Design and Fracture Analysis of Thin Plate

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***Abstract* - The basic aim of this project is to study the deformation and the way of failure of a plate which is having crack height, width of 10&20mm respectively at one end of plate , and to determine the stress intensity factor(k) for failure. For doing the fracture analysis we are using FINITE ELEMENT ANALYSIS package, ANSYS software, the geometric model with crack is created and then the model is converted in to finite element model by meshing. By taking boundary conditions (Displacement & Pressure). S.I.F is calculated for both mechanical loading and and stress by using ANSYS software. Finally the validification of the software is also concluded by theoretical comparison which displayed in this project. The conclusion and the limitations of the project are listed at the end of the documentation. However, ANSYS software is predominant in finite element analysis which is unbounded and it is applicable to number of problems. So, my project will be limited for above mentioned limitations.**

***Key words* - S.I.F, FEA, ANSYS, Plane Stress Analysis, Structural Analysis.**

I. INTRODUCTION

History of Fracture Mechanics

Fracture is a problem that society has faced for as long as there have been man-made structures. The problem may actually be worse today than in previous centuries, because more can go wrong in our complex technological society. Major airline crashes, for instance, would not be possible without modern aerospace technology.

Fortunately, advances in the field of fracture mechanics have helped to offset some of the potential dangers posed by increasing technological complexity. Our understanding of how materials fail and our ability to prevent such failures has increased considerably since World War II. Much remains to be learned, however, and existing knowledge of fracture mechanics is not always applied when appropriate.

While catastrophic failures provide income for attorneys and consulting engineers, such events are detrimental to the economy as a whole. An economic study [1] estimated the annual cost of fracture in the U.S. in 1978 at $119 billion (in 1982 dollars), about 4% of the gross national product. Furthermore, this study estimated that the annual cost could be reduced by $35 billion if current technology were applied, and that further fracture mechanics research could reduce this figure by an additional $28 billion.

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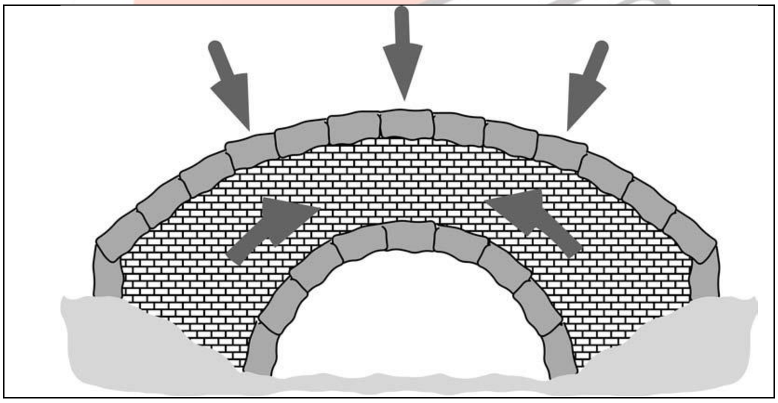


Fig 1 Schematic Roman bridge design. The arch shape of the bridge causes loads to be transmitted through the structure as compressive stresses.

II. WHY STRUCTURES FAIL

The cause of most structural failures generally falls into one of the following categories:

1. Negligence during design, construction, or operation of the structure.
2. Application of a new design or material, which produces an unexpected (and undesirable) result.

In the first instance, existing procedures are sufficient to avoid failure, but are not followed by one or more of the parties involved, due to human error, ignorance, or willful misconduct. Poor workmanship, inappropriate or substandard materials, errors in stress analysis, and operator error are examples of where the appropriate technology and experience are available, but not applied.

The second type of failure is much more difficult to prevent. When an ‘‘improved” design is introduced, invariably, there are factors that the designer does not anticipate. New materials can offer tremendous advantages, but also potential problems. Consequently, a new design or material should be placed into service only after extensive testing and analysis. Such an approach will reduce the frequency of failures, but not eliminate them entirely; there may be important factors that are overlooked during testing and analysis.

The Fracture Mechanics Approach To Design

Figure 2 contrasts the fracture mechanics approach with the traditional approach to structural design and material selection. In the latter case, the anticipated design stress is compared to the flow properties of candidate materials; a material is assumed to be adequate if its strength is greater than the expected applied stress. Such an approach may attempt to guard against brittle fracture by imposing a safety factor on stress, combined with minimum tensile elongation requirements on the material. The fracture mechanics approach (Figure 2(b)) has three important variables, rather than two as in Figure 2(a). The additional structural variable is flaw size, and fracture toughness replaces strength as the relevant material property. Fracture mechanics quantifies the critical combinations of these three variables.

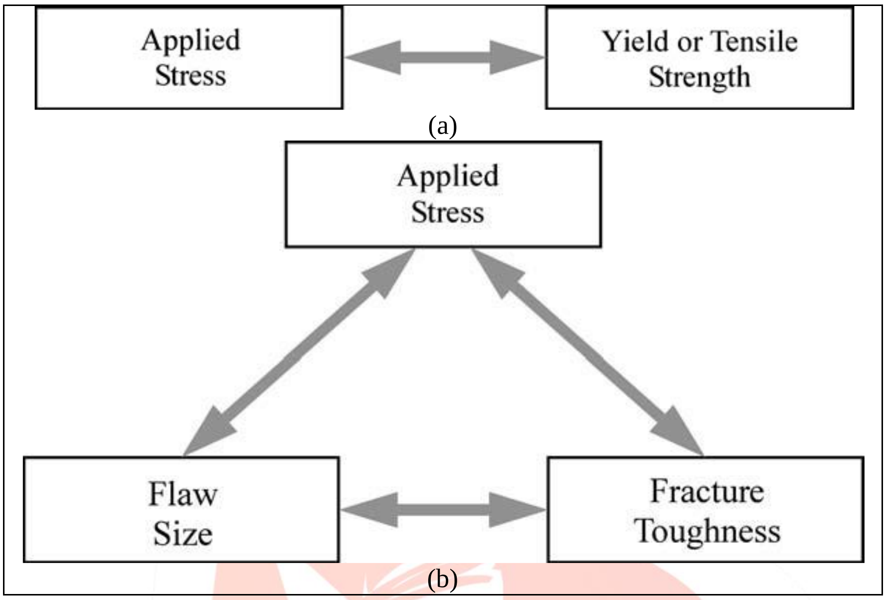


Fig 2 Comparison of the fracture mechanics approach to design with the traditional strength of materials approach: (a) the strength of materials approach and (b) the fracture mechanics approach.

Effect of Material Properties on Fracture

Figure 3 shows a simplified family tree for the field of fracture mechanics. Most of the early work was applicable only to linear elastic materials under quasistatic conditions, while subsequent advances in fracture research incorporated other types of material behaviour. Elastic-plastic fracture mechanics considers plastic deformation under quasistatic conditions, while dynamic, viscoelastic, and viscoplastic fracture mechanics include time as a variable. A dashed line is drawn between linear elastic and dynamic fracture mechanics because some early research considered dynamic linear elastic behaviour.

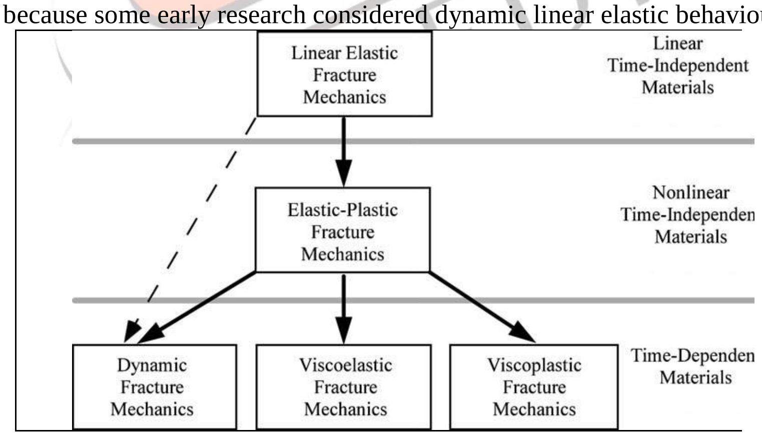


Fig 3 simplified family tree of fracture mechanics.

Typical Fracture Behaviour Of Selected Materials

Material	Typical Fracture Behaviour
High strength steel	Linear elastic
Low- and medium-strength steel	Elastic-plastic/fully plastic
Austenitic stainless steel	Fully plastic
Precipitation-hardened aluminium	Linear elastic
Metals at high temperatures	Viscoplastic
Metals at high strain rates	Dynamic/viscoplastic
Polymers (below T_g)[b]	Linear elastic/viscoelastic

Polymers (above T_g)[b]	Viscoelastic
Monolithic ceramics	Linear elastic
Ceramic composites	Linear elastic
Ceramics at high temperatures	Viscoplastic

Temperature is ambient unless otherwise specified.
[b]T_g—Glass transition temperature.

Table 1**.** typical Fracture Behaviour Of Selected Materials.

III. CRACK THEORY

Modes Of Fracture Failure

Three types of crack propagations are recognized: opening, sliding, and tearing. These types are called modes I, II, and III, respectively. A flaw may propagate in a particular mode or in a combination of these modes. These modes of fracture are explained in detail in the following section.

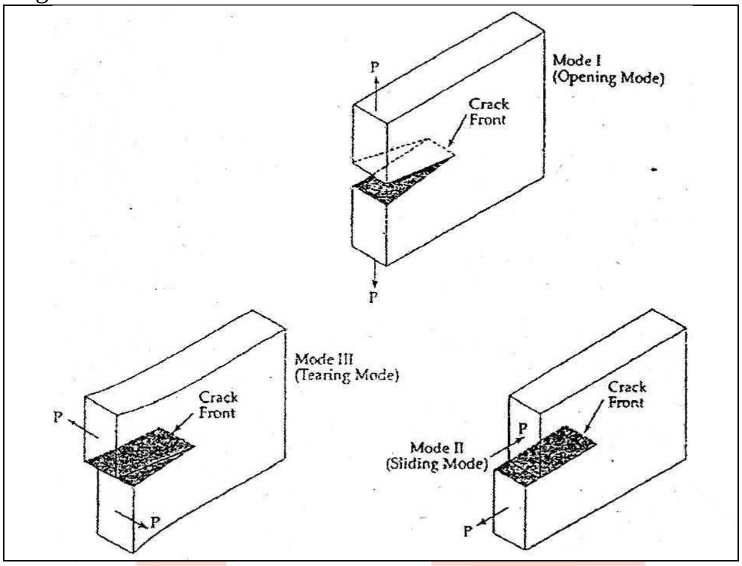


fig.4 modes of fracture failure

Stress Intensity Factor

Introduction

Knowing stress or displacement field in the vicinity of crack tip is useful in many ways. An experimentalist can think of methods of characterizing cracks by measuring stresses are strains near the crack tip. One of the biggest advantages is that stress analysis leads to define parameter stress intensity factor to characterize a crack. In comparison to energy release rate SIF is more handy for a designer and easier to measure in laboratory for determining material properties.

The credit goes to Irwin who defined the new variable, stress intensity factor, and used the symbol K after the name of his collaborator Kies. He defined K as

$$K_1 = \sigma(\pi a)^{\frac{1}{2}} \quad \text{.......................... (1.1)}$$

However the stress intensity factor

$$K_1 = (2\pi r)^{\frac{1}{2}} \sigma_{22}(r, \theta = 0) \quad \text{.......................... (1.2)}$$

The stress and displacement equations may be written in terms of the stress intensity factor for more one problems of plane strain. They become

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \quad \text{.......................... (1.3a)}$$

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \quad \text{.......................... (1.3b)}$$

$$\sigma_{12} = \frac{K_1}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad \text{.......................... (1.3c)}$$

$$u_1 = \frac{K_1}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \cos\frac{\theta}{2}\left[1 - 2\nu + \sin^2\frac{\theta}{2}\right] \quad \text{.......................... (1.3d)}$$

$$u_1 = \frac{K_1}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[1 - 2\nu + \cos^2\frac{\theta}{2}\right] \quad \text{(1.3e)}$$

Stress and displacement equation for the centre crack body are similar for other modes. For Mode-Il in plane strain and far field stress $\sigma_{12} = \tau$ with

$$K_{11} = \tau\sqrt{\pi\, a}$$

Then

$$\sigma_{11} = -\frac{K_{11}}{(2\pi\, r)^{1/2}} \sin\frac{\theta}{2}\left[2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

$$\sigma_{22} = \frac{K_{11}}{(2\pi\, r)^{1/2}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{12} = \frac{K_{11}}{(2\pi\, r)^{1/2}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2 - 2\nu + \cos^2\frac{\theta}{2}\right]$$

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2 - 2\nu + \cos^2\frac{\theta}{2}\right]$$

$$U_2 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \cos\frac{\theta}{2}\left[-1 + 2\nu + \sin^2\frac{\theta}{2}\right]$$

$$U_3 = 0$$

For Mode III for field stress $\sigma_{23} = \tau$ with $K_{III} = \tau\sqrt{\pi a}$

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{12} = 0$$

$$\sigma_{13} = -\frac{K_{III}}{(2\pi\, r)^{1/2}} \sin\frac{\theta}{2}$$

$$\sigma_{23} = \frac{K_{III}}{(2\pi\, r)^{1/2}} \cos\frac{\theta}{2}$$

$$U_1 = U_2 = 0$$

$$U_3 = \frac{K_{III}}{\mu}\left(\frac{2r}{\pi}\right)^{1/2} \sin\frac{\theta}{2}$$

IV. EFFECTIVE CRACK LENGTH

The appearance of the plastic zone at the tip does not allow material to bear high stresses predicted by the elastic analysis. In fact owing to the presence of the plastic zone the stiffness of the component decreases or the compliance increases. Consequently the crack is equivalent to a length that is longer than actual length.

EFFECT OF PLATE THICKNESS

For a plate having its thickness less than or equal to the size of plastic zone the crack is loaded on the plane stress. Fig. 2.3a shows the case of plane stress with a section to the plastic zone.

The thick plates (Fig. 2.3c) correspond; to plane, strain showing smaller plastic zone. Even in this case some effect of free surface exists where the plastic zone is larger. However, the thick region of plane strain dominates and the surface effects can be neglected. A plate having thickness greater or equal to *2.5* $\frac{K_I^2}{\sigma_{ys}^2}$ is regarded as a case of plane strain. In the transitional cases (Fig. 2.3b) the interior of the plate as well as its surface have mixed effects on the plastic zone.

It is evident from Fig. 2.3 that critical SIF of a plate depends upon its thickness. Typical nature of critical SIF dependence on the thickness is shown in Fig. 2.4.

For $B > 2.5\frac{K_{IC}^2}{\sigma_{ys}^2}$ critical SIF remains constant and then we can regard the critical stress intensity factor as the material property.

For $B > 2.5\frac{K_{IC}^2}{\sigma_{ys}^2}$ critical stress intensity factor depends on the thickness *B*. The relation between critical SIF and thickness may be regarded as a behavior of material and be provided to designers.

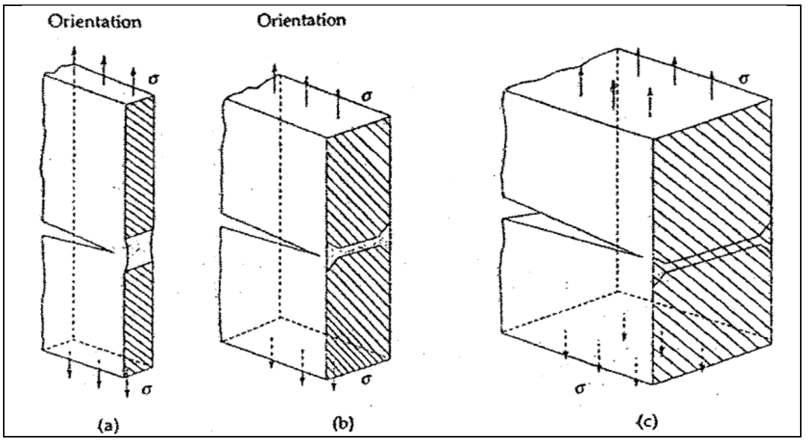


Fig 5. Plastic zone size for a) plane stress b) transitional zone c) plane strain

V. CALCULATIONS

Therfore The stress intensity factor

$$K_1 = \sigma(\pi a)^{\frac{1}{2}}$$

1. 1000 pa stresss applied at top and end lines of thin plate , then The stress intensity factor K_1=1000(0.496)= **496Pa (m)$^{0.5}$.**

And the stress in X-direction is

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Therefore σ_{11} =496(0.2833)=140.05E+06 pa

=**0.140E+09 N/mm^2.**

And stress in Y-direction is

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Then σ_{22} = 496(0.5930)=294.12E+06 pa

=**0.294E+09 N/mm^2.**

And stress in Z-direction is negligible so we can neglect that stress

The displacement of crack- tip in X-direction

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]$$ μ is shear modulus and 33E+03 pa

U_1= 496(1.3161E-05)= 0.0065m.

=**6.52mm.**

2. 1500 pa stresss applied at top and end lines of thin plate , then The stress intensity factor K_1=1500(0.496)= **744 Pa (m)$^{0.5}$.**

And the stress in X-direction is

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Therefore σ_{11} =744(0.2833)=210.77E+06 pa

=**0.210E+09 N/mm^2.**

And stress in Y-direction is

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Then σ_{22} = 992(0.5930)=441.19E+06 pa

=**0.441E+09 N/mm^2.**

And stress in Z-direction is negligible so we can neglect that stress

The displacement of crack- tip in X-direction

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]$$ μ is shear modulus and 33E+03 pa

U_1= 744(1.3161E-05)= 0.0097m.

=**9.79mm.**

3. 2000 pa stresss applied at top and end lines of thin plate , then The stress intensity factor K_1=2000(0.496)= **992 Pa (m)$^{0.5}$.**

And the stress in X-direction is

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Therefore σ_{11} =992(0.2833)=281.06E+06 pa

=**0.281E+09 N/mm^2.**

And stress in Y-direction is

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Then σ_{22} = 992(0.5930)=588.29E+06 pa

=**0.588E+09 N/mm^2.**

And stress in Z-direction is negligible so we can neglect that stress

The displacement of crack- tip in X-direction

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]$$ μ is shear modulus and 33E+03 pa

U_1= 992(1.3161E-05)= 0.0130m.

=**13.055mm.**

4. 2500 pa stresss applied at top and end lines of thin plate , then The stress intensity factor K_1=2500(0.496)=**1240 Pa (m)$^{0.5}$.**

And the stress in X-direction is

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Therefore σ_{11} =1240(0.2833)=351.29E+06 pa

=**0.351E+09 N/mm^2.**

And stress in Y-direction is

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Then σ_{22} = 1240(0.5930)=735.32E+06 pa

=**0.735E+09 N/mm^2.**

And stress in Z-direction is negligible so we can neglect that stress

The displacement of crack- tip in X-direction

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]$$ μ is shear modulus and 33E+03 pa

U_1= 1240(1.3161E-05)= 0.0163m

=**16.31mm.**

5. 3000 pa stresss applied at top and end lines of thin plate , then The stress intensity factor K_1=3000(0.496)=**1488 Pa (m)$^{0.5}$.**

And the stress in X-direction is

$$\sigma_{11} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Therefore σ_{11} =1488 (0.2833)=421.55E+06 pa

=**0.421E+09 N/mm²**.

And stress in Y-direction is

$$\sigma_{22} = \frac{K_1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$ at angle 45°, and r= 0.7070

Then σ_{22} = 1488(0.5930)=882.38E+06 pa

=**0.882E+09 N/mm²**.

And stress in Z-direction is negligible so we can neglect that stress

The displacement of crack- tip in X-direction

$$U_1 = \frac{K_{11}}{\mu}\left(\frac{r}{2\pi}\right)^{1/2} \sin\frac{\theta}{2}\left[2-2\nu+\cos^2\frac{\theta}{2}\right]$$ μ is shear modulus and 33E+03 pa

U_1= 1488(1.3161E-05)= 0.01935m.

=**19.58mm.**

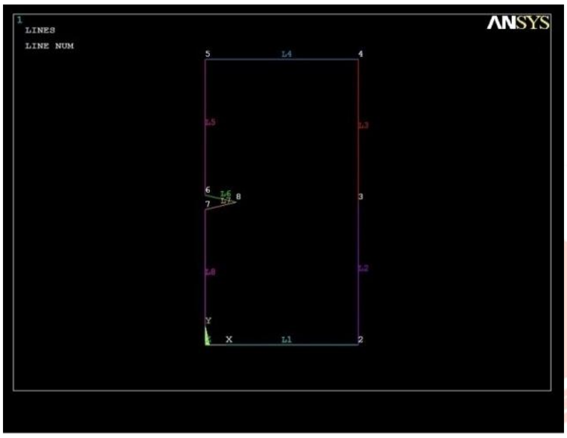


fig. creating geometrical model using key points

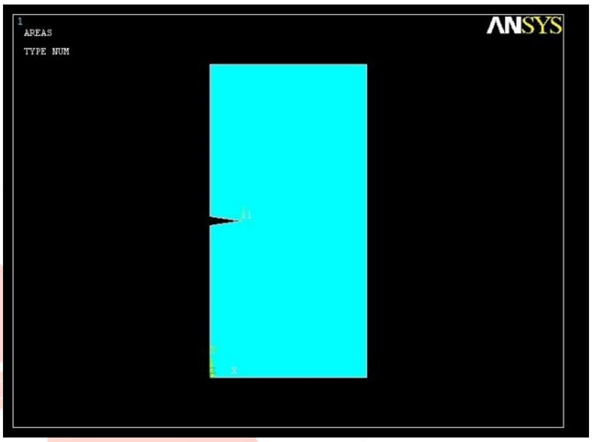


fig. solid model

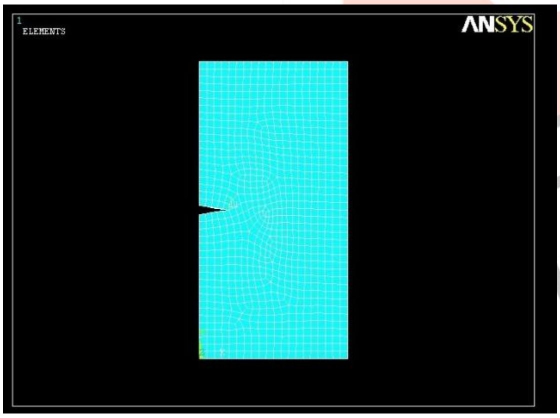


fig. after meshing

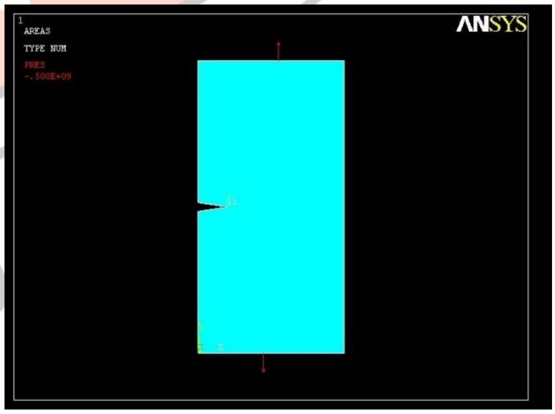


fig. applying load

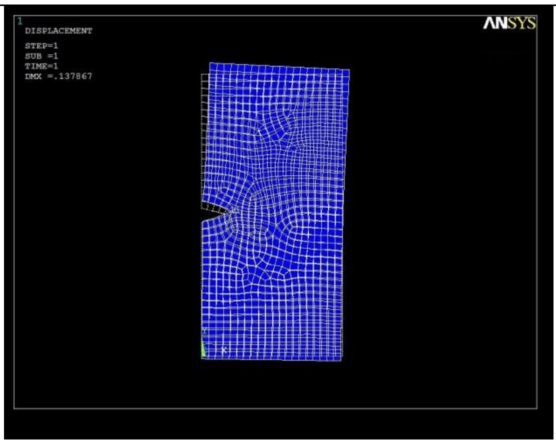


fig. deformed and un-deformed modes of structure

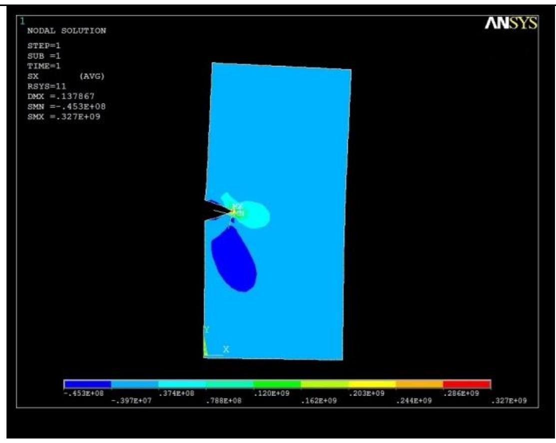


fig.stress in x direction

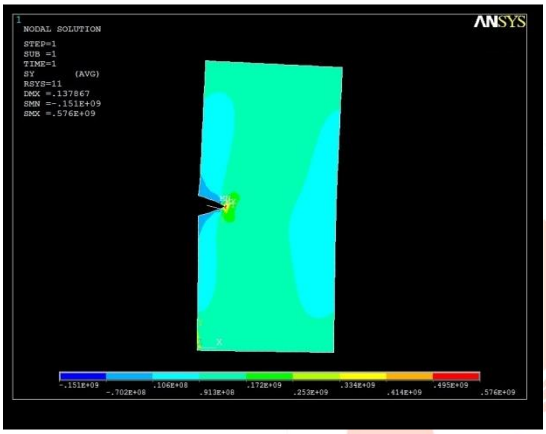


fig. stress in y direction

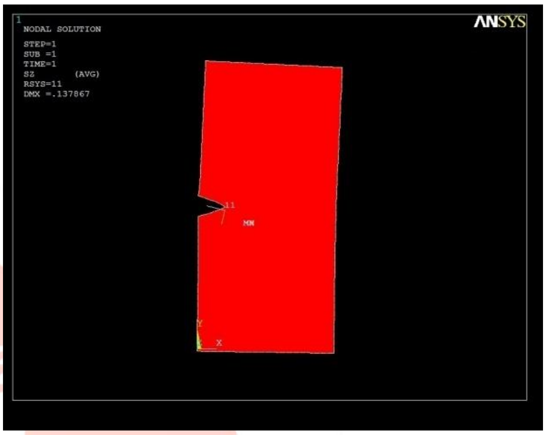


fig. stress in z direction

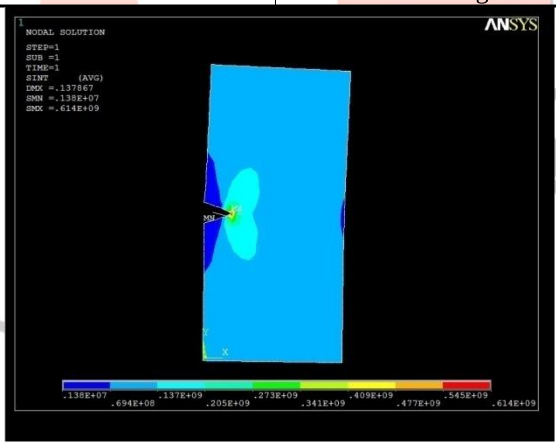


fig. stress intensity

practical stress vs stress intensity factors

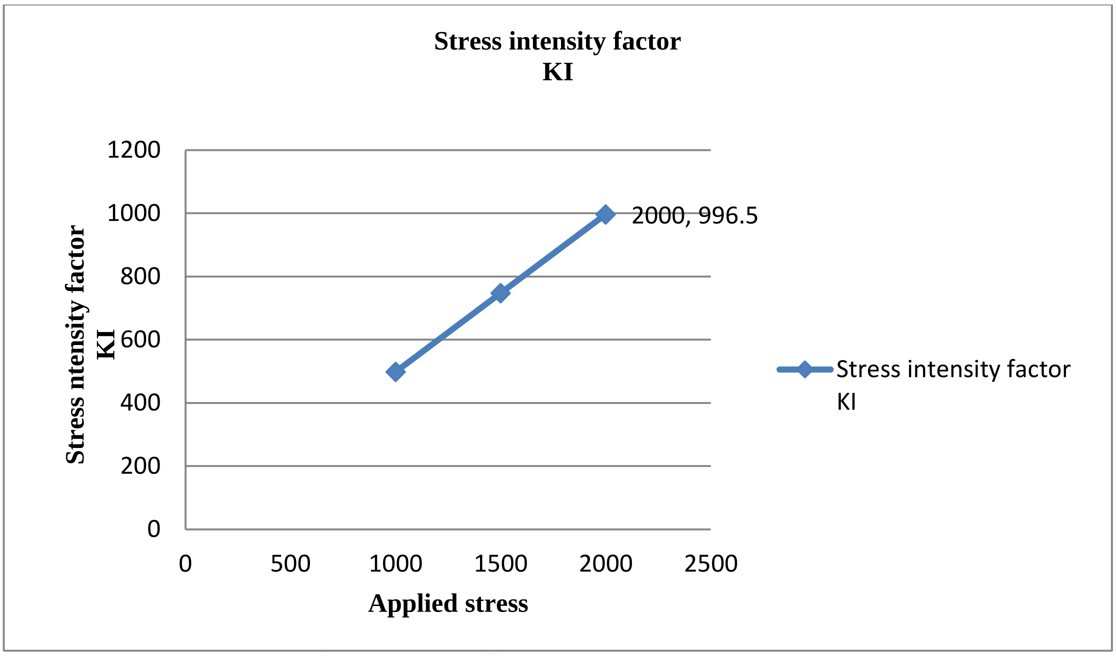
S.No	Stress in Pa	Stress ntensity factor KI	Stress Intensity factor KII
1	1000	498.25	263.55
2	1500	747.37	395.32
3	2000	996.50(Crack tip opened)	572.10
4	2500	1195.80	632.52
5	3000	1494.75	790.65

table. practical stress vs stress intensity factors

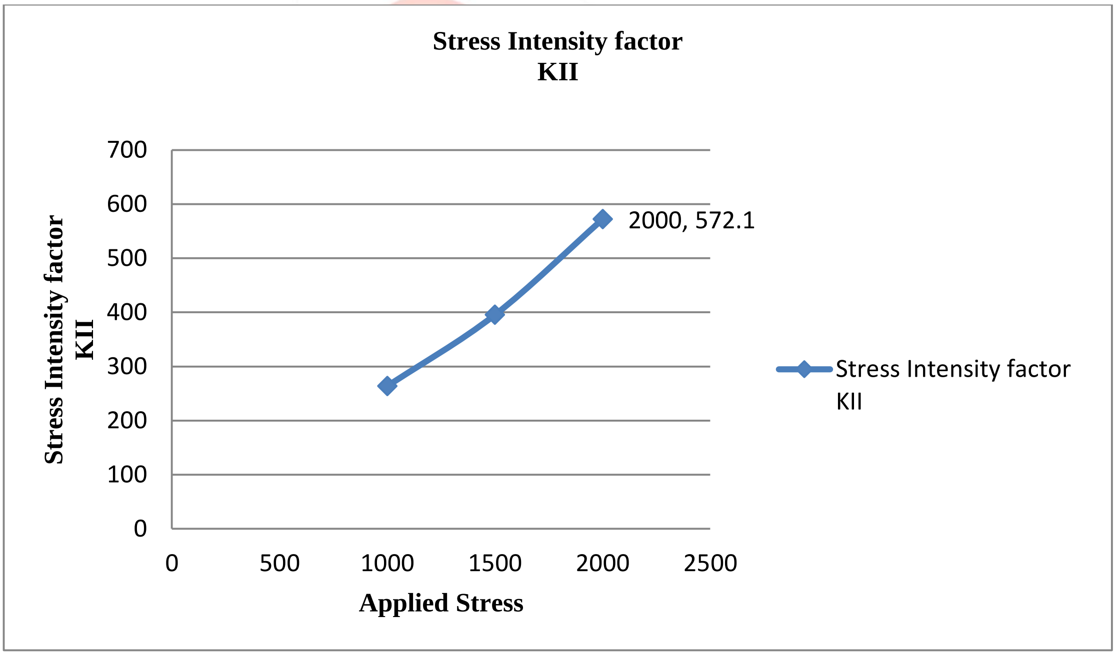
theoretical values of stress vs displacement vs stress intensity factor

S.No	Stress in Pa	Displacement in mm	Stress Intensity factor, K_1 Pa $(m)^{0.5}$
1	1000	6.52	496
2	1500	9.79	744
3	2000	13.05	992
4	2500	16.31	1240
5	3000	19.58	1488

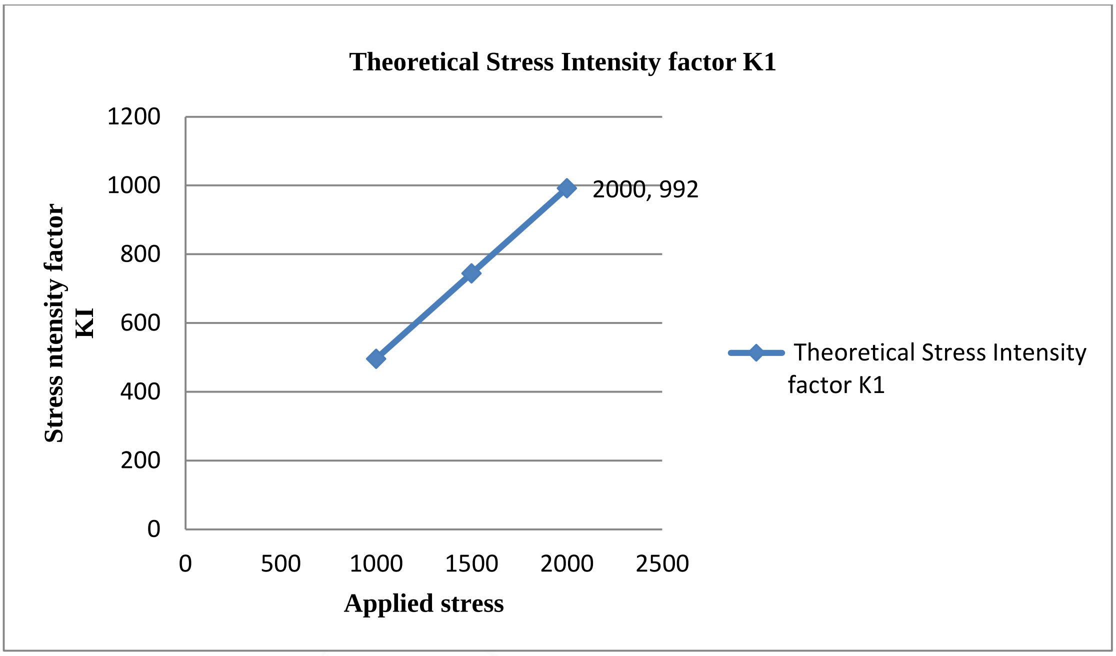
table. theoretical values of stress vs displacement vs stress intensity factor



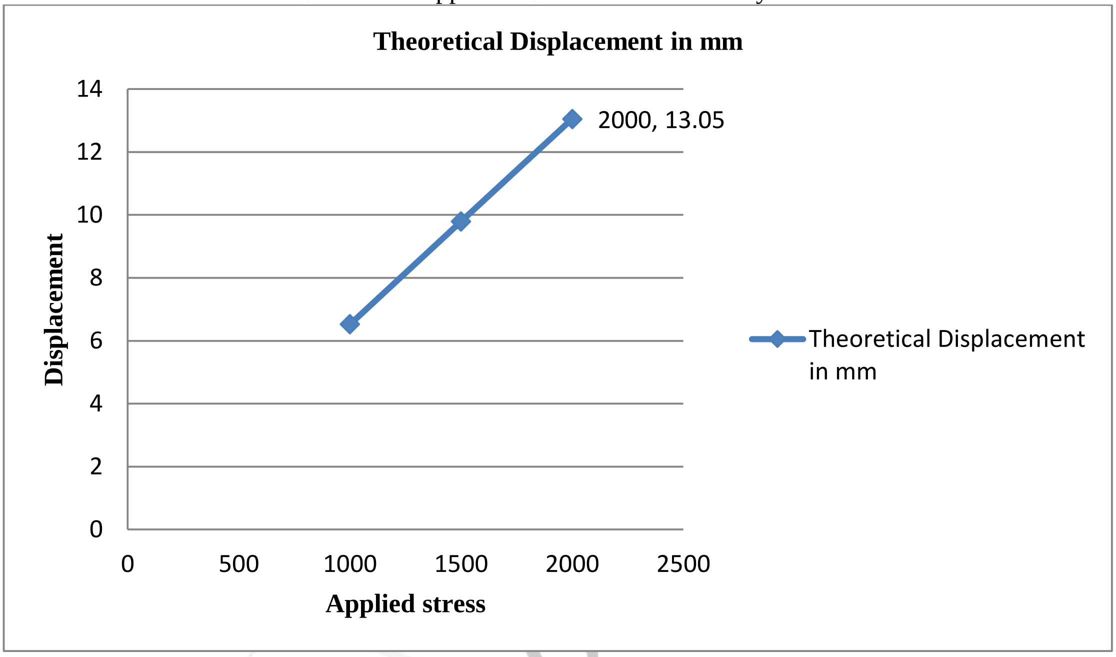
graf. practical applied stress vs stress intensity factor ki



graf. practical applied stress vs stress intensity factor kii



Graf. theoretical applied stress Vs stress intensity factor KI

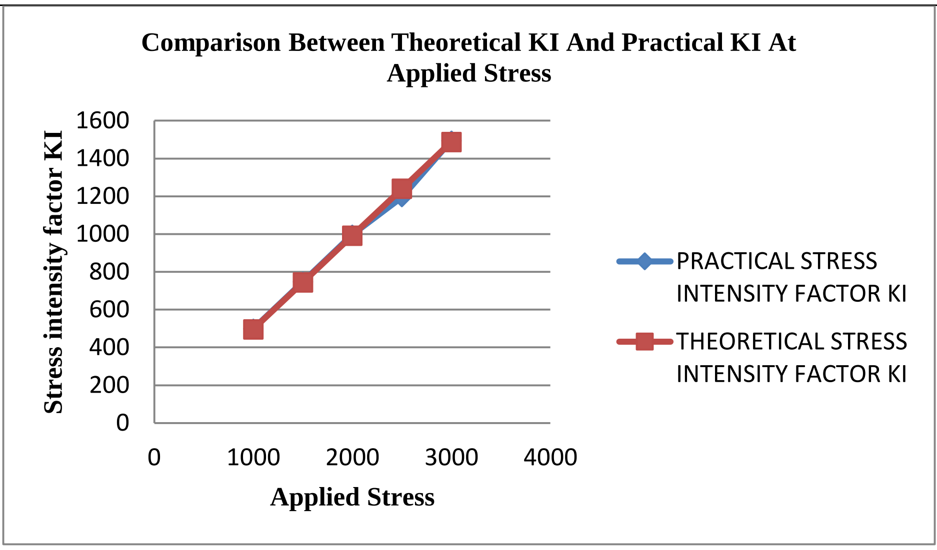


graf. theoretical applied stress vs displacement

Comparison Between Theoretical KI And Practical KI At Applied Stress

S.NO	APPLIED STRESS	Practical stress intensity factor KI	Theoretical stress intensity factor KI
1	1000	498.25	496
2	1500	747.37	744
3	2000	996.50(Crack tip opened)	992
4	2500	1195.80	1240
5	3000	1494.75	1488

table. comparison between theoretical ki and practical ki at applied stress.



graf .comparison between theoretical KI and practical KI at applied stress

VI. CONCLUSIONS

We created the two dimensional model of thin plate in ANSYS by defining key points. The size of plate is 0.2m *0.2 m with centered crack length a= 20 mm as a semi full crack mode. The thick ness of model is restricted to 20mm in order to calculate the value of K_{IC} as a property in plain stress calculations. we calculate theoretically the stresses in X,Y,Z directions and displacements also.

In ANSYS program we analysis the specimen module by giving the pressure on both top and bottom lines. We observed at crack tip, the module get deformed at particular pressure. And compare the theoretically calculated results of stresses and displacement and practically calculated results. There may be 99% accuracy in results.

This work clearly demonstrates the robustness of the finite element method in handling real life problems. The numerical results obtained using the finite element meshed are in good agreement with previous experimental work on done on crack geometry.

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