bearing system, till that this model will help to understand the overall behavior of the bearing system. Also we have to select a model which will not get affected by speed and load acting on the rotating element. In the same way, if we look at the response of the bearing model with Nb=7, one can understand that the amplitude of vibration comes lesser than other models and is desirable.

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