A Region based Active contour Approach for Liver CT Image Analysis driven by fractional order image fitting energy

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***Abstract*— Computer tomography images are widely used in the diagnosis of liver tumor analysis because of its faster acquisition and compatibility with most life support devices. Accurate image segmentation is very sensitive in the field of medical image analysis.Active contours plays an important role in the area of medical image analysis.It constitute a powerful energy minimization criteria for image segmentation. This paper presents a region based active contour model for liver CT image segmentation based on variational level set formulation driven by fractional order image fitting energy. The neighbouring intensities of image pixels are described in terms of Gaussian distribution. The mean and variances of intensities in the energy functional can be estimated during the energy minimization process.The addition of the fractional order fitting term to the energy functional makes the segmentation more accurate and be robustness to noise.In order to ensure the stable evolution of the level set function a penalty term is added into the model. Also this model has been compared with different active active contour models. Our results shows that the presented model achieves superior performance in CT liver image segmentation.**

***IndexTerms*—Active Contours, Fractional Order differentiation, Level Sets.**

I. INTRODUCTION

Active Contours plays an important role in the field of medical image segmentation.Liver tumor segmentation in CT images is a key problem in medical image processing scenario.The goal is to exact separation of the tumor regions from the background organs inorder to visualize and analyse the physicians to predict the benign and malignancy conditions.Active contour model proposed by Kass et al.[1] , has been proved to be an efficient framework for image segmentation. These models can be formulated under an energy minimization framework based on the theory of curve evolution. The solutions to these models can be obtained by using the level set method [2], whose basic idea is to represent a contour or surface as the zero level set of an implicit function defined in a higher dimension, and to formulate the motion of the contour or surface as the evolution of the level set function. The main advantage of this method is to handle the topological changes automatically. Active contour models can be generally categorized into two groups : the edge-based models[3-9] and region-based models [10-14]. Edge-based active contour models utilize local image gradients to attract contours toward object boundaries,whereas region-based models employ global image information in each region, such as the distribution of intensities, colors, textures and motion , to move contours toward the boundaries. In the field of medical image segmentation, we use an energy functional from a mathematical model and minimizing this energy functional to track the tumor regions.The popular piece- wise constant (PC) model is a typical variational level set method,which aims to minimize the Mumford_Shah functional .This model makes use of inside and outside regions global image properties with reference to the evolving curve into their energy functional as constraints.The PC model fails to segment the intensity inhomogenity images because it assumes that the intensities in each region always maintain constant.Li et al. proposed an implicit active contours model based on local binary fitting energy, which used the local information as constraint, and works very well on the images with intensity inhomogeneities [15]. This model outperformed both PC and PS models in segmentation accuracy and computational efficiency. Wang et al. [13] employed Gaussian distributions to model local image information, and proposed local Gaussian distribution fitting (LGDF) energy.Zhang et al. proposed a novel region-based active contour model for image segmentation, which combined the merits of the traditional GAC and PC models . Zhang et al. exploited local image region statistics to present a level set method for segmenting images with intensity inhomogeneity [14]. This model utilize a Gaussian filtering scheme to regularize the level set function.The LIF model can achieve similar segmentaton accuracy to the LBF model with less computational costs.He et al.proposed an improved region-scalable fitting model based on the “modifying” kernel and local entropy Two piecewise smooth (PS) active contour models were developed under the framework of minimizing the Mumford–Shah functional.Local intensity crteria has been extensively used in active contour models to improve the tumor boundary accuracy for the images corrupted by noise and intensity inhomogenity. Fractional order differentiation have been widely used in the field of image processing[16]. Mathieu et al. proposed an edge detector based on fractional differentiation . Nakib et al. gave a new geometric interpretation of two dimensional fractional order differentiation and is applied into threshold segmentation [17]. Ren et al. present a novel image up-sampling algorithm based on fractional-order bidirectional diffusion [18].Fractional order differentiation such as Riemann-Liouville fractional order differentiation, Grünwald-Letnikov fractional order differentiation and frequency-domain fractional order differentiation etc are the methods that can define the fractional order differentiation. The main advantage of fractional order differentiation over first order differentiation is the former preserves the low frequency information.This is very significant in liver CT image analysis criteria. Regularization should be imposed on the variational level set to cure the influence of noise and smooth the level set function.In the Mumford–Shah (MS)functional, length regularization is a constraint to penalize the length of contours.This regularization is less robustness to noise. Li et al. use the variable exponent p-Dirichlet integral to form a novel weighted regularization to diminish the influence of noise. Based on these methods,we adopt a weighted regularization based on a novel edge indicator function in this paper.

In this paper we makes use of a fractional order fitting term to construct a image fitting energy functional,and is implemented this functional using variational level set formulation for image segmentation..This functional is implemented using the variational level set fomulation with a regularized term.In order to ensure the stable evolution of the level set function, apenalty term is added into the presented model. An adaptive function is incorporated into the length regularization to smooth the level set function. The presented model has been compared with different active contour models.

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II. MUMFORD-SHAH MODEL

Mumford-Shah variational methods have been widely studied in image processing because of their numerical advantages and flexibility. Let C be the evolving curve and the boundary in a region Ω, and ω be the open subset of Ω. i.e. $\omega \subset \Omega$ and $C=\partial\omega$. Also, inside C denotes region ω and outside C denotes $\Omega\backslash\varpi$.If an image u_0 (x,y) is formed by two regions of approximately piecewise-constant intensities, considering the fitting term,

$$F_1(C)+F_2(C)=\int_{inside(C)}\left|u_0(x,y)-c_1\right|^2 dxdy + \int_{outside(C)}\left|u_0(x,y)-c_2\right|^2 dxdy \quad (1)$$

where c_1 andc_2 the constants depending on C are the averages of u_0 inside C. C_0, the boundary of the object is the minimizer of the fitting term.

III. PIECEWISE CONSTANT MODEL

The piecewise constant model is an alternative solution to the Mumford Shah model, proposed by Chan and Vese. They fit the original image u_0 by a piecewise constant function. For image u_0 on the image domain Ω, PC model is formulated as the following energy functional.

$$E^{PC}=\lambda_1\int_{inside(C)}\left|u_0-c_1\right|^2 dxdy+\lambda_2\int_{outside(C)}\left|u_0-c_2\right|^2 dxdy+\mu\left|C\right|,\ \ x\in\Omega \quad (2)$$

where λ_1, λ_2 and μ are positive constants. Inside(C) and outside (C) denote the region inside and outside of curve C, respectively. The constants c_1 and c_2, depending on C, are the average of u_0 inside C and outside C. Generally, we choose $\lambda_1 = \lambda_2 = \lambda$ in practice. In order to solve this minimization problem, they replace the unknown curve C by the level set function ϕ, and then rewrite the above function as follows

$$E^{PC}(\phi,c_1,c_2)=\lambda_1\int_\Omega\left|u_0-c_1\right|^2 H(\phi)dxdy+\lambda_2\int_\Omega\left|u_0-c_2\right|^2(1-H(\phi))dxdy + \mu\int_\Omega \delta(\phi)\left|\nabla\phi\right|dxdy,\ \ x\in\Omega \quad (3)$$

H(Φ) is the Heaviside function and $\delta(\phi)$ is the Dirac function. Generally, the regularized versions are selected as follows:

$$H_\varepsilon(z)=\frac{1}{2}(1+\frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)),\ \ z\in R \quad (4)$$

$$\delta_\varepsilon(z)=\frac{1}{\pi}\frac{\varepsilon}{\varepsilon^2+z^2},\quad z\in R \quad (5)$$

By minimizing Eq.(3), we solve c_1 and c_2 as follows:

$$c_1(\phi)=\frac{\int_\Omega u_0(x,y).H(\phi)dxdy}{\int_\Omega H(\phi)dxdy} \quad (6)$$

$$c_2(\phi)=\frac{\int_\Omega u_0(x,y).(1-H(\phi))dxdy}{\int_\Omega(1-H(\phi))dxdy} \quad (7)$$

The gradient descent flow equation for functional (3) is obtained as follow

$$\frac{\partial\phi}{\partial t}=\delta(\phi)\left[\mu\nabla\left(\frac{\nabla\phi}{|\nabla\phi|}\right)-\nu-\lambda_1(u_0-c_1)^2+\lambda_2(u_0-c_2)^2\right] \tag{8}$$

In PC model, the optimal image u_0 is approximated by a piecewise constant function.However,this fitting term is not accurate, which limit the application of PC model. In addition, re-initialization step is adopted for maintaining stable curve evolution and ensuring more precise results, which lead to the extraordinary time consumption

IV. FRACTIONAL ORDER DIFFERENTIATION

Fractional order differentiation is proposed by Leibniz and Hospital in 1965.Compared to the integer order differentiation, the fractional order differentiation have the non-local property. The enhancement and preservation of low frequency information by fractional order differentiation is superior to that by first order one. Since these advantages, fractional order differentiation is widely used in many image processing problems. Fourier domain analysis is very much useful in image processing problems. So in this paper we use Fourier domain fractional order differentiation since it is easy to implement. Consider a two dimensional function $f(x,y)\in L^2(R^2)\cap C(R^2)$ and its fourier transform is defined as

$$\hat{f}(w_1,w_2)=\int_{R^2} f(x,y)\exp(-j(w_1x+w_2y)dxdy \tag{9}$$

The fractional order partial derivatives of $f(x,y)\in L^2(R^2)\cap C(R^2)$ are defined as follows

$$D_x^{\vee} f(x,y)=F^{-1}((jw_1)^{\vee}\hat{f}(w_1,w_2))$$

$$D_y^{\vee} f(x,y)=F^{-1}((jw_1)^{\vee}\hat{f}(w_1,w_2)) \tag{10}$$

The enhancement and preservation of low frequency information by fractional order differentiation is superior to that by first order one. Since these advantages, fractional order differentiation is widely used in image processing area.

A. THE FRACTIONAL ORDER MODEL

A fractional order fitting model is implemented in the level set model for image segmentation, in which an adaptive regularization term and a fractional order fitting term are used. By using this model, we fit the original image by capturing global and fractional information, and is applied in segmentation to capture the low frequency information also. A penalty term is add into this model to keep the evolving level set function as an approximate signed distance. In this approach the energy functional consists of three parts: fitting term E^F , regularization term E^R and the penalty term E^P

B. THE FITTING TERM

The new fitting term is derived by using the global image fitting term which is based on the Piecewise constant model and the fractional order fitting term which is based on the fractional order differentiation in the Fourier domain applied in the piecewise constant (PC) model. The new fitting term E_{New}^F consist of two parts: global fitting term E^{GF} and the fractional order fitting term E^{FF}.

$$E_{New}^F=E^{GF}+E^{FF} \tag{11}$$

In order to obtain the gradient information of the global image fitting term the energy term in Eq. (2) is converted in to the level set model by using level set function and assume $\lambda_1=\lambda_2=\lambda$, keep c_1 and c_2 keep constant

$$\nabla E^{GF}=\lambda\delta(\phi)\left[\left(u_0-c_1\right)^2-\left(u_0-c_2\right)^2\right] \tag{12}$$

The global fitting term assumes that the image intensity is piecewise constant, which is not well in the images with high noise level or intensity inhomogeneity. In order to fit the image well the fractional order fitting term is incorporate into the global term. The fractional order fitting term is defined as flows

$$E^{FF}(C,d_1,d_2)=\beta\int_{inside(C)}\left|D_x^{\vee}u_0-d_1\right|^2dxdy+\beta\int_{outside(C)}\left|D_x^{\vee}u_0-d_2\right|^2dxdy \tag{13}$$

There are two basic ideals behind the selection of fractional order differentiation .First, the intensity inhomogeneity is slowly varying in the image domain. Its spectrum infrequency domain will be concentrated in the low-frequency area. When the value of ν varies from0 to 1,fractional order differentiation is able to preserve and enhance the low frequency information, which is superior to the first order differentiation [39]. Second, fractional order differentiation can improve immunity to noise, which can be interpreted in terms of robustness to noise in general. This property is very important for medical images and infrared images that suffer from noise. According to these reasons, the fractional order fitting term is used in the proposed model. In the same manner the gradient obtained in the global fitting term, to obtain the gradient information of the fractional order fitting term E^{FF} using the flowing level set equation

$$E^{FF}(\phi, d_1, d_2) = \beta\int_{\Omega}\left|D_x^{\vee}u_0 - d_1\right|^2 H(\phi)dxdy + \beta\int_{\Omega}\left|D_x^{\vee}u_0 - d_2\right|^2(1 - H(\phi))dxdy \tag{14}$$

The terms d_1 and d_2 are defined as follows

$$d_1(\phi) = \frac{\int_{\Omega} D_x^{\vee}u_0.H(\phi)dxdy}{\int_{\Omega} H(\phi)dxdy} \tag{15}$$

$$d_2(\phi) = \frac{\int_{\Omega} D_x^{\vee}u_0.(1 - H(\phi))dxdy}{\int_{\Omega}(1 - H(\phi))dxdy} \tag{16}$$

By keeping d_1 and d_2 fixed, the gradient obtained is

$$\nabla E^{FF} = \beta\delta(\phi)\left[\left(D_x^{\vee}u_0u_0 - d_1\right)^2 - \left(D_x^{\vee}u_0u_0 - d_2\right)^2\right] \tag{17}$$

Thus, the overall fitting term for the proposed model is be further described as follows in term of the level set function

$$E^F(\phi, c_1, c_2, d_1, d_2) = E^{GF}(\phi, c_1, c_2) + E^{FF}(\phi, d_1, d_2) \tag{18}$$

This fitting term include the global fitting term and the fractional fitting term. The parameters λ and β controls the weighting of two fitting terms to form a suitable fitting force. The corresponding gradient is

$$\nabla E = \nabla E^{GF} + \nabla E^{FF} \tag{19}$$

This new fitting term fits the original image more accurately, which is useful for the extraction of the tumor region.

C. REGULARIZATION TERM

In image segmentation, regularization is imposed on the level set curve to diminish the influence of noise and smooth the level set function. An adaptive length regularization is proposed in the fractional order fitting model.

$$E^R(\phi) = \int_{\Omega}\delta(\phi)p\left(\left|\nabla G_{\sigma} * u_0\right|\right)\left|\nabla\phi\right|dxdy \tag{20}$$

The gradient of E^R is given by

$$\nabla E^R = -\delta(\phi)div\left(p\left(\left|\nabla G_{\sigma} * u_0\right|\right).\frac{\nabla\phi}{\left|\nabla\phi\right|}\right) \tag{21}$$

This regularization can be a geometric weighted regularization on the curve C, which can perform the suitable smooth force on the level set function.

D. LEVEL SET FORMULATION

In many situations, the level set function will develop shocks, very sharp and/or flat shape during the processing of the evolution, which in turn makes further computation highly inaccurate in numerical approximations . To solve this problem, it is necessary to keep the level set function as an approximate signed distance function. The distance regularization term is used to penalize the deviation of the level set function ϕ from a signed distance function is given by

$$E^p = \frac{1}{2}\int_{\Omega}\left(\left|\nabla\phi\right| - 1\right)^2 dxdy \tag{22}$$

The gradient of E^P is given by

$$\nabla E^p = div\left(\frac{\nabla\phi}{\left|\nabla\phi\right|}\right) - \nabla\phi \tag{23}$$

This extra internal energy avoids directly using the re-initialization step to keep the level set function as a signed distance function, which plays an important role in this model. With the above defined forces, i.e. fitting term E^F , adaptive regularization term E^R and penalty term E^P, the overall energy functional is defined in terms of the level set formulation as follows

$$E(\phi, c_1, c_2, d_1, d_2) = E^F(\phi, c_1, c_2, d_1, d_2) + E^P(\phi) + E^R(\phi) \tag{24}$$

where μ is positive parameter. Thus, image segmentation problem is solved by finding a solution that minimizes the functional E by evolving the level set function ϕ. By the linearity of gradient operator, the gradient of the function E is given by

$$\nabla E = \nabla E^{P} + \nabla E^{F} + \mu \nabla E^{R} \qquad (25)$$

Using Eqns. (19), (21) and (23), the gradient descent flow equation for the energy functional E is

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[-\left(\lambda\left(u_0 - c_1\right)^2 + \beta\left(D_x^{v} u_0 - d_1\right)^2\right) + \left(\lambda\left(u_0 - c_2\right)^2 + \beta\left(D_x^{v} u_0 - d_2\right)^2\right)\right]$$
$$+ \Delta\phi - div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + \mu\delta(\phi) div\left(p\left(|\nabla G_{\sigma} * u_0|\right).\frac{\nabla\phi}{|\nabla\phi|}\right) \qquad (26)$$

V. Results and Discussion

To demonstrate the performance of the proposed fractional order level set scheme, compared to the classical level set techniques, we use CT liver images. Sometimes the images accompany background noise, which potentially has a negative impact on correct settlements of the contours. Due to the existence of other organs adjacent to liver with same intensity the contour searching may be strongly affected by the combination of noise and other edges. To achieve ideal image segmentation the method will have to make great efforts to get through the interference of image noise and neighboring structures. In this section three methods are employed for performance and comparison, i.e. the original level set, the level set without re-initialization by Li et al. and the fractional order approach. The whole implementation is run on a PC with a 1.5GHz Intel(R) Pentium(R) CPU.

Fig. 1 demonstrates that the level set approach without re-initialization by Li et al. outperforms the original level set method by Osher and Sethian in tumor detection. Although the approach by Li et al. has achieved a great success, evidence shows that this algorithm requires further improvements. For example, it has been observed that in a noisy image with ambiguous boundaries this approach cannot ideally locate the tumor boundaries (Fig. 2).

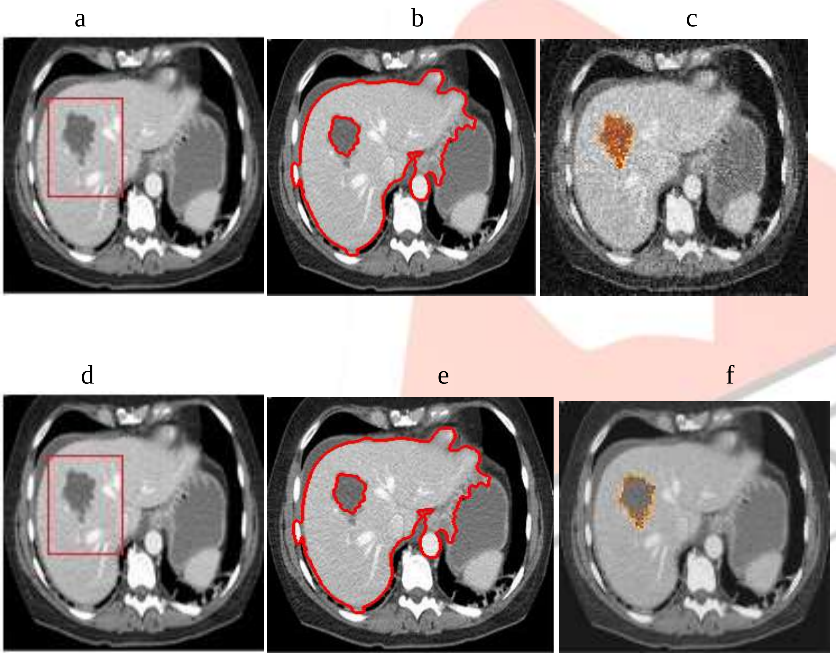

Fig. 1. Performance comparison between the original level set method (first row) and the level set scheme without re-initialization (second row). λ=2.5,μ=0.4,ν=1 and time step = 3. (a) Initial, (b) iteration 200, (c) iteration 350, (d) initial, (e) iteration 200, (f) iteration 300.

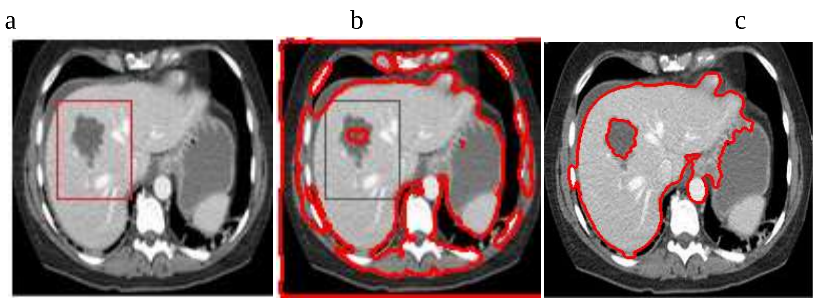

Fig. 2. Performance of the level set scheme without re-initialization. λ= 2.5, μ= 0:4, ν= 1, and time step = 3.
(a) Initial, (b) iteration 200, (c) iteration 300.

Fig. 3 illustrates individual performance of using the original level set algorithm and the fractional order approach, where the former is shown on the first row while the latter is demonstrated on the second row. The original level set method is distracted by the near edges of the liver tumors. Fractional order method gets around this problem, and finally addresses on the ideal tumor boundary. This indicates that our approach is able to ignore the interference of neighboring organs edges, and fast approach to the tumor boundary .

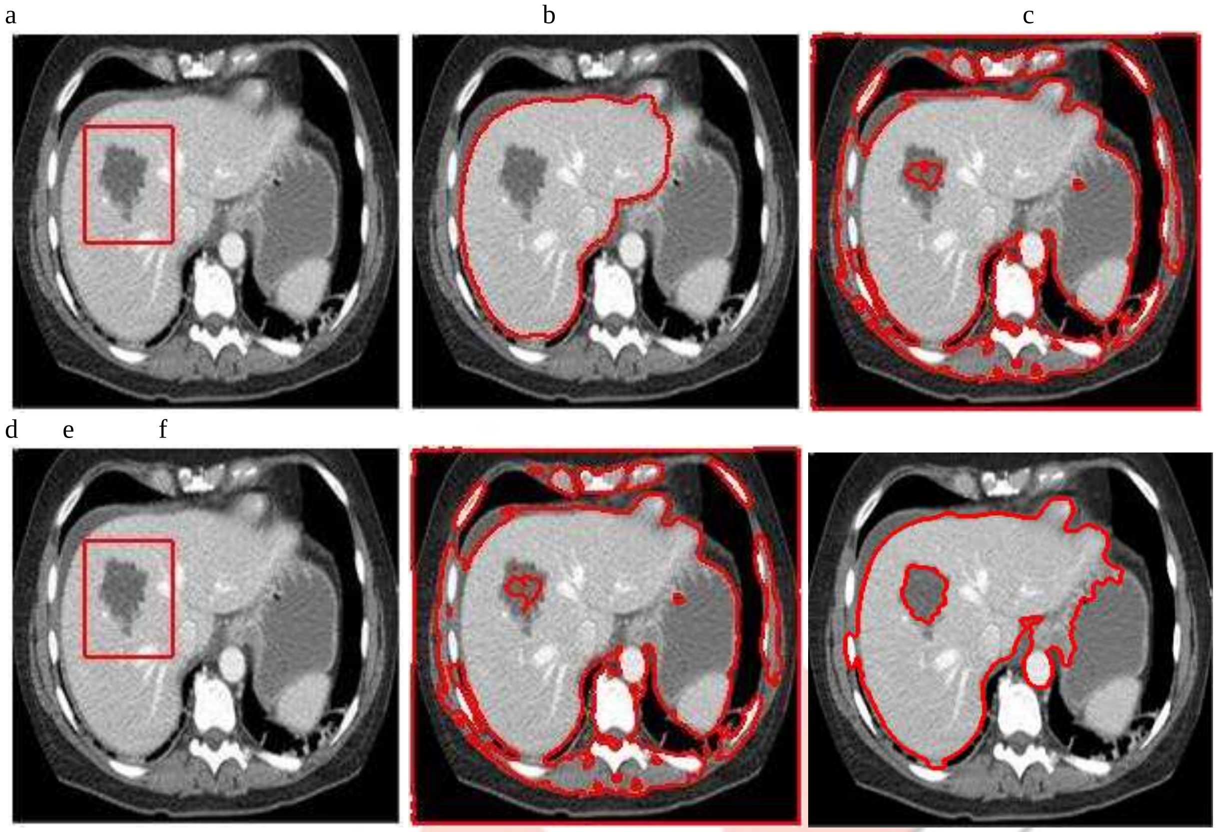

Fig. 3. Performance comparison of the original level set scheme and the fractional order approach .λ=3,μ=0.3, ν=2, and time step =2.5. (a) Initial, (b) iteration 150, (c) iteration 300, (d) initial, (e) iteration 150, (f) iteration 250.

Segmentation of multiple tumors is shown in Fig. 4. Multiple tumors exist in the image, and different tumors have different intensities. As seen in this figure, the original level set scheme cannot locate the edges of the tumor regions in the liver CT image. The edges have not been correctly outlined. These edges are vague somehow so that the energy minimization in the classical level set function cannot be ideally achieved. The model proposed by Li et al. has a better outcome of tumor detection. The accuracy of the edge detection still needs to be improved.. This may need more efforts to optimize the contour settlement. On the other hand, it is clear that the fractional order approach has a better performance than these two methods in terms of edge detection (images on the third row).

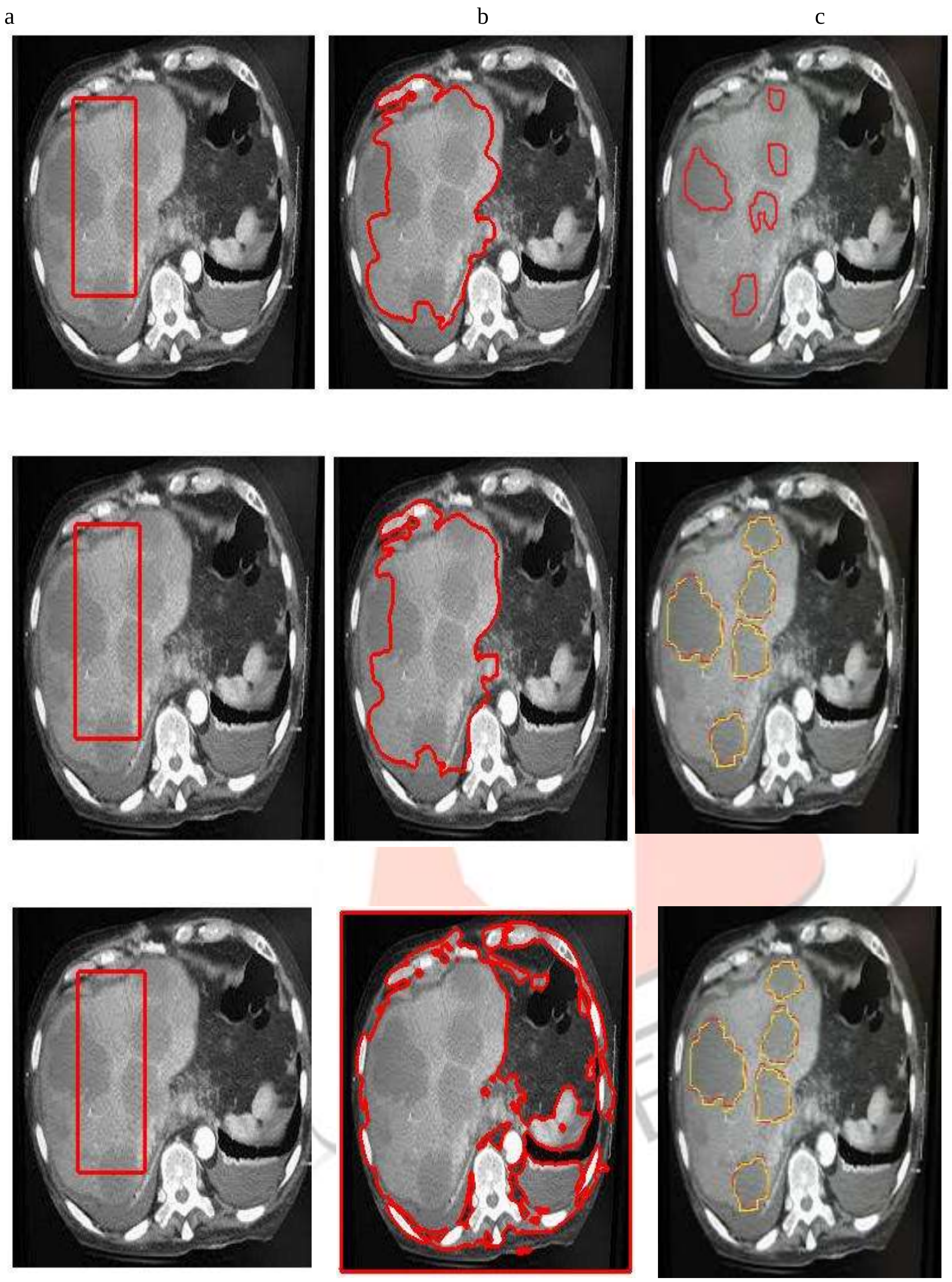

Fig. 4. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and the Fractional order approach (third row). λ=2.5,μ=0.4, ν=2, and time step=2. 5 (a) Initial, (b) iteration 150, (c) iteration300.

This experiment utilizes a CT liver tumor image where the tumor region is much brighter. The target is to outline the correct tumor region using the available methods. Fig. 5 demonstrates that the Fractional order level set scheme has the best performance in tumor detection, where Li's method leads to errors in detecting the exact segmentation of the tumor region . Meanwhile, the original level set method cannot correctly locate the tumor boundary, resulting from the side-effects raised by the image noise.

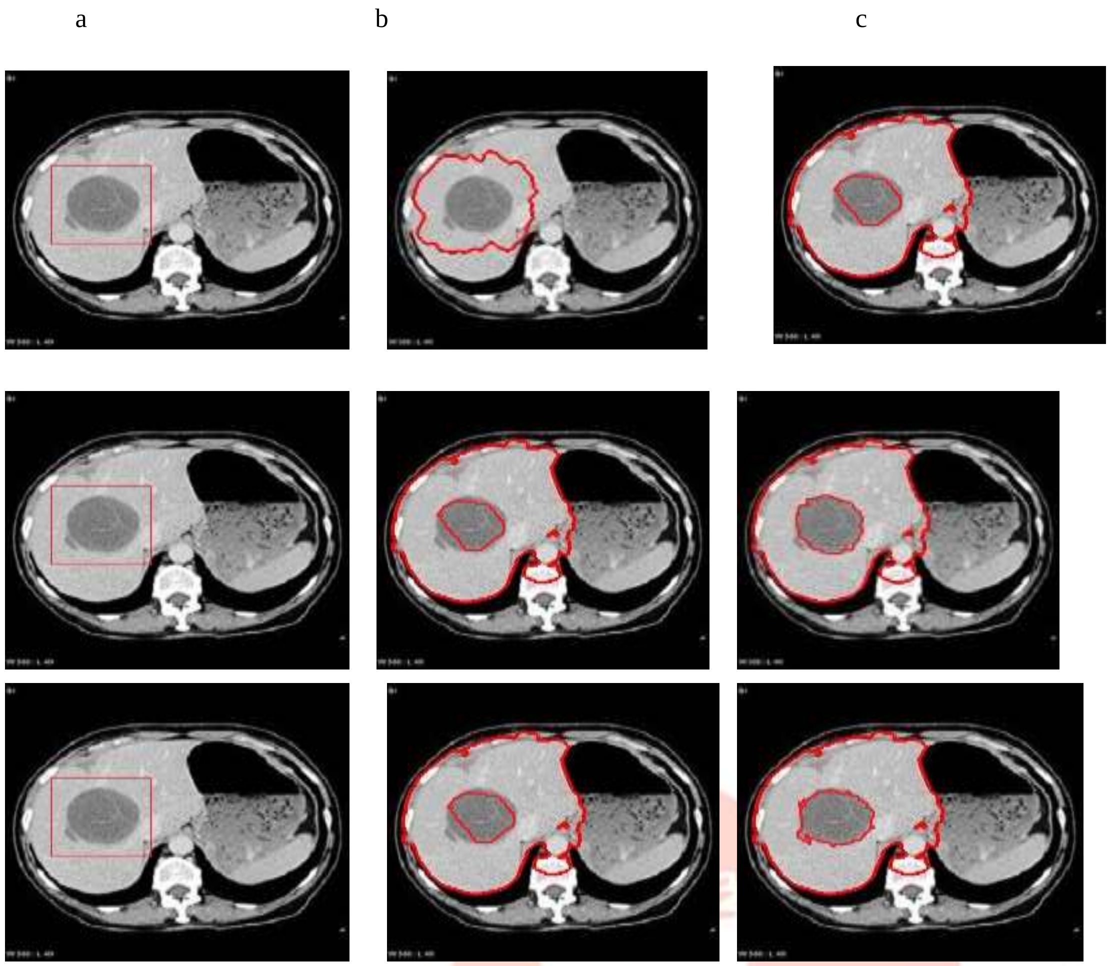

Fig. 5. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and the fractional order approach (third row). λ=2.5, μ=0.4, ν=2, and time step =2. 5. (a) Initial, (b) iteration 250, (c) iteration 350.

Fig.6 demonstrate how these methods cope with the noisy background. The original level set method failed to segment the tumor region. Meanwhile, Li's model and the Fractional order method have been successfully segment the tumor region.. Interestingly, we observe that at iteration 150 fractional order scheme seems reluctant to pick up a concave area. However, it recovers very soon and successfully addresses on the exact tumor region at iteration 350. This may be due to the oscillation of the energy terms during the evolution (before iteration 350). Taking a closer look at the results from Li's model, we observe that this model has less detecting accuracy on the block corners of the tumor region than the fractional model approach.

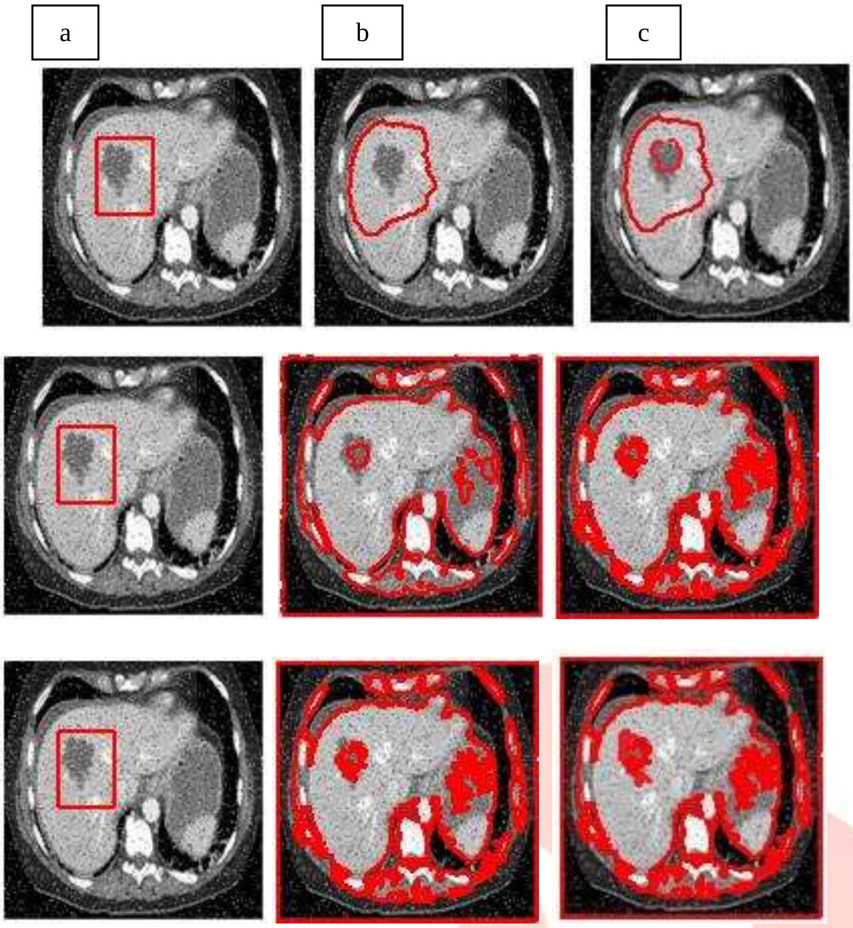

Fig. 6. Performance comparison between the original level set method (first row), the level set scheme without re-initialization (second row), and fractional model approach (third row). λ=2.5, μ=0.4, ν=2, and time step =2. 5. (a) Initial, (b) iteration 350, (c) iteration

III. CONCLUSION

An effective segmentation tool for liver CT images based on active contour model with fractional order fitting term is presented. Level set scheme is used for the analysis. A fractional order fitting term is included in the energy functional model. By incorporating the fractional order fitting term, the novel fitting term can describe the original image more accurately, and be robustness to noise. This model is efficient for CT liver images, and intensity inhomogeneous images.

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