A Parallel System with Priority to Preventive Maintenance over Replacement Subject To Maximum Operation and Repair Times

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***Abstract* - The present study deals with profit analysis of a parallel system of two identical units by giving priority to preventive maintenance of one unit over replacement of the other. Each unit has two modes- operative and complete failure. A single server is provided immediately to conduct the repair activities whenever needed. The preventive maintenance of the unit is done after a maximum operation time up to which no failure occurs. The failed unit is replaced by new one in case its repair is not possible by the server in a given maximum repair time. The unit performs with full efficiency as new after repair and preventive maintenance. All random variables are statistically independent. The distributions for failure time, replacement time and the rate by which unit undergoes for preventive maintenance are taken as negative exponential while that of preventive maintenance, repair and replacement rates are assumed as arbitrary with different probability density functions. The semi-Markov process and regenerative point technique are adopted to derive the expressions for some measures of system effectiveness in steady state. The variation of mean time to system failure (MTSF), availability and profit function has been observed graphically for arbitrary values of various parameters and costs.**

***Key Words* - Parallel system, preventive maintenance, replacement, priority and profit analysis.**

I. INTRODUCTION

The main intension of the manufacturers is to get maximum profit by selling their products with minimum efforts. And, they have managed it up to a considerable level by using proper operational and repair techniques in their systems. The method of parallel redundancy has been considered as one of the effective strategies for improving performance of the systems and thus profit. Several studies have been conducted on parallel systems under a common assumption that system can work for a long time without requiring any maintenance. But, this assumption seems to be unrealistic as continued operation and ageing of operable systems reduce their performance, reliability and safety. In such a situation preventive maintenance can be conducted after a specific operation time in order to slow the deterioration process. Kishan and Kumar (2009) studied a parallel system with preventive maintenance. The system of parallel units can be made more profitable by making replacement of the failed unit in case server fails to get its repair in a fixed time. Malik and Gitanjali (2012) obtained reliability measures of a parallel system with replacement of the unit subject to maximum repair time. Furthermore, the concept of priority in repair disciplines is one of the best ideas to enhance the profit of the system. Malik and Nandal (2010), Malik and Sureria (2012) and Kumar et al. (2012) have developed reliability models for the standby systems using the concept of priority. However, the idea of priority to preventive maintenance over replacement has not been introduced while analyzing system reliability models of two or more units.

Thus, the focus of the present study is to fill up this gap while carrying out profit analysis of a parallel system of two identical units. Each unit has two modes- operative and complete failure. A single server is provided immediately to conduct the repair activities whenever needed. The preventive maintenance of the unit is done after a maximum operation time up to which no failure occurs. The failed unit is replaced by new one in case its repair is not possible by the server in a given maximum repair time. Priority is given to preventive maintenance of one unit over the replacement of the other unit. The unit works as new after repair and preventive maintenance. All random variables are statistically independent. The distributions for failure time, replacement time and the rate by which unit undergoes for preventive maintenance are taken as negative exponential while that of preventive maintenance, repair and replacement rates are assumed as arbitrary with different probability density functions. The semi-Markov process and regenerative point technique are adopted to derive the expressions for some measures of system effectiveness in steady state. The variation of mean time to system failure (MTSF), availability and profit function has been observed graphically for arbitrary values of various parameters and costs.

II. NOTATIONS

E/ $\bar{E}$	:	Set of regenerative/ non-regenerative states
λ	:	Constant failure rate
α_0	:	The rate by which system undergoes for preventive maintenance (called maximum constant rate of operation time)
β_0	:	The rate by which system undergoes for replacement (called maximum constant rate of repair

time)

FUr /FWr : The unit is failed and under repair/waiting for repair
FURp : The unit is failed and under replacement
UPm : The unit is under preventive maintenance
WPm : The unit is waiting for preventive maintenance
FUR/FWR : The unit is failed and under repair / waiting for repair continuously from previous state
FURP : The unit is failed and under replacement continuously from previous state
UPM : The unit is under preventive maintenance continuously from previous state
WPM : The unit is waiting for preventive maintenance continuously from previous state
g(t)/G(t) : pdf/cdf of repair time of the unit
f(t)/F(t) : pdf/cdf of preventive maintenance time of the unit
r(t)/R(t) : pdf/cdf of replacement time of the unit
q_{ij} (t)/ Q_{ij}(t) : pdf / cdf of passage time from regenerative state Si to a regenerative stateSj or to a failed state Sj without visiting any other regenerative state in (0, t]
$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t) : pdf/cdf of direct transition time from regenerative state Si to regenerative state Sj or to a failed state Sj visiting state Sk, Sr once in (0, t]
M_i(t) : Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
W_i(t) : Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i : The mean sojourn time in state S_i which is given by
$\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, where T denotes the time to system failure.
m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that
$\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\,'}(0)$
&/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/** : Symbol for Laplace Transformation /Laplace Stieltjes Transformation

The possible transition states of the system model are shown in fig.1

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements as

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

$$p_{01} = \frac{2\lambda}{2\lambda+\alpha_0},\ p_{02} = \frac{\alpha_0}{2\lambda+\alpha_0},\ p_{10} = g^*(\lambda+\alpha_0+\beta_0),\ p_{13} = \frac{\lambda}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{40} = r^*(\lambda+\alpha_0),\ p_{48} = p_{41.8} = \frac{\lambda}{(\lambda+\alpha_0)}(1-r^*(\lambda+\alpha_0)),\ p_{6,11} = p_{66.11}\frac{\alpha_0}{(\lambda+\alpha_0)}(1-f^*(\lambda+\alpha_0)),$$

$$p_{31} = p_{56} = g^*(\beta_0),\ p_{49} = \frac{\alpha_0}{(\lambda+\alpha_0)}(1-r^*(\lambda+\alpha_0)),\ p_{6,10} = p_{61.10} = \frac{\lambda}{(\lambda+\alpha_0)}(1-f^*(\lambda+\alpha_0)),$$

$$p_{14} = \frac{\beta_0}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),\quad p_{11.3} = \frac{\lambda}{(\lambda+\alpha_0+\beta_0)}g^*(\beta_0)(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{15} = \frac{\alpha_0}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),\ p_{16.5} = \frac{\alpha_0}{(\lambda+\alpha_0+\beta_0)}g^*(\beta_0)(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{37} = p_{31.7} = p_{59} = 1-g^*(\beta_0),\ p_{19.5} = \frac{\alpha_0}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\beta_0))(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{60} = f^*(\lambda+\alpha_0),\ p_{11.37} = \frac{\lambda}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\beta_0))(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{26} = p_{71} = p_{81} = p_{94} = p_{10,1} = p_{11,6} = 1 \tag{2}$$

It can be easily verified that

$$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} + p_{15} = p_{40} + p_{48} + p_{49} = p_{60} + p_{6,10} + p_{6,11} = 1$$

$$p_{10} + p_{14} + p_{11.3} + p_{11.37} + p_{16.5} + p_{19.5} = p_{40} + p_{41.8} + p_{49} = p_{60} + p_{6,10} + p_{66.11} = 1$$

The mean sojourn times (μ_i) is in the state S_i are

$$\mu_0 = m_{01} + m_{02}, \quad \mu_1 = m_{10} + m_{13} + m_{14} + m_{15}, \quad \mu_2 = m_{26}, \quad \mu_4 = m_{40} + m_{48} + m_{49},$$
$$\mu_6 = m_{60} + m_{6,10} + m_{6,11}, \quad \mu_9 = m_{94}, \quad \mu_1^{'} = m_{10} + m_{14} + m_{11.3} + m_{11.37} + m_{16.5} + m_{19.5},$$
$$\mu_4^{'} = m_{40} + m_{41.8} + m_{49}, \quad \mu_6^{'} = m_{60} + m_{6,10} + m_{66.11} \qquad (3)$$

IV. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let ϕ_i(t) be the cdf of first passage time from regenerative state Si to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i(t):

$$\phi_0(t) = Q_{01}(t)\&\phi_1 + Q_{02}(t)$$
$$\phi_1(t) = Q_{10}(t)\&\phi_0 + Q_{14}(t)\&\phi_4(t) + Q_{13}(t) + Q_{15}(t)$$
$$\phi_4(t) = Q_{40}(t)\&\phi_0 + Q_{48}(t) + Q_{49}(t) \qquad (4)$$

Taking LST of above relation (4) and solving for Φ_0^{**}(s), we have

$$R^*(s) = \frac{1-\phi^{**}(s)}{s} \qquad (5)$$

The reliability of the system model can be obtained by taking Inverse Laplace transform of (5). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0}\frac{1-\phi^{**}(s)}{s} = \frac{N}{D} \qquad (6)$$

Where

$$N = \mu_0 + p_{01}\mu_1 + p_{01}p_{14}\mu_4 \quad and \quad D = 1 - p_{01}p_{10} - p_{01}p_{14}p_{40} \qquad (7)$$

V. STEADY STATE AVAILABILITY

Let A_i(t) be the probability that the system is in up-state at instant 't' given that the system entered regenerative state Si at t = 0.The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{14}(t)©A_4(t) + (q_{11.3}(t) + q_{11.37}(t))©A_1(t) + q_{16.5}(t)©A_6(t) + q_{19.5}(t)©A_9(t)$$
$$A_2(t) = q_{26}(t)©A_6(t)$$
$$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{41.8}(t)©A_1(t) + q_{49}(t)©A_9(t)$$
$$A_6(t) = M_6(t) + q_{60}(t)©A_0(t) + q_{61.10}(t)©A_1(t) + q_{66.11}(t)©A_6(t)$$
$$A_9(t) = q_{94}(t)©A_4(t) \qquad (8)$$

Where

$$M_0(t) = e^{-(2\lambda+\alpha_0)t}, M_1(t) = e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G(t)}, M_4(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)}, M_6(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)} \qquad (9)$$

Taking LT of above relations (8) and solving for A_0*(s). The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \qquad (10)$$

Where

$$N_1 = \mu_0 X + \{\mu_1(1-p_{01}) + \mu_4(p_{14} + p_{19.5})\}Y + \mu_6 Z \quad \text{and}$$
$$D_1 = (\mu_0 + \mu_2\, p_{02})X + \{\mu_1^{'}(1-p_{49}) + \mu_4^{'}(p_{14} + p_{19.5}) + \mu_9(p_{14}p_{49} + p_{19.5})\}Y + \mu_6^{'}Z \qquad (11)$$

VI. BUSY PERIOD ANALYSIS FOR SERVER

(a) Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repair the unit at an instant't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^R(t)$ are as follows:

$$B_0^R(t) = q_{01}(t)©B_1^R(t) + q_{02}(t)©B_2^R(t)$$

$$B_1^R(t) = W_1(t) + q_{10}(t)©B_0^R(t) + q_{14}(t)©B_4^R(t) + (q_{11.3}(t) + q_{11.37}(t))©B_1^R(t) + q_{16.5}(t)©B_6^R(t) + q_{19.5}(t)©B_9^R(t)$$
$$B_2^R(t) = q_{26}(t)©B_6^R(t)$$
$$B_4^R(t) = q_{40}(t)©B_0^R(t) + q_{41.8}(t)©B_1^R(t) + q_{49}(t)©B_9^R(t)$$
$$B_6^R(t) = q_{60}(t)©B_0^R(t) + q_{61.10}(t)©B_1^R(t) + q_{66.11}(t)©B_6^R(t)$$
$$B_9^R(t) = q_{94}(t)©B_4^R(t) \quad (12)$$

Where

$$W_1(t) = e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G(t)} + (\lambda e^{-(\lambda+\alpha_0+\beta_0)t}©1)\overline{G(t)} + (\alpha_0 e^{-(\lambda+\alpha_0+\beta_0)t}©1)\overline{G(t)} \quad (13)$$

Taking LT of above relations (12) and solving for $B_0^{R*}(s)$ .The time for which server is busy due to repair is given by

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R*}(s) = \frac{N_2}{D_1} \quad (14)$$

Where

$N_2 = W_1^*(0)(1-P_{49})Y$ and D_1 is already mentioned. (15)

(b) Due to Replacement

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement the unit at an instant 't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^{Rp}(t)$ are as follows:

$$B_0^{Rp}(t) = q_{01}(t)©B_1^{Rp}(t) + q_{02}(t)©B_2^{Rp}(t)$$
$$B_1^{Rp}(t) = q_{10}(t)©B_0^{Rp}(t) + q_{14}(t)©B_4^{Rp}(t) + (q_{11.3}(t) + q_{11.37}(t))©B_1^{Rp}(t) + q_{16.5}(t)©B_6^{Rp}(t) + q_{19.5}(t)©B_9^{Rp}(t)$$
$$B_2^{Rp}(t) = q_{26}(t)©B_6^{Rp}(t)$$
$$B_4^{Rp}(t) = W_4(t) + q_{40}(t)©B_0^{Rp}(t) + q_{41.8}(t)©B_1^{Rp}(t) + q_{49}(t)©B_9^{Rp}(t)$$
$$B_6^{Rp}(t) = q_{60}(t)©B_0^{Rp}(t) + q_{61.10}(t)©B_1^{Rp}(t) + q_{66.11}(t)©B_6^{Rp}(t)$$
$$B_9^{Rp}(t) = q_{94}(t)©B_4^{Rp}(t) \quad (16)$$

Where

$$W_4(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)} + (\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{R(t)} \quad (17)$$

Taking LT of above relations (16) and solving for $B_0^{Rp*}(s)$ .The time for which server is busy due to replacement is given by

$$B_0^{Rp}(s) = \lim_{s\to 0} sB_0^{Rp*}(s) = \frac{N_3}{D_1} \quad (18)$$

Where

$N_3 = W_4^*(0)(p_{14} + p_{19.5})Y$ and D_1 is already mentioned. (19)

(c) Due to Preventive Maintenance

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance the unit at an instant 't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^P(t)$ are as follows:

$$B_0^P(t) = q_{01}(t)©B_1^P(t) + q_{02}(t)©B_2^P(t)$$
$$B_1^P(t) = q_{10}(t)©B_0^P(t) + q_{14}(t)©B_4^P(t) + (q_{11.3}(t) + q_{11.37}(t))©B_1^P(t) + q_{16.5}(t)©B_6^P(t) + q_{19.5}(t)©B_9^P(t)$$
$$B_2^P(t) = W_2(t) + q_{26}(t)©B_6^P(t)$$
$$B_4^P(t) = q_{40}(t)©B_0^P(t) + q_{41.8}(t)©B_1^P(t) + q_{49}(t)©B_9^P(t)$$
$$B_6^P(t) = W_6(t) + q_{60}(t)©B_0^P(t) + q_{61.10}(t)©B_1^P(t) + q_{66.11}(t)©B_6^P(t)$$
$$B_9^P(t) = W_9(t) + q_{94}(t)©B_4^P(t) \quad (20)$$

Where

$$W_2(t)=W_9(t)=\overline{F(t)} \text{ and } W_6(t)=e^{-(\lambda+\alpha_0)t}\overline{F(t)}+(\alpha_0 e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)}+(\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)} \qquad (21)$$

Taking LT of above relations (20) and solving for $B_0^{P*}(s)$ .The time for which server is busy due to preventive maintenance is given by

$$B_0^P(\infty)=\lim_{s\to 0} sB_0^{P*}(s)=\frac{N_4}{D_1} \qquad (22)$$

Where

$$N_4=W_2^*(0)\,p_{02}X+W_6^*(0)\,\mathrm{Z}+W_9^*(0)(\mathrm{p}_{14}\,p_{49}+p_{19.5})\,\mathrm{Y} \text{ and } \mathrm{D}_1 \text{ is already mentioned.} \qquad (23)$$

VII. EXPECTED NUMBER OF REPAIRS

Let $R_i(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $R_i(t)$ are given as:

$$R_0(t)=Q_{01}(t)\&R_1(t)+Q_{02}(t)\&R_2(t)$$
$$R_1(t)=Q_{10}(t)\&(1+R_0(t))+Q_{14}(t)\&R_4(t)+Q_{11.3}(t)\&(1+R_1(t))$$
$$+Q_{11.37}(t)\&R_1(t)+Q_{16.5}(t)\&(1+R_6(\mathrm{t}))+Q_{19.5}(t)\&R_9(\mathrm{t})$$
$$R_2(t)=Q_{26}(t)\&R_6(t)$$
$$R_4(t)=Q_{40}(t)\&R_0(t)+Q_{41.8}(t)\&R_1(t)+Q_{49}(t)\&R_9(t)$$
$$R_6(t)=Q_{60}(t)\&R_0(t)+Q_{61.10}(t)\&R_1(t)+Q_{66.11}(t)\&R_6(t)$$
$$R_9(t)=Q_{94}(t)\&R_4(t) \qquad (24)$$

Taking LST of above relations (24) and solving for $R_0^{**}(s)$ .The expected no. of repairs per unit time by the server are giving by

$$R_0(\infty)=\lim_{s\to 0} sR_0^{**}(\mathrm{s})=\frac{N_5}{D_1} \qquad (25)$$

Where

$$N_5=(P_{10}+P_{11.3}+P_{16.5})(1-P_{49})Y \text{ and } \mathrm{D}_1 \text{ is already mentioned.} \qquad (26)$$

VIII. EXPECTED NUMBER OF REPLACEMENTS

Let $Rp_i(t)$ be the expected number of replacements by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $Rp_i(t)$ are given as:

$$Rp_0(t)=Q_{01}(t)\&Rp_1(t)+Q_{02}(t)\&Rp_2(t)$$
$$Rp_1(t)=Q_{10}(t)\&Rp_0(t)+Q_{14}(t)\&Rp_4(t)+Q_{11.3}(t)\&Rp_1(t)$$
$$+Q_{11.37}(t)\&(1+Rp_1(t))+Q_{16.5}(t)\&Rp_6(\mathrm{t})+Q_{19.5}(t)\&Rp_9(\mathrm{t})$$
$$Rp_2(t)=Q_{26}(t)\&Rp_6(t)$$
$$Rp_4(t)=Q_{40}(t)\&(1+Rp_0(t))+Q_{41.8}(t)\&(1+Rp_1(t))+Q_{49}(t)\&Rp_9(t)$$
$$Rp_6(t)=Q_{60}(t)\&Rp_0(t)+Q_{61.10}(t)\&Rp_1(t)+Q_{66.11}(t)\&Rp_6(t)$$
$$Rp_9(t)=Q_{94}(t)\&Rp_4(t) \qquad (27)$$

Taking LST of above relations (27) and solving for $Rp_0^{**}(s)$.The expected number of replacements per unit time by the server is giving by

$$Rp_0(\infty)=\lim_{s\to 0} sRp_0^{**}(s)=\frac{N_6}{D_1} \qquad (28)$$

Where

$$N_6=Y(1-p_{49})(p_{14}+p_{11.3}+p_{19.5}) \text{ and } \mathrm{D}_1 \text{ is already mentioned.} \qquad (29)$$

IX. EXPECTED NUMBER OF PREVENTIVE MAINTENANCES

Let $P_i(t)$ be the expected number of preventive maintenance by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $P_i(t)$ are given as:

$$P_0(t) = Q_{01}(t) \& P_1(t) + Q_{02}(t) \& P_2(t)$$
$$P_1(t) = Q_{10}(t) \& P_0(t) + Q_{14}(t) \& P_4(t) + (Q_{11.3}(t) + Q_{11.37}(t)) \& P_1(t)$$
$$+ Q_{16.5}(t) \& \ P_6(t) + Q_{19.5}(t) \& P_9(t)$$
$$P_2(t) = Q_{26}(t) \& (1 + P_6(t))$$
$$P_4(t) = Q_{40}(t) \& P_0(t) + Q_{41.8}(t) \& P_1(t) + Q_{49}(t) \& P_9(t)$$
$$P_6(t) = Q_{60}(t) \& (1 + P_0(t)) + Q_{61.10}(t) \& (1 + P_1(t)) + Q_{66.11}(t) \& (1 + P_6(t))$$
$$P_9(t) = Q_{94}(t) \& (1 + P_4(t)) \quad (30)$$

Taking LST of above relations (30) and solving for $P_0^{**}(s)$. The expected number of preventive maintenances per unit time by the server is giving by

$$P_0(\infty) = \lim_{s \to 0} sP_0^{**}(s) = \frac{N_7}{D_1} \quad (31)$$

Where

$N_7 = p_{02}X + (p_{14}p_{49} + p_{19.5})Y + Z$ and D_1 is already mentioned. (32)

Where

$$X = (1 - p_{49})\{(1 - p_{11.3} - p_{11.37})(1 - p_{66.11}) - p_{61.10}\ p_{16.5}\} - p_{41.8}(1 - p_{66.11})(p_{14} + p_{19.5})$$
$$Y = p_{01}(1 - p_{66.11}) + p_{61.10}p_{02}$$
$$Z = (1 - p_{49})\{p_{01}p_{16.5} + (1 - p_{11.3} - p_{11.37})p_{02}\} - p_{02}p_{41.8}(p_{14} + p_{19.5}) \quad (33)$$

X. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2B_0^{Rp} - K_3B_0^P - K_4R_0 - K_5Rp_0 - K_6P_0 \quad (34)$$

Where

P = Profit of the system model
K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due to repair
K_2 = Cost per unit time for which server is busy due to replacement
K_3 = Cost per unit time for which server is busy due to preventive maintenance
K_4 = Cost per unit time repair
K_5 = Cost per unit time replacement
K_6 = Cost per unit time preventive maintenance

XI. CONCLUSION

The results for some important reliability measures have been evaluated for the particular case $g(t) = \theta e^{-\theta t}$, $r(t) = \beta e^{-\beta t}$, $f(t) = \alpha e^{-\alpha t}$. Graphs are drawn to show the behavior of MTSF, availability and profit with respect to failure rate (λ) as shown in figures 2, 3 and 4 respectively. It is observed that MTSF, availability and profit go on decreasing with the increase of failure rate (λ) and the rate (α0) by which unit undergoes for preventive maintenance while their values increase with the increase of repair rate (θ) and replacement rate (β). MTSF and availability keep on increasing as the rate (β0) by which unit undergoes for replacement increases while system becomes less profitable. Also, there is no effect of preventive maintenance rate (α) on MTSF whereas system becomes more profitable with the increase of preventive maintenance rate (α). Thus, the study reveals that a parallel system of two identical units in which priority to preventive maintenance is given over replacement can be made more reliable and profitable to use by increasing repair, preventive maintenance and replacement rates.

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State Transition Diagram

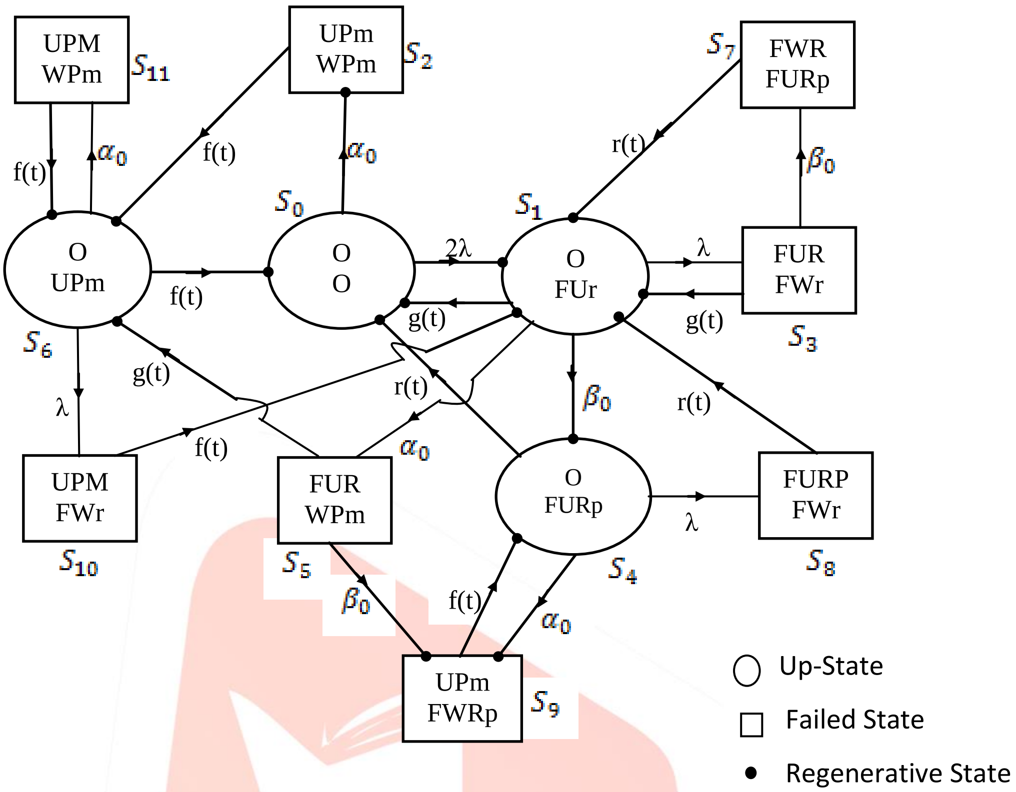


Fig. 1

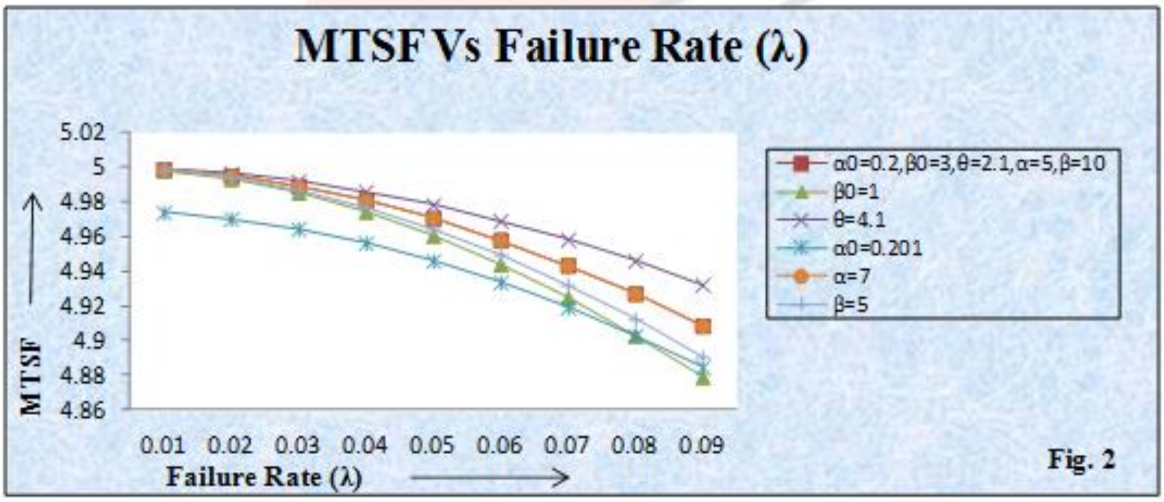
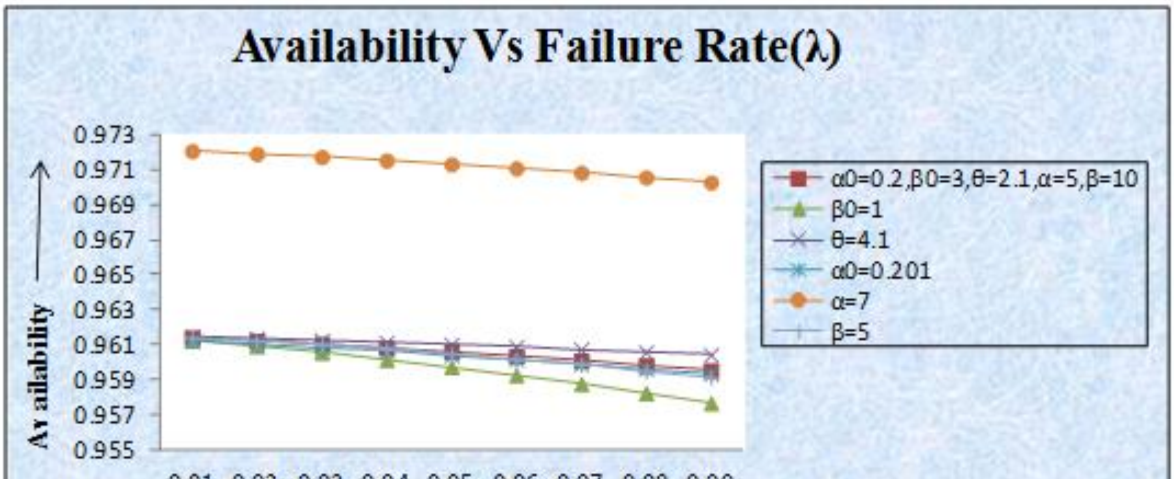
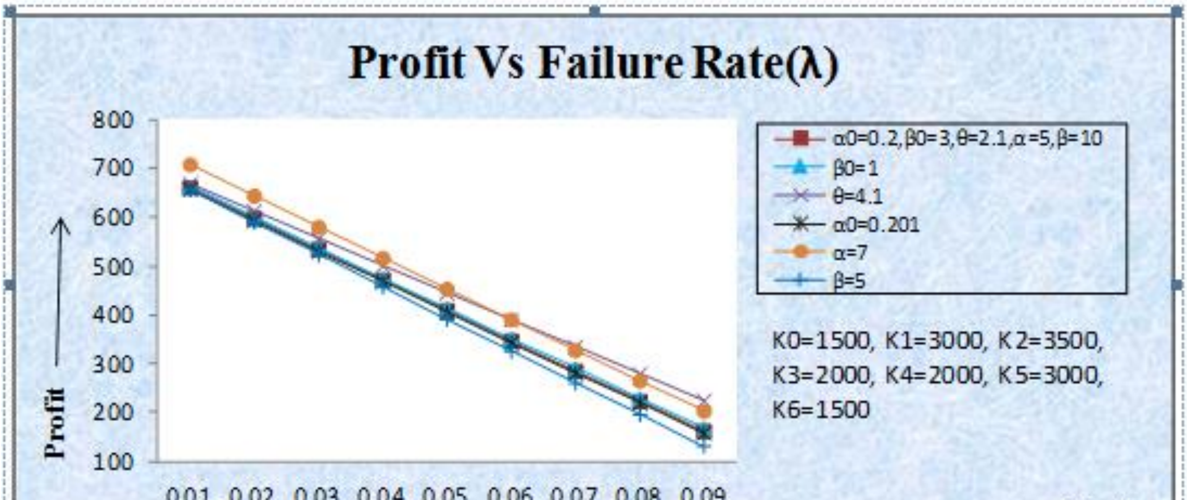


Fig. 2


Availability Vs Failure Rate(λ)
0.973
0.971
0.969
0.967
0.965
0.963
0.961
0.959
0.957
0.955
Availability
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
Failure Rate(λ)
α0=0.2,β0=3,θ=2.1,α=5,β=10
β0=1
θ=4.1
α0=0.201
α=7
β=5
Fig. 3


Profit Vs Failure Rate(λ)
800
700
600
500
400
300
200
100
Profit
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
Failure Rate(λ)
α0=0.2,β0=3,θ=2.1,α=5,β=10
β0=1
θ=4.1
α0=0.201
α=7
β=5
K0=1500, K1=3000, K2=3500,
K3=2000, K4=2000, K5=3000,
K6=1500
Fig. 4