© 2014 IJEDR | Volume 2, Issue 4 | ISSN: 2321-9939

# Assessment of Seismic Vibration Response of Structure to Expected Peak Ground Acceleration

[1]Mr. Mandar Sardesai, [2]Dr Atul Desai,
[1] Research Scholar, [2] Professor Applied Mechanics Department
[1] Sardar Vallabhbhai National Institute of Technology, Surat, India

***Abstract* - Cable supported structures have distinctive dynamic behaviour. Extradosed bridge, which is intermediate to Girder Bridge and cable stayed bridge, owing to its shallow cables, the structure behaviour of Extradosed Bridge differs from that of cable stayed bridge. Forced vibration of structure for given Earthquake time history is governed by peak acceleration. For cable stayed structures such as Extradosed cable stayed bridge it is difficult to predict dynamic response using usual methods of dynamic analysis as applied to some other bridge structures like response spectrum analysis, accurate analysis like time history analysis is time consuming and has time and cost effects. Nonlinearities can only be considered in time history analysis. The proposed method correlates the peak ground acceleration (PGA) and earthquake deformation ratio (EDR) which can be used for simplified dynamic analysis and can prove handy tool for structural engineers to know earthquake related serviceability without much complicated analysis at initial stages. This ratio can be used to present seismic damage indices. The method is proposed considers Extradosed bridge for example.**

***Index Terms* - Extradosed cable stayed bridge; Structural Behavior; Earthquake vibrations; Dynamic response; PGA-EDR; Seismic Damage index.**

## I. INTRODUCTION

The recent research[7] has shown that a Extradosed bridge, which is intermediate to Girder Bridge and a cable stayed bridge, adds substantial prestress to the deck because of the shallow pylon, are found to be economical for spans upto 250m. Dynamic response prediction has been the matter of research for many authors, in particular as the structural design of many structures is governed by the earthquake load cases or combinations thereof. The commonly used simplified methods used for analysis are based of theory of dynamics pertaining to SDOF systems. Rules of modal combinations viz SRSS, CQC are used for MDOF systems. These combinations rules are fairly accurate and helpful since exact methods such as time history analysis involves significant skill, time and cost. Although it is acceptable that having temporal response parameters proves helpful, the structural design is always governed by peak response. The peak values correspond to peak ground motions. The approach as presented here gives fairly accurate estimation of earthquake response and may prove helpful for quick response prediction at the time of preliminary designs and further serves as a tool for seismic damage index for higher intensity ground motions for which the structure is not designed.

The intrados is defined as the interior curve of an arch, or in the case of cantilever-constructed girder bridge, the soffit of the girder. Similarly, the extrados is defined as the uppermost surface of the arch. The term 'Extradosed' was coined by Jacques Mathivat in 1988 [9] to appropriately describe an innovative cabling concept he developed for the Arrêt-Darré Viaduct , in which external tendons were placed above the deck instead of within the cross-section as would be the case in a girder bridge. To differentiate these shallow external tendons, which define the uppermost surface of the bridge, from the stay cables found in a cable-stayed bridge, Mathivat called them 'Extradosed' prestressing.

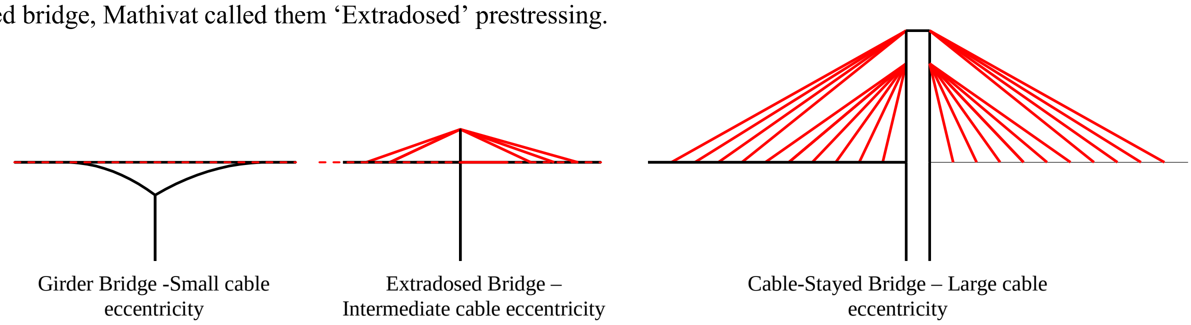


Fig. 1 Figure showing Cable arrangements in Girder Bridge, Extradosed Bridge and Cable stayed bridge

Some features of Extradosed bridge as given below;

- External appearance resembles cable-stayed bridge – but structural characteristics are comparable to those of conventional girder bridge

- The Girder Depth are lesser than that of conventional girder bridges
- The stay cables (prestressing tendons outside the girder) need no tension adjustment necessary for cable-stayed bridges, and can be treated as usual tendons as in girder bridges
- The height of pylon is half as that of cable-stayed bridge and hence easier to construct
- With small stress fluctuation under live load the anchorage method for stay cables can be same as that of tendons inside girder and thereby achieve economy

With the rapid increase in span length, combined trend and also trend of using high strength materials have resulted in slender structures and a concern is being raised over dynamic behavior of such structures, in case of cable supported structures it is more pronounced as this further includes vibrations of cable elements also. An accurate analysis of natural frequencies is fundamental to the solution of its dynamic responses due to seismic and wind and traffic loads. Seismic analysis of structure is important aspect of structural design as overall economy and safety of structure is most of the time governed by the load cases involving seismic forces. Many studies associated with this have been performed over last few decades. However, number of researches on dynamic behavior of Extradosed Bridge is very limited.

To gauge the exact dynamic response of structure, time history analysis is used. But this method is found to be time consuming and involves considerable cost. Further it is sometimes required to know the approximate results beforehand prior to proceeding to extensive analysis such as time history analysis.

The Proposed “Earthquake- Displacement Ratio” (EDR) is defined as ratio of maximum seismic displacement to maximum static displacement. These both displacements measured at the same point. Earthquake-displacement ratio can be used as a tool to gauge the seismic response of the structure according to expected Peak Ground Acceleration.

The aim of this paper is to present work relating to dynamic behavior of Extradosed bridge and the proposed PGA – EDR relationship.

## II. EXISTING METHODS OF SEISMIC RESPONSE PREDICTION

Dynamic analysis of structures has been studied by many researchers and as a result following two basic methods are used for seismic analysis of different structures. Viz. Response Spectrum method and Time History analysis

### Response Spectrum Method

For three dimensional seismic motions, the typical modal Equation is written as

$$\ddot{y}(t)_n + 2\varsigma\omega\dot{y}(t)_n + \omega_n{}^2 y(t)_n = P_{nx}\ddot{u}(t)_{gx} + P_{ny}\ddot{u}(t)_{gy} + P_{nz}\ddot{u}(t)_{gz} \qquad \text{Eq. (1)}$$

Where, the three *Mode Participation Factors* are defined by P*ni* = -ϕ*n Mi* in which i is equal to x, y or z. Two major problems must be solved in order to obtain an approximate response spectrum solution to this equation. First, for each direction of ground motion maximum peak forces and displacements must be estimated. Second, after the response for the three orthogonal directions is solved it is necessary to estimate the maximum response due to the three components of earthquake motion acting at the same time. The modal combination problem due to one component of motion is as given below.

For input in one direction only, Equation above is written as

$$\ddot{y}(t)_n + 2\varsigma\omega\dot{y}(t)_n + \omega_m{}^2 y(t)_n = P_{ni}\ddot{u}(t)_g \qquad \text{Eq. (2)}$$

Given a specified ground motion, $\ddot{u}_{(t)g}$ damping value and assuming *pni* =1.0, it is possible to solve above Equation at various values of ω and plot a curve of the maximum peak response *y*(ω)*MAX* . For this acceleration input, the curve is by definition the ***displacement response spectrum*** for the earthquake motion. A different curve will exist for each different value of damping.

### *Calculation of Modal Response*

The maximum modal displacement, for a structural model, can now be calculated for a typical mode *n* with period *Tn* and corresponding spectrum response value *S n* (ω). The maximum modal response associated with period *Tn* is given by

$$y(T_n)_{\max} = \frac{s(\omega_n)}{\omega_n{}^2} \qquad \text{Eq. (3)}$$

The maximum modal displacement response of the structural model is calculated from

$$u_n = y(T_n)_{\max}\phi_n \qquad \text{Eq. (4)}$$

The corresponding internal modal forces, *f kn* , are calculated from standard matrix structural analysis using the same equations as required in static analysis. Rules of modal combinations viz SRSS, CQC are used for MDOF systems.

The response spectrum method of dynamic analysis must be used carefully. The CQC method should be used to combine modal maxima in order to minimize the introduction of avoidable errors. The increase in computational effort, as compared to the SRSS method, is small compared to the total computer time for a seismic analysis. The CQC method has a sound theoretical basis and has been accepted by most experts in earthquake engineering. The use of the absolute sum or the SRSS method for modal combination cannot be justified. In order for a structure to have equal resistance to earthquake motions from all directions, the CQC3 method should be used to combine the effects of earthquake spectra applied in three dimensions. The percentage rule methods have no theoretical basis and are not invariant with respect to the reference system. Engineers, however, should clearly understand that the response spectrum method is an approximate method used to estimate maximum peak values of displacements and forces and that it has significant limitations. It is restricted to linear elastic analysis in which the damping properties can only be estimated with a low degree of confidence. The use of nonlinear spectra, which are commonly used, has very little theoretical background and should not be used for the analysis of complex three dimensional structures. For such structures, true nonlinear time-history response should be used.

***Time History Analysis***

A simple single degree of freedom system (a mass, M, on a spring of stiffness, k for example) has following equation of motion:

$$m\ddot{u} + ku = f(t) \qquad \text{Eq. (5)}$$

Where, u is the displacement.

Ground velocities and displacements can then be calculated from the integration of accelerations and velocities within each time step.

$$\begin{aligned}
\dddot{u} &= \frac{1}{\Delta t}(\ddot{u}_i - \ddot{u}_{i-1}) \\
\ddot{u}(t) &= \ddot{u}_{i-1} + t\dddot{u} \\
\dot{u}(t) &= \dot{u}_{i-1} + t\ddot{u}_{i-1} + \frac{t^2}{2}\dddot{u} \\
u(t) &= u_{i-1} + t\dot{u}_{i-1} + \frac{t^2}{2}\ddot{u}_{i-1} + \frac{t^3}{6}\dddot{u}
\end{aligned} \qquad \text{Eq. (6)}$$

The evaluation of those equations at τ = Δt produces the following set of recursive equations

$$\begin{aligned}
\dddot{u} &= \frac{1}{\Delta t}(\ddot{u}_i - \ddot{u}_{i-1}) \\
\ddot{u}(i) &= \ddot{u}_{i-1} + \Delta t\dddot{u} \\
\dot{u}(i) &= \dot{u}_{i-1} + \Delta t\ddot{u}_{i-1} + \frac{\Delta t^2}{2}\dddot{u} \\
u(i) &= u_{i-1} + \Delta t\dot{u}_{i-1} + \frac{\Delta t^2}{2}\ddot{u}_{i-1} + \frac{\Delta t^3}{6}\dddot{u}
\end{aligned} \qquad \text{Where, I = 1, 2, 3…..} \qquad \text{Eq. (7)}$$

***IS 1893 / IRC-6 Method of analysis*** [20] [21]

Every structure has finite number of modes of vibrations. For example, a 2-DOF system has two mode shapes as shown in Figure (2) below,

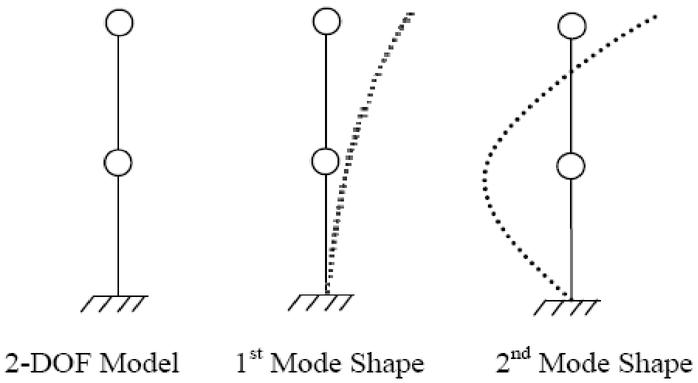


*Fig. 2* Degree of freedom and mode shapes

The natural time period corresponding to the k$^{th}$ mode is called natural time period (Tk) of k$^{th}$ mode. For obtaining the natural frequencies and mode shapes, free vibration analysis (also called eigen value analysis) of the structure is to be carried out. In seismic coefficient method (single mode method), only one mode of vibration was considered. The time period for this mode was obtained in a very simplistic fashion without performing the free vibration analysis. In response spectrum method, the natural periods and mode shapes obtained using free vibration analysis are used to obtain seismic force. Sufficient number of modes shall be used so that sum of modal mass of considered modes is more than 90% of the total mass of the structure.

The elastic seismic acceleration coefficient *Ak* for mode *k* shall be determined by:

Ak = Z/2 . Sa/g . I/R     Eq. (8)

For the fundamental mode of vibration i.e., the first mode of vibration, the shape of acceleration response spectrum is same as the one used in the seismic coefficient method. However, for higher modes (i.e., $k > 1$), the ascending part of the spectrum between 0 to 0.1 sec can be used. Since, the fundamental mode makes the most significant contribution to the overall response and the contribution of higher modes is relatively small, this is now permitted by several codes. Damping factor for higher modes, the value of acceleration response spectrum at T = 0 will remain unity irrespective of the damping value. Ordinates for other values of damping can be obtained by multiplying the value for 5 percent damping with the factors given by the code. Note that the acceleration spectrum ordinate at zero period equals peak ground acceleration regardless of the damping value. Hence, the multiplication should be done for $T \geq 0.1$sec only. For $T = 0$, multiplication factor will be 1, and values for $0 \leq T < 0.1$sec should be interpolated accordingly, see fig.3

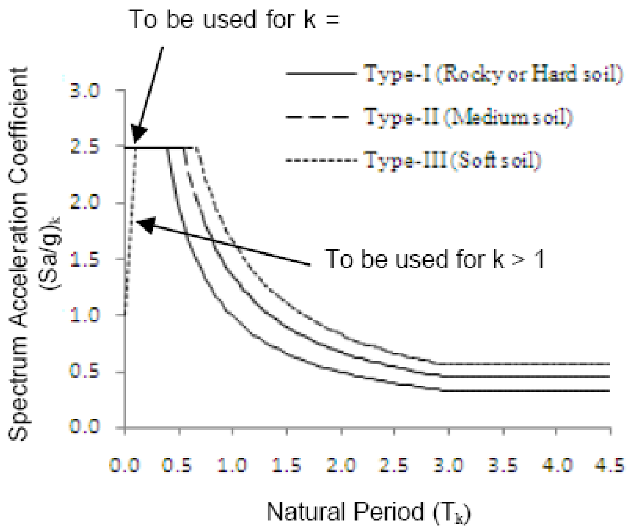


*Fig.* 3 Acceleration response for 5% damping used for response spectrum method

The seismic zone map and zone factors are taken from IS 1893 (Part 1): 2002. The seismic zoning map broadly classifies India into zones where one can expect earthquake shaking of the more or less the same maximum intensity. The zoning criterion of the map is based on likely intensity. It does not give us any idea regarding how often a shaking of certain intensity may take place in a location (that is, probability of occurrence or return period). For example, say area A experiences a maximum intensity VIII every 50 years and area B experiences a maximum intensity VIII every 300 years. But both these areas will be placed in zone IV, even though area A has higher seismicity. The current trend worldwide is to specify the zones in terms of ground acceleration that has a certain probability of being exceeded in a given number of years.

Zone factor ($Z$) accounts for the expected intensity of shaking in different seismic zones Efforts have been made to specify $Z$ values that represent a reasonable estimate of PGA in the respective zone. For instance, $Z$ value of 0.36 in zone V implies that a value of 0.36$g$ is reasonably expected in zone V. But it does not imply that acceleration in zone V will not exceed 0.36$g$. For example, during 2001 Bhuj earthquake, peak ground acceleration of approximately 0.6$g$ was inferred from data obtained from the Structural Response Recorder located at Anjar, 44kms away from the epicenter.

Seismic design philosophy assumes that a structure may undergo some damage during severe shaking. However critical and important facilities must respond better in an earthquake than an ordinary structure. Importance factor is meant to account for this by increasing the design force level for critical and important structures. This is taken in to consideration by using Importance factor (I).

R is the response reduction factor. The basic philosophy of earthquake resistant design is that a structure should not collapse under strong earthquake shaking, although it may undergo some structural as well as non-structural damage. Thus, a bridge is designed for much less force than what would be required if it were to be necessarily kept elastic during the entire shaking. Clearly, structural damage is permitted but should be such that the structure can withstand these larger deformations without collapse. Thus, two issues come into picture, namely (a) ductility, *i.e.*, the capacity to withstand deformations beyond yield, and (b) over strength. Over strength is the total strength including the additional strength beyond the nominal design strength considering actual member dimensions and reinforcing bars adopted, partial safety factors for loads and materials, strain hardening of reinforcing steel, confinement of concrete, presence of masonry in fills, increased strength under cyclic loading conditions, redistribution of forces after yield owing to redundancy, *etc*. Hence, the response reduction factors $R$ used to reduce the maximum elastic forces to the design forces reflect these above factors. Clearly, the different bridge components have different ductility and over strength. For example, the superstructure has no or nominal axial load in it, and hence its basic behavior is that of flexure. However, the substructure (piers) which is subjected to significant amount of axial load undergoes a combined axial load-flexure behavior. It is well known that piers are much more ductile than the superstructures. Also, the damage to the substructure is more detrimental to the post-earthquake functioning of the bridge than damage to the superstructure. In the second case, the span alone may have to be replaced, while the first requires replacement of the entire bridge and minor modifications may not help. Thus, the R factors for superstructures are kept at a lower value than those for substructures. The superstructure is essentially expected to behave elastically and hence R value is taken as unity. A similar argument can be given

for the *R* values of foundations which are also lower than those for substructures. An important issue is that of connections, which usually do not have any significant post-yield behavior that can be safely relied upon. Also, there is no redundancy in them. Besides, there is a possibility of the actual ground acceleration during earthquake shaking exceeding the values reflected by the seismic zone factor *Z*. In view of these aspects, the connections are designed for the maximum elastic forces (and more) that are transmitted through them. Thus, the *R* factors for connections are recommended to have values less than or equal to 1.0. The R values for ductile frame type pier is taken as 3.25 as against 2.5 for single pier. For ductile RC buildings, the value of R is 5.0 (IS1893 (Part 1:2002)). The lower value of R for pier is due to less redundancy as compared to buildings and non-availability of alternate load path. In American code the value for ductile frame type pier is 5.0 as compared to R = 8 for ductile RC building frames. In Eurocode the behavior factor, q is taken as 3.5 for ductile RC pier as against 5.0 for ductile RC building.
Various components of the bridge do not enjoy the same level of ductility and over strength. Hence, the level of design seismic force *vis-à-vis* the maximum elastic force that will be experienced by the component if the entire bridge were to behave linearly elastic, varies for different bridge components.

The response spectrum values as given by IS code for medium soil are
For medium soil sites (Type II)

$$\left(\frac{Sa}{g}\right)_k = \begin{cases} 2.50 - for - T_k \le 0.55 \\ 1.36/T_k - for - 0.55 \le T_k < 3.0 \\ 0.45 - for - T_k \ge 3.0 \end{cases} \qquad \text{Eq. (9)}$$

## III. Formulation of Problem

STIFFNESS AND MASS MATRIX FORMULATION FOR EXTRADOSED BRIDGE

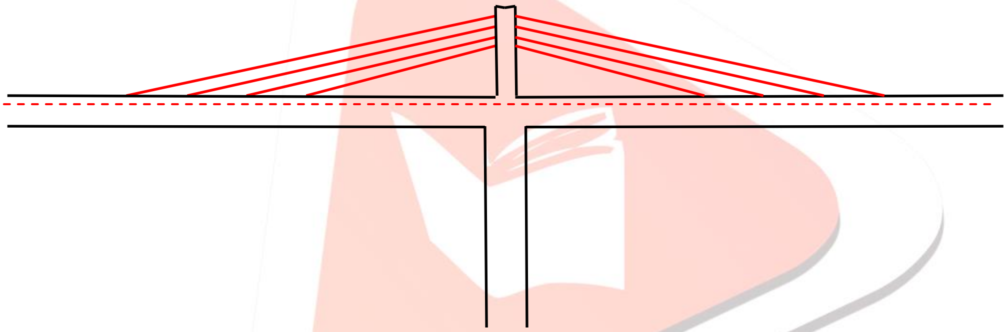

*Fig.4* Elevation – Typical Extradosed bridge

Consider a typical Extradosed bridge as shown in figure 4, let us take a small section as shown in the figure 5 below. The boundary conditions for this element can be considered as that of beam on elastic foundation to relate effect of elastic support provided by cable. Further this beam will be subjected to prestressing force due to horizontal component of cable forces, as shown in figure 5

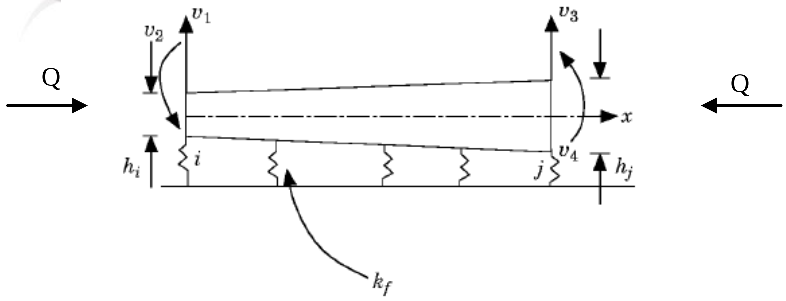


*Fig.5* Beam on elastic foundation

Now, consider an element *ij* of length *L* of a beam on an elastic foundation as shown in Figure.5 having a uniform width *b* and a linearly varying thickness *h(x)* It will be a simple matter to consider an element having a linearly varying width if the need arises. Neglecting axial deformations this beam on an elastic foundation element has two degrees of freedom per node a lateral translation and a rotation about an axis normal to the plane of the paper and thus possesses a total of four degrees of freedom. The (4x4) stiffness matrix *k* of the element is obtained by adding the (4x4) stiffness matrices *kB*, *kF* and *kQ* pertaining to the usual beam bending stiffness and foundation stiffness and stiffness due to prestressing force (Q) respectively
Since, there are four end displacements or degrees of freedom a cubic variation in displacement is assumed in the form

$$v = Aa \qquad \text{Eq. (10)}$$

Where, $A = (1\ x\ x^2\ x^3)$ and $a^T = (a_1\ a_2\ a_3\ a_4)$ (Displacement variation within element)
The four degrees of freedom corresponding to the displacements $v_1$, $v_3$ and the rotations $v_2$ $v_4$ at the longitudinal nodes are given by

$q=Ca$ (Nodal displacements) Eq. (11)

Where $q^T=(v_0\ v_1\ v_2\ v_3)$ and $C$ is the connectivity matrix for an element $ij$ between $x=0$ and $x=L$ *as given in* Figure 5

From equations (Eq.10) and (Eq.11)

$V=AC^{-1}q$ Eq. (12)

If $E$ is the Young|s modulus and $I=bh(x)^3/12$ is the second moment of area of the beam Cross-section about an axis normal to the plane of the paper\ the bending moment $M$ in the element is given by

$$M = D\frac{\partial^2 v}{\partial x^2} = DBC^{-1}q \qquad \text{Eq. (13)}$$

Where $D=EI(x)$ and $B=d^2A/dx^2=(0, 0, 2, 6x)$

A. Stiffness due to bending

The potential energy $U_B$ due to bending is

$$U_B = \frac{1}{2}\int_0^l \frac{d^2 v}{dx^2} M dx \qquad \text{Eq. (14)}$$

And the stiffness is given by

$$kb = \frac{\partial^2 U_B}{\partial d^2} \qquad \text{Eq. (15)}$$

From equations (Eq.14) and (Eq.15) we get,

$$\bar{kb} = \int_0^l B^T DBdx \text{ (Elemental)} \qquad \text{Eq. (16)}$$

$$Kb = (C^{-1})^T \bar{kb} C^{-1} \text{ (Assmebled)} \qquad \text{Eq. (17)}$$

B. Stiffness due to elastic foundation

The potential energy of foundation stiffness is given by,

$$U_f = \frac{1}{2}\int_0^l V^T kfVdx \qquad \text{Eq. (18)}$$

and then the stiffness is given by,

$$kf = \frac{\partial^2 U_f}{\partial d^2} \qquad \text{Eq. (19)}$$

From equations (Eq.18) in (Eq.19) we get,

$$\bar{kf} = \int_0^l A^T kfAdx \text{ (Elemental)} \qquad \text{Eq. (20)}$$

$$Kf = (C^{-1})^T \bar{kf} C^{-1} \text{ (Assembled)} \qquad \text{Eq. (21)}$$

C. Stiffness due to Prestressing force

The potential energy of prestressing force is given by,

$$U_Q = \frac{1}{2}\int_0^l Q\left(\frac{\partial v}{\partial x}\right)^2 dx \qquad \text{Eq. (22)}$$

Then the stiffness is given by,

$$k_Q = \frac{\partial^2 U_Q}{\partial d^2} \qquad \text{Eq. (23)}$$

Substituting equation (22) in (23) we get,

$$\bar{k}_Q = \int_0^l A^T k_Q A dx \qquad \text{Eq. (24)}$$

$$K_Q = (C^{-1})^T k_Q C^{-1} \qquad \text{Eq. (25)}$$

Finally complete stiffness is given by,

$$K = K_B + K_f + K_Q \qquad \text{Eq. (26)}$$

Element mass matrix is the equivalent nodal mass that dynamically represents the actual distributed mass of the element. This is kinetic energy of the element.

$$T = \frac{1}{2}\int_0^l (\dot{v})^T \rho dV \dot{v} \qquad \text{Eq. (27)}$$

Where, $\dot{v}$ = Lateral velocity and $\rho$ = mass density

$$T = \frac{\rho}{2}(\dot{q})^T\left(C^{-1}\right)^T\left\{\int_0^l A^T hxAdx\right\}(C)^{-1}q \qquad \text{Eq. (28)}$$

Then, the mass matrix is given by,

$$m = \left(C^{-1}\right)^T \overline{m} C^{-1} \qquad \text{Eq. (29)}$$

and $\overline{m} = \rho\int_0^l A^T hxAdx$ Eq. (30)

for free vibration of this beam,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \qquad \text{Eq. (31)}$$

and for forced vibration,

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f\} = [N]^{-1} f_0 \qquad \text{Eq. (32)}$$

For Extradosed bridge, since the cable are shallower and the effect of prestressing force is more the effecting of prestress shall be taken in to account as shown in the equation above.

**B) Vibration of Cables**

**i) With equivalent modulus**

In global analysis of cable stayed / Extradosed bridges, one common practice is to model each cable as a single truss element with an equivalent modulus to allow for sag. The element stiffness matrix in local coordinates for such a cable element can be written as,

$$k_c = \frac{A_c E_{eq}}{l_c}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{Eq. (33)}$$

The equivalent modulus of elasticity is given be

$$E_{eq} = \frac{E_c}{1 + (wH_c)^2 A_c E_c / 12T^3} \qquad \text{Eq. (34)}$$

Where, $l_c$ is chord length, $H_c$ is the horizontal projection length, $A_c$ is the cross-sectional area, $E_c$ is the effective material modulus of elasticity, $w$ is the weight per unit length and $T$ is the updated cable tension of the cable. A certain cable profile has been assumed to account for the effect of cable sag. However, once the equivalent modulus has been obtained, the profile will not have a role to play in the final analysis, and hence the method cannot model transverse vibrations of the cable.

**ii) With original modulus**

Another approach for accounting for the transverse vibrations of cables is to model each cable by number of cables elements with the original modulus. Following the sign conventions adopted by Broughton and Ndumbara (1994), the element incremental stiffness matrix in local coordinates can be written as

$$K_c = \frac{E_c A_c}{L_0 (L_0 + e)^2}$$

$$\begin{bmatrix} (L_0+u_c)^2 & v(L_0+u_c) & -(L_0+u_c)^2 & -v(L_0+u_c) \\ v(L_0+u_c) & v_c^{\,2} & -v(L_0+u_c) & -v_c^{\,2} \\ -(L_0+u_c)^2 & -v(L_0+u_c) & (L_0+u_c)^2 & v(L_0+u_c) \\ -v(L_0+u_c) & -v_c^{\,2} & v(L_0+u_c) & v_c^{\,2} \end{bmatrix}$$

$$+\frac{T}{(L_0+e)^3}$$

$$\begin{bmatrix} v_c^{\,2} & -v(L_0+u_c) & -v_c^{\,2} & v(L_0+u_c) \\ -v(L_0+u_c) & (L_0+u_c)^2 & v(L_0+u_c) & -(L_0+u_c)^2 \\ -v_c^{\,2} & v(L_0+u_c) & v_c^{\,2} & -v(L_0+u_c) \\ v(L_0+u_c) & -(L_0+u_c)^2 & -v(L_0+u_c) & (L_0+u_c)^2 \end{bmatrix} \qquad \text{Eq. (35)}$$

Where the updated element basic tension T and the element extension e along the deformed element longitudinal axis are given, respectively, by

$$T = T_0 + E_c A_c / L_0 e \qquad \text{Eq. (36)}$$

$$e = \sqrt{(L_0 + u_c)^2 + v_c{}^2} - L_0 \qquad \text{Eq. (37)}$$

$T_0$ is the original cable element pre-tension, $L_0$ is the original cable element length, and $u_c$ and $v_c$ are, respectively, the relative displacements of one node acting along and perpendicular to the cable chord with respect to the other node.

**iii) Mass matrix for cable**

The cable element mass matrix is the same for both the single-element and multiple-element modeling methods. The mass matrix is given as follows,

$$m_c = \frac{m_{cc}}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \qquad \text{Eq. (29)}$$

In which $m_{cc}$ is the total mass of the cable element

**C) Vibration of stay cables**

To demonstrate the abilities of various methods in predicting local cable vibrations, each stay cable was analyzed as an inclined stay cable fixed/pinned at both ends to evaluate the natural frequencies of local vibrations. It is noted however that the real situation is slightly different, as the end anchorages themselves are movable. The first symmetric and anti-symmetric in-plane transverse vibration frequencies $\omega$ in radians per second can be computed, respectively, as

$$\omega = \frac{\omega^*}{l}\sqrt{\frac{T_\theta}{m}}$$ For symmetric in-plane vibration and,

$$\omega = \frac{2\pi}{l}\sqrt{\frac{T_\theta}{m}}$$ For anti-symmetric in-plane vibration,

Where, $$\tan\left(\frac{\omega^*}{2}\right) = \left(\frac{\omega^*}{2}\right) - \frac{4}{\lambda^2}\left(\frac{\omega^*}{2}\right)^3 \qquad \text{Eq. (30)}$$

Where $l$ denotes the chord length, $T_\theta$ is static cable tension, $m$ is the cable mass per unit length.

## IV. ANALITICAL STUDY – FORCED VIBRATION BEHAVIOUR

To study static as well as dynamic behavior of Extradosed Bridge, 3 numbers of models with variable parameters are prepared. Basic span configuration as applicable for Extradosed span is selected to be 120, 200 and 260m main span, the side span is about 0.45 of main span. The pylon height is varied from 8 to 12 to account for the effect of varying cable inclinations. The cable inclination varies from 17 to 30 degrees. The requirement of cable area and prestressing is as per preliminary design. Box beam superstructure is adopted with solid rectangular pylon designed by working stress method. For details of model refer table-1.

Since, Extradosed bridges take part in an intermediate zone between prestressed bridges and cablestayed bridges, their structural behavior may be similar to these kinds of typologies, depending on design criteria adopted during the project stage. Generally a rigid deck Extradosed bridge shall have a similar behavior to the prestressed bridges, thus avoiding high stress oscillations of stay cables and, consequently, avoiding fatigue conditions associated with anchorages and tendons present in a slender deck extradosed bridge, which behavior is quite close to the cable-stayed bridge. Its construction demands the acquaintance of technologies currently applied on straight course-prestressed concrete bridges and cable-stayed bridges, which is generally developed by means of the consecutive cantilever method but counting with the assistance of tension rods that are not placed on temporary, but on permanent basis.

In order to analytically study structural behavior of Extradosed bridges with respect to ground acceleration numerical studies were conducted for forced vibration on typical Extradosed bridge. The parameters were non-dimentionalised using equivalent factors. The earthquake time histories used are having Koyna (PGA-0.49g), Bhuj (PGA-0.3g) and El-Centro (PGA-1.61g).

Table-1 Details of Earthquake time histories used for this study;

| Sr No | Span (L) configuration | Pylon ht (H) | Remarks |
|---|---|---|---|
| 1 | 48+100+48 | 10 | On pile foundation, 12.5 wide deck |
| 2 | 90+200+90 | 20 | On pile foundation, 14 wide deck |
| 3 | 110+260+110 | 26 | On pile foundation, 18 wide deck |

Non-Dimensionalizing of parameters

Forced vibration analysis for three earthquake time histories having different characteristics are undertaken. To compare the results all parameter have been non dimensionalised using equivalent factors as mentioned below;

- $V = \rho * g * A * L$ Eq. (31)
- $M = \rho * g * A * L^2$ Eq. (32)

Where, V & M are non dimensioning factors for shear force, bending moment. Where, $\rho$ = Mass Density, g= Gravitational acceleration, A= Cross section area of component and L= Half span length of the component.

Span and pylon height are non-dimensionlalized by using parametric length. The results obtained from the time history analysis in terms of bending moment and shear forced in the structure are non-dimensionalised and superimposed and presented in Fig 6 to 11

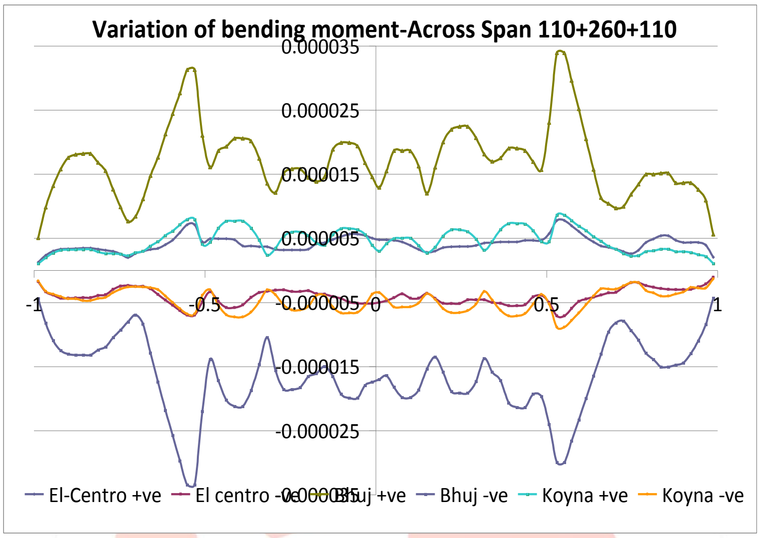


*Fig.6* Variation of bending moment across span for 110+260+110m span (Non-dimentionalized)

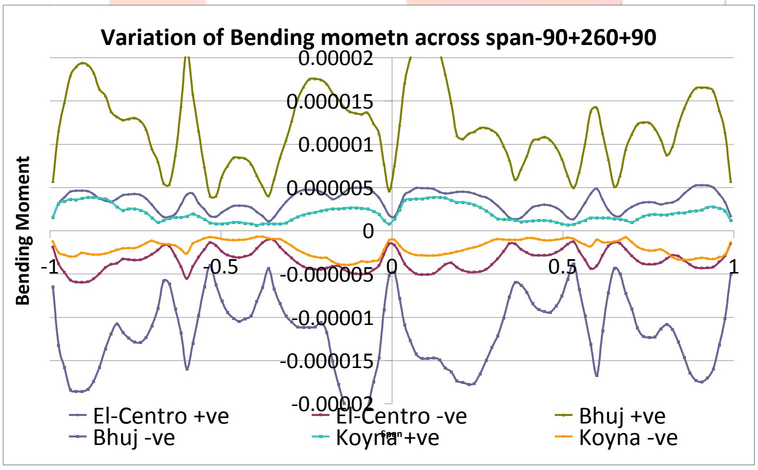


*Fig.7* Variation of bending moment across span for 90+200+90m span (Non-dimentionalized)

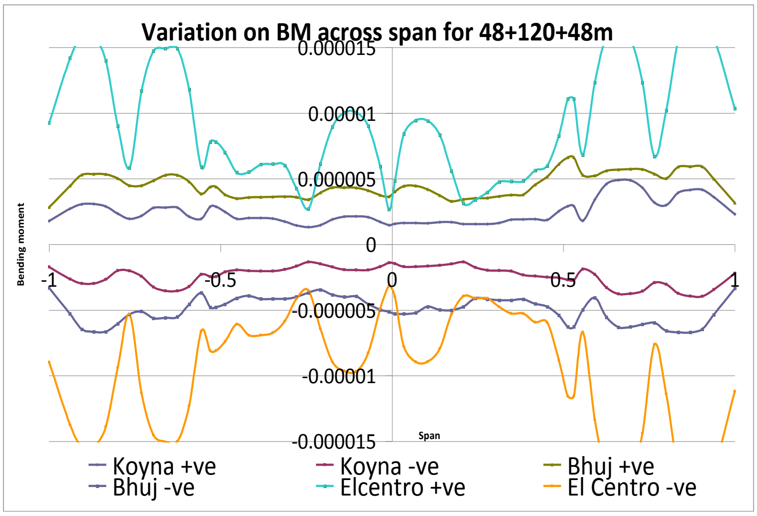


*Fig.8* Variation of bending moment across span for 48+120+48m span (Non-dimentionalized)

Fig 9 to 11 gives results for BM across pylon height

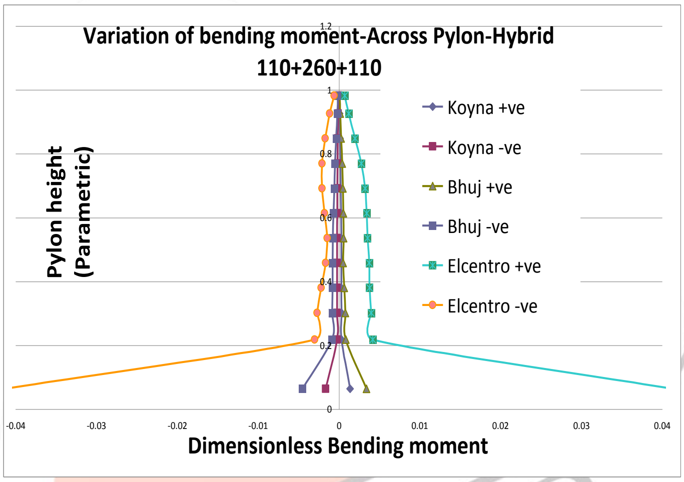


*Fig.9* Variation of bending moment across pylon height for 110+260+110m span (Non-dimentionalized)

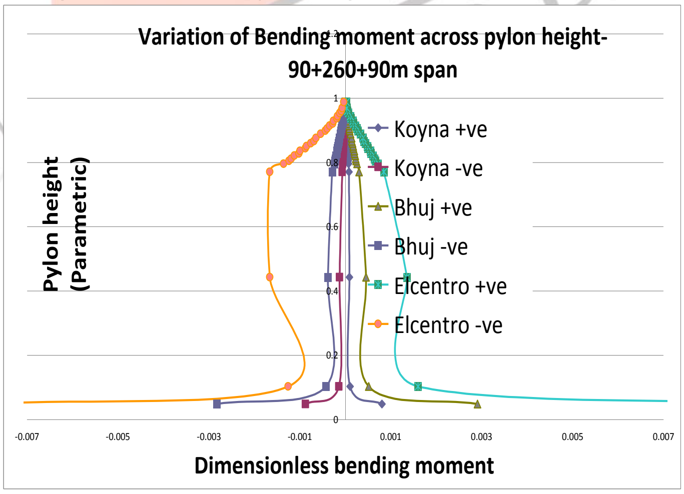


*Fig.10* Variation of bending moment across pylon height for 90+200+90m span (Non-dimentionalized)

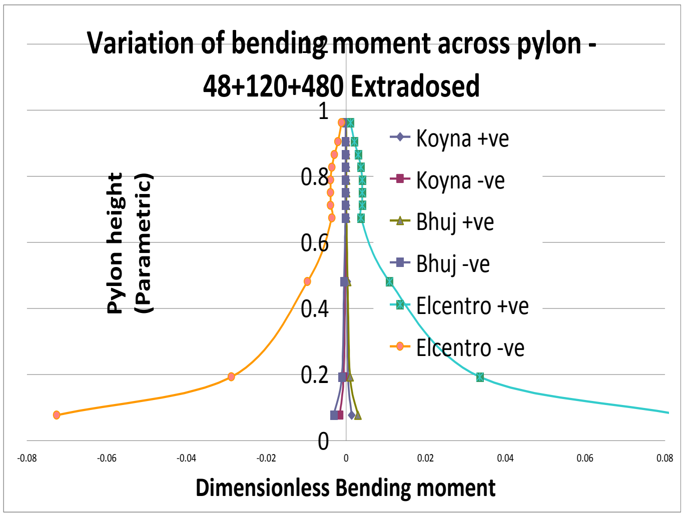


*Fig.11* Variation of bending moment across pylon height for 48+120+48m span (Non-dimentionalized)

Forced vibration studies of deck and pylon of three types of bridges reaffirms following facts for extradosed bridge,

- Magnitude of bending moment / shear force is directly proportional to the magnitude of forcing function / Peak Ground Acceleration (PGA).
- With increase in the distance between cables supports the shear force in deck also increases.
- Pylon stiffness does not have any effect on the deck moments/shear.
- It is observed that only for cable stayed bridge with harp shape cable arrangement the shear force reduces at the junction of deck.

## V. ANALITICAL STUDY – PGA-EDR RELATIONSHIPS

It is seen in above sections that for time history analysis generally the magnitude of structural actions are directly proportional to magnitude of forcing function ie. Its Peak ground acceleration. Hence it will be possible to relate these quantities as a function of forcing frequency. The term EDR as explained below is used for this purpose.

All data of earthquake records are now days processed using “Seismosignal” software. Seismosignal processes strong motion seismic data. It will generate strong motion parameter such as Fourier and Power spectra, Elastic response spectra and pseudo spectra, over damped and constant ductility in elastic response spectra. Spatial earthquake represents ie. Time history components or all the three direction viz. longitudinal, transverse and vertical are specified. This is data can be obtained from various agencies and can be classified by the Peak Ground Accelerations.

**Earthquake Deformation ratio (EDR)** Every structure has its stiffness characteristics which controls the behaviour of structure. To account for this variable, the ratio EDR is defined as ratio of maximum seismic displacement to maximum static displacement, shall be used. These both displacements measured at the same point.

The values of PGA and EDR can be plotted against each other to get correlation by curve fitting to obtain the values of EDR for various PGA it is required to simulate earthquake Ground accelerations on analytical model in computer analysis using Time histories which is the time variation of ground acceleration. It is the most useful way of defining the shaking of ground during earthquake. This ground acceleration is descriptive by numerical values at discrete time intervals. Integration of this time acceleration history gives velocity history, integration of which intern gives displacement history.

The proposed equation is of the form of

**EDR = A1 + B1 (PGA) + B2 (PGA)^2 + B3 (PGA)^3** Eq. (33)

Where, A1, B1, B2 B3 are the constants as obtained from curve fitting data. The constants need to be calculated for each response parameters to be considered eg. Deck / pylon deflection (longitudinal, transverse, vertical etc). The accuracy of the method may be compared with existing codal methods.

## VI. DISCUSSIONS AND CONCLUSIONS

Forced vibration studies of three different Extradosed bridges are done. It is noted that;

1. Forced vibration is governed by peak accelerations
2. Design of structures is most often governed by seismic cases and combinations thereof.
3. For cable stayed structures it is difficult to predict dynamic response using usual method
4. Accurate analysis viz. Time history analysis involves time, cost and skill requirements
5. Effect of seismic transmission units or isolators can not be considered in codal methods.
6. The commonly used simplified methods used for analysis are based of theory of dynamics pertaining to SDOF systems. Rules of modal combinations viz SRSS, CQC are used for MDOF systems.
7. It is observed that there is some relationship between Peak Ground Accelerations and the response of structure, relationship between these needs to be established by further study.

Forced vibration studies of deck and pylon of three types of bridges need to be done to find out correlation between peak ground acceleration and Earthquake displacement ratio.

The correlations can be used to quickly predict the expected dynamic response of the highly indeterminate structures like Extradosed cable stayed bridge during preliminary design as exact dynamic analysis at this stage is time and cost consuming and also not very much desired. Exact time histories at all locations are not available and design can be done for expected PGA. Using these relations displacement based design can be done for structure ie. By applying obtained deflections on static structural model. Apart from this the EDR values can be used as a tool for seismic damage index for earthquakes beyond designed values and provides less conservative approach than response spectrum analysis. Thus it will provide a quicker tool to assess the seismic vibrations response of the structure against the design PGA.

## VII. ACKNOWLEDGEMENTS

Authors acknowledge the immense help received from the scholars whose articles are cited and included in references of this manuscript. The authors are also grateful to authors / editors / publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed.

## REFERENCES

[1] Au F. T. K., Chang Y. S., Cheung Y. K., Zheng D. Y., "*On the Determination of Natural Frequencies and Mode Shapes of Cable Stayed Bridges*", Applied Mathematical Modeling, 2001, Vol-25, 1099-1115.
[2] Chio Cho, G., and Aparicio Bengoechea, A. (2002). "El Puente Pretensado Extradosado. Un Nuevo Tipo Estructural." Uis Ingenierias, 1(1), 67 - 73.
[3] Diego Cobo del Acro, Aparicio A, Mari R Antonio, "Preliminary design of prestressed concrete stress ribbon bridge", Journal of bridge engineering, Aug-01
[4] Fleming J. E., Engin A. E., "Dynamic Behaviour of Cable Stayed Bridge" , Earthquake Engineering & Structural Dynamics , 1980, Vol-8, 1-16
[5] Hong Mamhee Kim, Chang Sung-Pil, Lee Seung-Chul. "Development of ANN-based preliminary structural design systems for cable-stayed bridges", Advances in Engineering Software 33 (2002) 85-96
[6] Huang Ling, Hikosaka Hiroshi, Komine Keizo, "Simulation of accordion effect in corrugated steel web with concrete flanges", Computers and Structures 82 (2004) 2061–2069
[7] Kasuga Akio, "Construction of extradosed bridges in japan" Seminar On Design & Construction PC Cable Stayed Bridges - Kaula Lampur – Aug 2002
[8] Komiya, M. (1999). "Characteristics and Design of PC Bridges with Large Eccentric Cables." Extradosed Bridge Technology in Japan and the New Pearl Harbor Memorial Bridge. Federal Highway Administration / U.S. Department of Transportation and The Connecticut Department of Transportation, Washington, DC. 55-80.
[9] Mathivat, J. (1988). "Recent developments in prestressed concrete bridges." FIP Notes, 1988(2), 15-21.
[10] Menn, C. (1987). "Consistent Design and New Systems for Concrete Bridges." International Conference on Cable-Stayed Bridges, Bangkok, Thailand, 833-840.
[11] Menn, C. (1994). "Conceptual Design of Cable-Stayed Bridges." International conference A.I.P.C.-F.I.P.: ponts suspendus et à haubans : cable-stayed and suspension bridges. Deauville, October 12-15. 181-194.
[12] Moona Jiho, Yi Jongwon, Choi Byung H, Lee Hak-Eun, "Shear strength and design of trapezoidally corrugated steel webs" Journal of Constructional Steel Research July-08
[13] Nakamura Shun-ichi, Tanakab Hioyasu, Kazutoshi Kato. "Static analysis of cable-stayed bridge with CFT arch ribs", Journal of Constructional Steel Research – May 2008
[14] Otsuka H, Wakasa T, Ogata J, Yabuki W, , Takemura D, "Comparison of structural characteristics for different types of cable-supported prestressed concrete bridges" Structural Concrete 2002 (fib journal)
[15] Reich Yoram, Fenves Steven, "System that learns to design cable-stayed bridges" Journal of structural engineering, July 1995
[16] Santos, D. M. dos (2006). "Comportamento estrutural de pontes com protensão no extradorso" thesis, prestented to Departamento de Engenharia de Estructuras e Fondaçoes - Escola Politécnica da Universidade de São Paulo in partial fulfillment of the requirements for a Master degree.
[17] Takami Katsuhi`ko, Hamada Sumia, "Behavior of extradosed bridge with composite girder" ASCE / Journal Of Bridge Engineering Aug 96
[18] M V Sardesai, A K Desai, Investigation into Cable-Structure Interaction For Extradosed Bridge, International Journal of Engineering Research and Applications 2013
[19] _M V Sardesai, A K Desai, Free and forced vibration analysis of Extradosed Bridge

**I. S. & IRC Codes:**

[20] IRC-6 Standard specifications and code of practice for road bridges, Loads and stresses.
[21] IS: 1893(Part 1):2002,"Criteria for Earthquake Resistant Design of structures (General Provisions & Buildings)".
[22] IS: 1893(Part 3):2002, "Draft Indian Standard on Criteria for Earthquake Resistant Design of Structures (Bridges & Retaining walls)".
[23] IS:875,"Code of practice for Design Loads (Other than Earthquake) for Buildings and Structures