Bilateral Control of Dynamical System

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***Abstract*- This paper proposed a novel frame work for bilateral teleoperation system with a Mass damper system with/without using time delay. A Bilateral Tele-operation system is composed of a local master site, which is driven by a human operator, and a remote slave site, which is in contact with the environment. In such a system, the slave follows the movement of master and the master receives feedback information from the slave. The slave do not track master properly due to instability caused by time delay present in communication channel. The presence of constant/variable time delay in communication channels can deteriorate system performance. A design feedback control scheme for bilateral telerobotic systems in the presence of time delay has been exploited. Bilateral teleoperator show instability in the presence of time delays. The instability occurs due to non-passive nature of communication channel. Scattering parameter based method has been exploited for designing a passivity based controller to study the effect of time delays of telerobotic system. In scattering parameter methods, communication channel assumed to be lossless transmission line and by this approach a passive control law has been designed that makes channel passive.**

***Index terms*- Bilateral teleoperation, position tracking, passivity, proportional- derivative (PD) control.**

I. INTRODUCTION

Telerobotic is perhaps one of the oldest fields in robotics [1]. Since its humble beginning in the 1940s when the first teleoperator was designed, the focus had been primarily on the nuclear, space, and underwater applications until the 1980s. The recent advances in computing power and in communications have led to the emergence of new applications such as telesurgery, semiautonomous telerobotic, live power line maintenance, and others. On the other hand, progress in bilateral control has been the key point for the development of new master–slave architectures [11], which are important in precise telemanipulation tasks. This pervasive interest has spawned the continuous development of the new telerobotic systems. With the recent advances in the telerobotic system, it reduced the human involvement in the various fields where it was dangerous to reach there, like exploratory missions, hazardous environment, Space exploration and operation in geosynchronous orbits. This not only works out economical and safe but also avoids unwarranted human exposure to potentially dangerous environment. When bilateral telerobotic system was first introduced, the main issues were precise tracking and stability. The stability of the system was found to be effected by time delays present in the communication channel. Different approaches have formulated to provide passivity of the teleoperator. One of the methods is based on designing PD-type controller [3]. It was found to be a useful technique for precise tracking and ensured stability of the system; however the system went unstable in the presence of time delay thus bringing the need for more robust controllers.[6]

II. PASSIVITY BASED STABILITY CRITERIA

Passivity is related to the property of stability in an input-output system, that is the system is stable if bounded input energy supplied to the system will yield bounded output energy.
A system is passive if it absorbs more energy than it produces. To formalize this, define the ''input power'' P_{in}, which is positive when entering a system, as the scalar product between the input vector x and the output y of the system.

$$P_{in} = x^T y \quad (1)$$

A system is passive, if it obeys [12]

$$\frac{d}{dt}[stored\ energy] = [external\ power\ input] + [internal\ power\ generation]$$

$$P_{in} = \frac{d}{dt}E_{store} + P_{diss} \quad (2)$$

$$\frac{d}{dt}E_{store} = P_{in} - P_{diss}$$

III. S-PARAMETER REPRESENTATION

Scattering parameters or S-parameters (the elements of a scattering matrix or S-matrix) describe the electrical behavior of linear electrical networks when undergoing various steady state excitations by electrical signals.
The other parameters are Y-parameters, Z-parameters, H-parameters, T-parameters or ABCD-parameters also used in electrical networks. The difference between S-parameters and these parameters in the sense that S-parameters does not use open circuit or short circuit condition to characterize a linear electrical network. In S-parameters matched loads conditions are used. These terminations are much easier to use at high signal frequencies than open-circuit and short-circuit terminations. Moreover, the quantities are measured in terms of power.

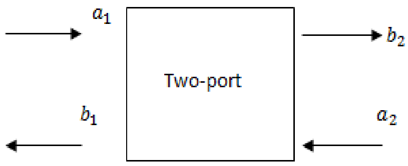


Fig.**1.** Two port network

The S-parameter matrix for the 2-port network is probably the most commonly used and serves as the basic building block for generating the higher order matrices for larger networks. In this case the relationship between the reflected, incident power waves and the S-parameter matrix [7] is given by:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{3}$$

By expanding given matrix,

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{4}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{5}$$

Each equation gives the relationship between the reflected and incident power waves at each of the network ports 1 and 2, the network's individual S-parameters, S_{11}, S_{12}, S_{21} *and* S_{22}.

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1^-}{V_1^+} \text{ and } S_{21} = \frac{b_2}{a_1} = \frac{V_2^-}{V_1^+} \tag{6}$$

$$S_{12} = \frac{b_1}{a_2} = \frac{V_1^-}{V_2^+} \text{ and } S_{22} = \frac{b_2}{a_2} = \frac{V_2^-}{V_2^+} \tag{7}$$

Each 2-port S-parameter has the following generic descriptions:
S_{11} is the input port voltage reflection coefficient.
S_{12} is the reverse voltage gain.
S_{21} is the forward voltage gain.
S_{22} is the output port voltage reflection coefficient.

a) General passivity condition using scattering parameter

A system is passive if it absorbs more energy than it produces. Systems passivity is sufficient to demonstrate stability for bounded input energies. Also, from the passivity properties, arbitrary combinations of multiple passive systems are passive again. Therefore passivity of a telerobotic system as a serial chain of network elements can be assured by examining of every element's passivity. An n-port is said to be passive if and only if for any independent set of n-port flows, **V,** injected into the system, and efforts F**,** applied across the system maintain the following relation[2]

$$\int_0^\infty F^T(t)\, v(t)\, dt \geq 0 \tag{8}$$

Again relation between *F* and *v* can be mapped the by scattering operator (S).

$$F - v = S(F + v) \tag{9}$$

For LTI systems, the scattering operator ***S*** can be expressed in the frequency domain as a scattering matrix S(s), where

$$F(s) - v(s) = S(s)[F(s) + v(s)] \tag{10}$$

In the case of a two-port, this scattering matrix [2] can be related to the hybrid matrix *H(s)* as follows:

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} [H(s) - I][H(s) + I]^{-1} \tag{11}$$

A system is passive if and only if the norm of its scattering operator is less than or equal to one.

$$\|S\| \leq 1 \tag{12}$$

IV. MECHANICAL TO ELECTRICAL CONVERSION

In this section, the force /effort are analogous to voltage and velocity/flow analogous to current. This conversion is described below elaborately [2].

a) The mass of the manipulator represented by an inductor.

In the mechanical system, the relation between force and acceleration is

$$F(t) = M\dot{v}(t) \tag{13}$$

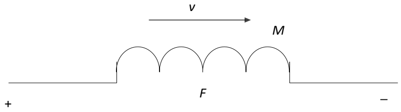
Where *M* is an inertial element.

In the electrical system, the relation between voltage and current is

$$V(t) = L\, i(t) \tag{14}$$

By comparing equation (13) & (14) then

$$M = L$$



b) The damping of the system can be represented by a resistor.

In mechanical system, damping force will be,

$$F(t) = Bv(t) \tag{15}$$

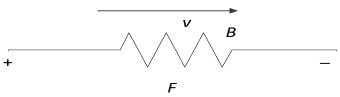
Where B is a damping constant

In electrical system, voltage applied to the system will be,

$$V(t) = Ri(t) \tag{16}$$

By comparing eq. (15) & (16) then

$$B = R$$



c) The spring constant or environmental stiffness can be represented by a capacitor.

In mechanical system, restoring force will be,

$$F(t) = K\int v(t)dt \tag{17}$$

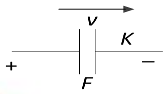
Where K is a stiffness element

In electrical system, voltage applied will be

$$V(t) = \frac{1}{C}\int i(t)dt \tag{18}$$

By comparing eq. (17) & (18) then

$$K = \frac{1}{C}$$



V. NETWORK REPRESENTATION OF BILATERAL TELEOPERATOR

Network representation[3] of teleoperator is shown in Fig 2. Here the master, communication block, and slave are represented by two-port network, and the operator and environment are represented by one-port network. The electrical network is characterized by the relationship between effort ***F*** (force, voltage), and flow ***v*** (velocity, current). For an LTI system, this relationship is specified by its impedance *Z(s)* according to

$$F(s) = Z(s)v(s) \tag{19}$$

where ***F (s)*** **,** ***v(s)*** are the Laplace transforms of F(t), V(t), respectively. LTI system be can be converted by the Hybrid matrix H(s).

$$\begin{bmatrix} F_1(s) \\ -v_2(s) \end{bmatrix} = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix} \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} = \mathrm{H(s)} \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} \tag{20}$$

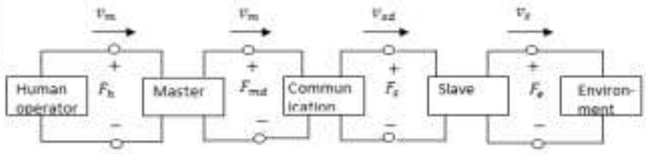


Fig.2. Network representation of teleoperator

The dynamics equation [2] for master and slave is shown below:

$$M_m\dot{v}_m = F_h + \tau_m \tag{21}$$

$$M_s\dot{v}_s = -F_e + \tau_s \tag{22}$$

Where v_m and v_s are the respective velocities for the master and slave, τ_m and τ_s are the respective motor torques, M_m and M_s are the respective inertias, F_h is the operating torque, and F_e is the environment torque.

Motor torques equation is given by

$$\tau_m = -B_m v_m - F_{md} \tag{23}$$

$$\tau_s = -B_{s2} v_s + F_s - \alpha_f F_e \tag{24}$$

Where F_{md} is the reflected force and F_s is the coordinating torque and is given by

$$F_s = K_s\int(v_{sd} - v_s)dt + B_{s1}(v_{sd} - v_s) \tag{25}$$

Where v_{sd} is the velocity set point.

The coordinating torque is used to describe a motor command which is error between the master and slave variables. Its purpose is to cause the master and slave to track each other. The circuit representation of bilateral teleoperator shown in Fig.3.

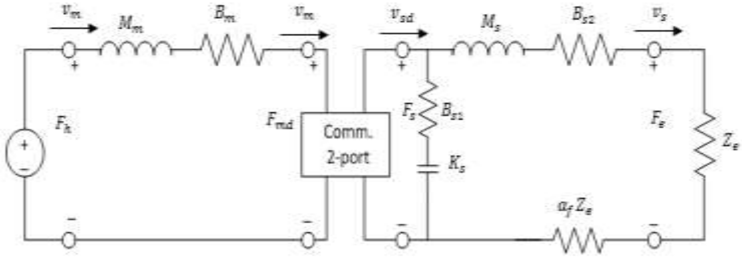


Fig.3. Circuit representation of Tele-operator

We can relate F_s to F_e by combining (22) to (24) and using the environments impedance[10] relationship $F_e = Z_e v_s$ as:

$$F_s = \left(\frac{M_s+B_{s2}}{(1+\alpha_f)} + 1\right)(1+\alpha_f)F_e \tag{26}$$

For large α_f

$$F_s \cong (1+\alpha_f)F_e) \tag{27}$$

a) Network representation of Bilateral teleoperation system without time delay

In this section, it is assuming that master of the system directly connected with slave and getting feedback from the slave to master i.e., there is no communication channel in between master and slave. Assuming there is not any loss/lag of information from master to slave.

From Fig.3.the communication block is

$$F_{md} = F_s \tag{28}$$

$$v_{sd} = v_m \tag{29}$$

From (21), (23) & (28)

$$M_m\dot{v}_m + B_m v_m = F_h - F_s \tag{30}$$

From (22), (24) & (29)

$$M_s\dot{v}_s + B_{s2}v_s = F_s - (1+\alpha_f)F_e \tag{31}$$

and the coordinating torque will be

$$F_s = K_s\int(v_m - v_s)dt + B_{s1}(v_m - v_s) \tag{32}$$

The above system can be represents in circuit form in Fig 5.

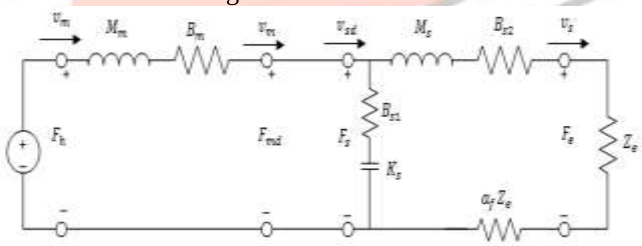


Fig.4. Circuit representation of Tele-operator without time delay

b) Bilateral teleoperation with time delay

In this section, there is a communication channel that creates delays in the system variables. The signal given to slave by master has delayed forms that create instabilities[4] in the system that will see in subsequent section.

When transmission delay exists, the communication block equations (28) & (29) becomes,

$$F_{md}(t) = F_s(t-T) \tag{33}$$

$$v_{sd}(t) = v_m(t-T) \tag{34}$$

The circuit representation of bilateral teleoperator in the presence of time delay is shown in Fig.5.

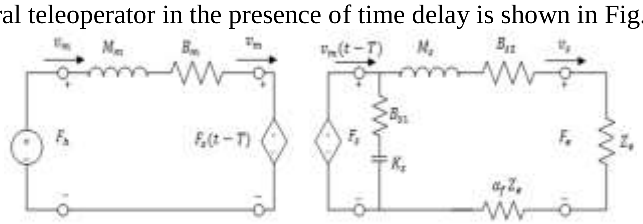


Fig.5. Circuit representation of system with time delay

The two port communication block for this system is non-passive and is cause of the instability.

VI. STABILITY ANALYSIS WITH TIME DELAY

From the passivity criteria we know that if two or more systems are connected in cascade then the entire system will passive. In bilateral teleoperation master and slave systems are both passive but due to communication channel the entire connected (cascade) system are not passive. It is signifies that transmission channel is not passive system. Mathematical representation of non-passive communication channel is shown below. To compensate instability of communication channel scattering parameter is used.
In time domain, from Fig.5, Communication block from equation (33) & (34) is

$$F_{md}(t) = F_s(t-T) \tag{35}$$

$$v_{sd}(t) = v_m(t-T) \tag{37}$$

In frequency domain,

$$F_{md}(s) = e^{-sT}F_s(s) \tag{38}$$

$$v_{sd}(s) = e^{-sT}v_m(s) \tag{39}$$

In matrix form, the above equation can be represents as,

$$\begin{bmatrix} F_{md}(s) \\ -v_{sd}(s) \end{bmatrix} = \begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix} \begin{bmatrix} v_m(s) \\ F_s(s) \end{bmatrix} \tag{40}$$

Hybrid matrix from the above representation is,

$$H(s) = \begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix} \tag{41}$$

Relation between scattering matrix $S(s)$ and hybrid matrix $H(s)$ from eq, (11)

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (H(s)-I)(H(s)+I)^{-1} \tag{42}$$

$$H(s) - I = \begin{bmatrix} -1 & e^{-sT} \\ -e^{-sT} & -1 \end{bmatrix}$$

$$H(s) + I = \begin{bmatrix} 1 & e^{-sT} \\ -e^{-sT} & 1 \end{bmatrix}$$

$$(H(s)+I)^{-1} = \begin{bmatrix} \frac{e^{sT}}{e^{sT}+e-^{sT}} & -\frac{1}{e^{sT}+e^{-sT}} \\ \frac{1}{e^{sT}+e^{-sT}} & \frac{e^{sT}}{e^{sT}+e^{-sT}} \end{bmatrix}$$

$$\tanh(sT) = \frac{e^{sT}-e^{-sT}}{e^{sT}+e^{-sT}} \tag{43}$$

$$\operatorname{sech}(sT) = \frac{2}{e^{sT}+e^{-sT}} \tag{44}$$

Using (43) & (44) and solving,

$$(H(s)-I)(H(s)+I)^{-1} = \begin{bmatrix} -\tanh(sT) & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & -\tanh(sT) \end{bmatrix} \tag{45}$$

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (H(s)-I)(H(s)+I)^{-1}$$

$$\boxed{S(s) = \begin{bmatrix} -\tanh(sT) & \operatorname{sech}(sT) \\ \operatorname{sech}(sT) & \tanh(sT) \end{bmatrix}}$$

Condition for system should be passive from eq (12) is,

$$\|S\| \le 1 \tag{46}$$

Now from the given result we can find the norm of scattering matrix, i.e.

$$\|S\| = sup_\omega\, \lambda^{\frac{1}{2}}\left(S^*(j\omega)S(j\omega)\right) \tag{47}$$

$$\|S\| = \sup_\omega \lambda^{\frac{1}{2}}\left(\begin{bmatrix} \tan^2(\omega T) + \sec^2(\omega T) & 2j\tan(\omega T)\sec(\omega T) \\ -2j\tan(\omega T)\sec(\omega T) & \tan^2(\omega T) + \sec^2(\omega T) \end{bmatrix}\right)$$

From above equation

$$\|S\| = \infty$$

This shows the scattering operator for given system is unbounded. Hence, the system is not passive with respect to time delay.

VII. TIME DELAY COMPENSATION USING SCATTERING PARAMETER

The basic idea is to choose the control law so that the two port characteristic of the communication block are identical to a two port lossless transmission line [2].

Fig.6. Two port lossless transmission line

Considering communication channel is lossless transmission lines the transmission lines equation becomes

$$f_1(s) = z_0 \tanh\left(\frac{sl}{V_0}\right) v_1(s) + \operatorname{sech}\left(\frac{sl}{V_0}\right) f_2(s) \tag{48}$$

$$-v_2(s) = -\operatorname{sech}\left(\frac{sl}{V_0}\right) v_1(s) + \frac{\tanh\left(\frac{sl}{V_0}\right)}{z_0} f_2(s) \tag{49}$$

Where, $z_0 = \sqrt{\frac{L}{C}}$ & $V_0 = \frac{1}{\sqrt{LC}}$

Let us take $z_0 = 1$ & $V_0 = \frac{l}{T}$

Where, z_0 is characteristic impedance, V_0 is frequency of the signal, l is the length of transmission line, L & C are passive circuit elements.

$$f_1(s) = \tanh(sT)\, v_1(s) + \operatorname{sech}(sT)\, f_2(s) \tag{50}$$

$$-v_2(s) = -\operatorname{sech}(sT)\, v_1(s) + \tanh(sT)\, f_2(s) \tag{51}$$

Compare the above communication channel with our tele-operator communication channel,

$$F_{md}(s) = f_1(s) \quad \& \quad F_s(s) = f_2(s)$$

$$v_m(s) = v_1(s) \quad \& \quad v_{sd}(s) = v_2(s)$$

The equation (50) & (51) will be,

$$F_{md}(s) = \tanh(sT)\, v_m(s) + \operatorname{sech}(sT)\, F_s(s) \tag{52}$$

$$-v_{sd}(s) = -\operatorname{sech}(sT)\, v_m(s) + \tanh(sT)\, F_s(s) \tag{53}$$

In matrix form, the above equation can be represent as,

$$\begin{bmatrix} F_{md} \\ -v_{sd} \end{bmatrix} = \begin{bmatrix} \tanh(sT) & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & \tanh(sT) \end{bmatrix} \begin{bmatrix} v_m(s) \\ F_s(s) \end{bmatrix}$$

Compare with hybrid matrix model,

$$H(s) = \begin{bmatrix} \tanh(sT) & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & \tanh(sT) \end{bmatrix} \tag{54}$$

Relation between scattering matrix $S(s)$ and hybrid matrix $H(s)$ is,

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (H(s) - I)(H(s) + I)^{-1}$$

$$H(s) - I = \begin{bmatrix} \tanh(sT) - 1 & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & \tanh(sT) - 1 \end{bmatrix}$$

$$H(s) + I = \begin{bmatrix} \tanh(sT) + 1 & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & \tanh(sT) + 1 \end{bmatrix}$$

$$(H(s) + I)^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -\frac{\operatorname{sech}(sT)}{1 + \tanh(sT)} \\ \frac{sech(sT)}{1 + \tanh(sT)} & 1 \end{bmatrix}$$

$$\tan^2 \mathrm{h(sT)} + \sec^2 \mathrm{h(sT)} = 1 \tag{55}$$

Using (55) and solving

$$(H(s) - I)(H(s) + I)^{-1} = \begin{bmatrix} 0 & \frac{\operatorname{sech}(sT)}{1 + \tanh(sT)} \\ -\frac{\operatorname{sech}(sT)}{1 + \tanh(sT)} & 0 \end{bmatrix}$$

$$\tanh(sT) = \frac{e^{sT} - e^{-sT}}{e^{sT} + e^{-sT}} \tag{56}$$

$$\operatorname{sech}(sT) = \frac{2}{e^{sT} + e^{-sT}} \tag{57}$$

Using (56) & (57) and solving

$$(H(s) - I)(H(s) + I)^{-1} = \begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix}$$

Scattering matrix,

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix}$$

$$\boxed{S(s) = \begin{bmatrix} 0 & e^{-sT} \\ e^{-sT} & 0 \end{bmatrix}}$$

Condition for system should be passive from eq (12) is,

$$\|S\| \leq 1$$

Now from the given result the norm of scattering matrix will be,

$$\|S\| = sup_{\omega}\, \lambda^{\frac{1}{2}}\left(S^{*}(j\omega)S(j\omega)\right)$$

$$\|S\| = sup_{\omega}\, \lambda^{\frac{1}{2}}\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right) \quad (3.34)$$

$$\|S\| = 1$$

Therefore by above condition, this show the system (52) & (53) is passive.
In the scattering matrix form, the system (52) & (53) can be represent as,

$$\begin{bmatrix}F_{md}(s) - v_m(s)\\ F_s(s) + v_{sd}(s)\end{bmatrix} = S(s)\begin{bmatrix}F_{md}(s) + v_m(s)\\ F_s(s) - v_{sd}(s)\end{bmatrix}$$

$$\begin{bmatrix}F_{md}(s) - v_m(s)\\ F_s(s) + v_{sd}(s)\end{bmatrix} = \begin{bmatrix}0 & e^{-sT}\\ e^{-sT} & 0\end{bmatrix}\begin{bmatrix}F_{md}(s) + v_m(s)\\ F_s(s) - v_{sd}(s)\end{bmatrix} \quad (58)$$

In time domain equation (58) will be,

$$\begin{bmatrix}F_{md}(t) - v_m(t)\\ F_s(t) + v_{sd}(t)\end{bmatrix} = \begin{bmatrix}F_s(t-T) + v_{sd}(t-T)\\ F_{md}(t-T) - v_m(t-T)\end{bmatrix}$$

Finally, the passive control law for the communication block circuit is,

$$F_{md}(t) = F_s(t-T) - v_{sd}(t-T) + v_m(t) \quad (59)$$

$$v_{sd}(t) = v_m(t-T) - F_s(t) + F_{md}(t-T) \quad (60)$$

Because the force and velocity signals may differs by order of magnitude, therefore control law (59) & (60) may have implementation problems.

By proper use of scaling can overcoming these problems. This can be done by adding equivalent of transformer with scaling factors of n & $\frac{1}{n}$,respectively at both ends of transmission lines.

By this scaling factor our control law becomes,

$$F_{md}(t) = F_s(t-T) + n^2\left(v_m(t) - v_{sd}(t-T)\right) \quad (61)$$

$$v_{sd}(t) = v_m(t-T) + \frac{1}{n^2}\left(F_{md}(t-T) - F_s(t)\right) \quad (62)$$

VIII. RESULTS

The following problems discussed above have been solved computationally.

a) Mass-damper system as bilateral teleoperator with time delays

In this section, two mass-damper [5]systems are used as master-slave teleoperation for analyzing the effect of time delay. Without using scattering parameter based approach, the system goes unbounded and is not passive in the presence of time delay. The block diagram of the mass-damper system as a master slave teleoperator shown in Fig.7.

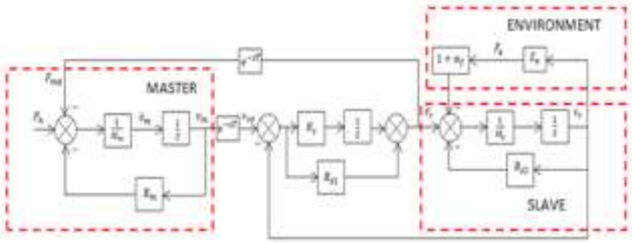


Fig.7. Time delay Tele-operator

Dynamics of master and slave are,
Master equation is given below,

$$M_m\dot{v}_m + B_m v_m = F_h - F_{md} \quad (63)$$

Slave equation is given below,

$$M_s\dot{v}_s + B_{s2}v_s = F_s - (1+\alpha_f)F_e \quad (64)$$

Where v_m and v_s are the respective velocities for the master and slave, M_m and M_s are the respective inertias, F_h is the operating torque, and F_e is the environment torque.

Where F_{md} is the reflected force[9] and F_s is the coordinating torque and is given by

$$F_s = K_s\int(v_{sd} - v_s)dt + B_{s1}(v_{sd} - v_s) \quad (65)$$

Where v_{sd} is the velocity set point.

The coordinating torque is used to describe a motor command which is error between the master and slave variables. Its purpose is to cause the master and slave to track each other.

In time delays,

$$F_{md}(t) = F_s(t-T) \quad (66)$$

$$v_{sd}(t) = v_m(t-T) \quad (67)$$

Simulation results:-

The values of parameters are, $M_m = 1, M_s = 1, M_h = 1,\ B_m = 0.1, B_{s1} = 0.1, B_{s2} = 1, z_e = 0.1, K_s = 16, \alpha_f = 0.1$. Initial condition, $x_0 = [0.1, 0.1]$. At delay=10ms, system is stable and delay =150ms, system is unstable. It is seen that system will become unstable due presence of time delays in the communication channel. At certain time bound the slave track master but as increasing the time delays slave will not follow properly and leads to instability of the system. The desired position trajectory of amplitude 2 units given to master as shown in Fig.8. The output of master position shown in Fig.9. and the output of slave position shown in Fig.10. First using same dynamics check for delays of 10ms, it has been shown in Fig.11. system was stable but as the time delays increases to 150ms the system goes unbounded as shown in Fig.12. The simulation of dynamics equation (63), (64), (65), (66) & (67) has been shown below.

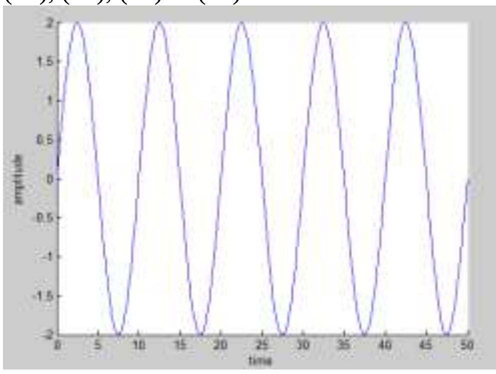


Fig.8. Desired position trajectory given to master

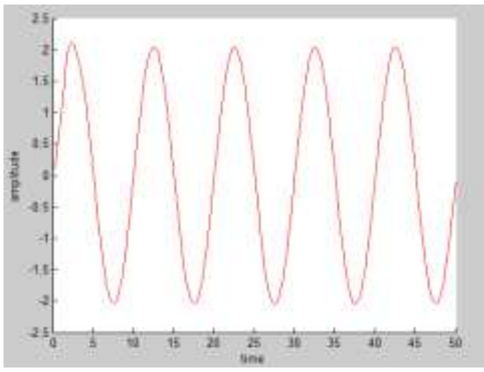


Fig.9. Master position output

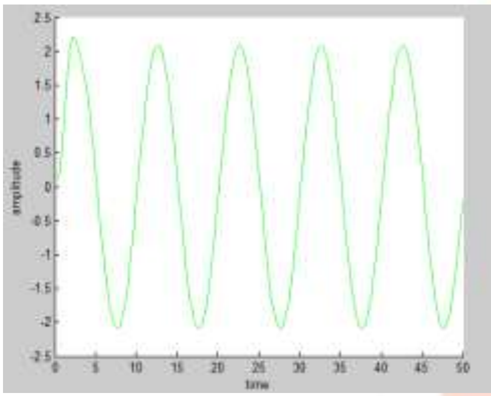


Fig.10. Slave position output

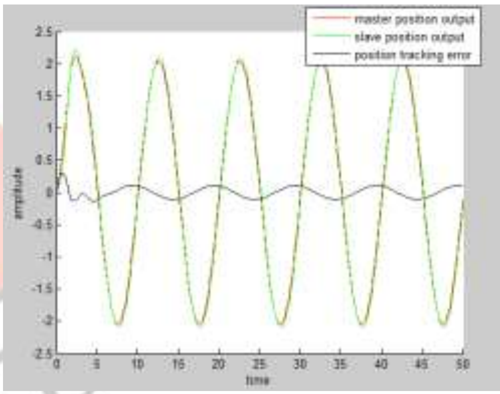


Fig.11. Position of master and slave with tracking error in 10ms delay

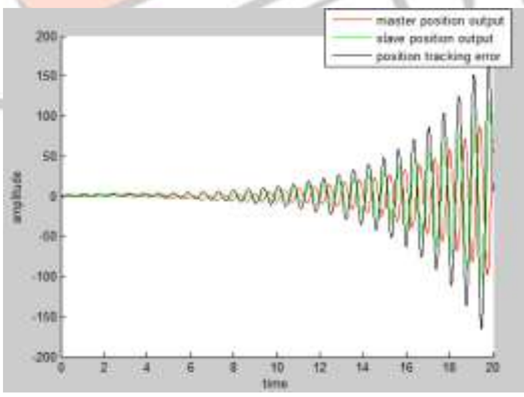


Fig.12. Position of master and slave with tracking error in 150ms delay

b) Mass-damper system as bilateral teleoperator with time delays compensation

In this section, a scattering parameter based method has been exploited for analyzing stability in the presence of time delays. The instability occurs in the system due to non-passive nature of communication channel. Communication channel assumed to be lossless transmission line and using scattering parameter based method for making channel passive. In this section, a passive control law is designed by using scattering parameter based approach that ensures the stability[8] of the system. The block diagram of time delay compensated bilateral teleoperator shown in Fig.13.

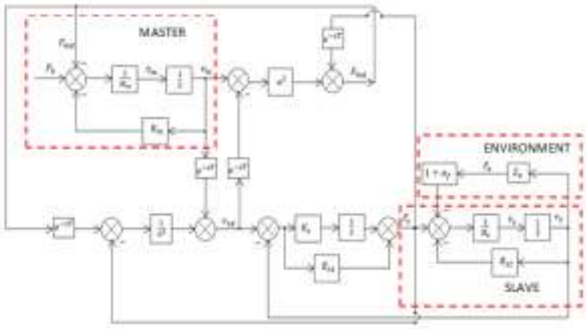


Fig.13. Time delay compensated bilateral Tele-operator

Dynamics of master and slave are,
Master equation is,

$$M_m \dot{v}_m + B_m v_m = F_h - F_{md} \quad (68)$$

Slave equation is,

$$M_s \dot{v}_s + B_{s2} v_s = F_s - (1 + \alpha_f) F_e \quad (69)$$

Where v_m and v_s are the respective velocities for the master and slave, M_m and M_s are the respective inertias, F_h is the operating torque, and F_e is the environment torque.
Where F_{md} is the reflected force and F_s is the coordinating torque and is given by

$$F_s = K_s \int (v_{sd} - v_s) dt + B_{s1}(v_{sd} - v_s) \quad (70)$$

Where v_{sd} is the velocity set point.
By using scattering based approach, the desired force to the master F_{md} and desired velocity to the slave v_{sd} shown below,

$$F_{md}(t) = F_s(t - T) - v_{sd}(t - T) + v_m(t) \quad (71)$$

$$v_{sd}(t) = v_m(t - T) - F_s(t) + F_{md}(t - T) \quad (72)$$

These equations are the passive control law equations that make communication channel to be passive.

Simulation results:-

The values of parameters are, $M_m = 1, M_s = 1, M_h = 1,\ B_m = 0.05, B_{s1} = 0.5, B_{s2} = 20, z_e = 0.1, K_s = 0.01, \alpha_f = 0.1$. Initial condition, $x_0 = [0.1, 0.1]$. We have checked for both time delays at delay=10ms, system is stable and delay =150ms, system is stable. It is seen that system will become stable by increasing time delays up to 150ms in the communication channel. It shows that by using scattering based method, it ensures the stability of the system by making communication channel passive. The desired position trajectory of amplitude 2 units given to master as shown in Fig.14. The output of master position has shown in Fig.15. and the output of slave position shown in Fig.16. The simulation of dynamics equation of bilateral teleoperator (68), (69). (70), (71) & (72) has been shown below. In Fig.17, it has been shown that slave tracking position of master and very small tracking error.

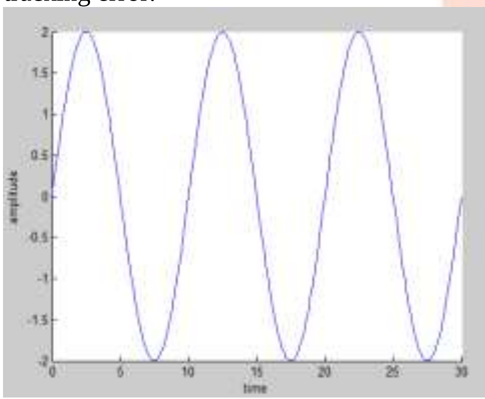


Fig.14. Desired position trajectory given to master

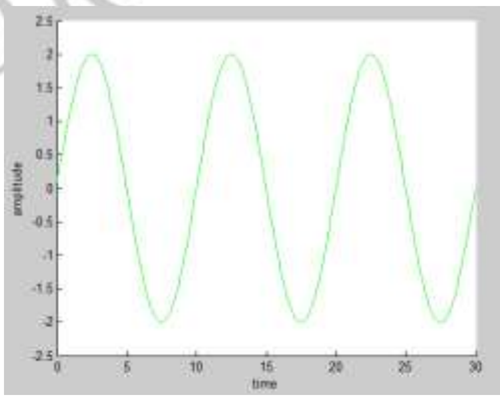


Fig.15. Master position output

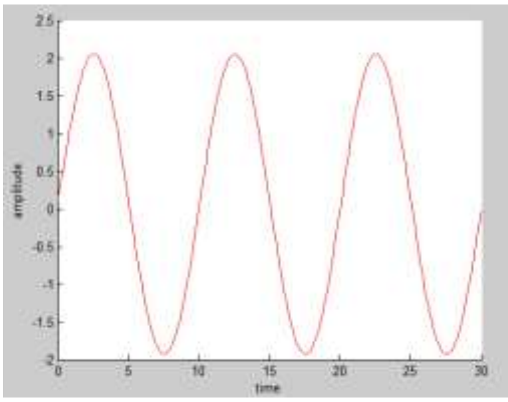


Fig.16. Slave position output

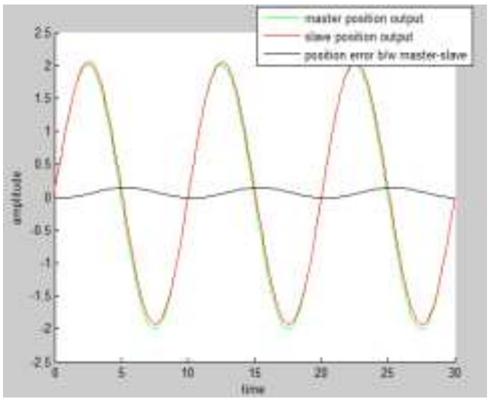


Fig.17. Position of master and slave With tracking error 0f 150ms delay

IX. CONCLUSION

- The presence of delay in transmission line makes the passive connected system non-passive which causes instability.
- To compensate time delay two mathematical tools (scattering method) is used. It is nothing but the scaling and rescaling of the transmitted and received signal.
- Time delay compensation has been done using scattering parameter using the concept of lossless transmission line but it established a delay dependent stability criteria. Simulation result (Fig.17) shows that at 150ms the system is stable. But time delay increasing beyond 150ms, the system is going to unstable.

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