© 2017 IJEDR | Volume 5, Issue 2 | ISSN: 2321-9939

A class of efficient Ratio type estimators for the Estimation of Population Mean Using the auxilliary information in survey sampling

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Abstract: - Estimation of the population mean is the persistent issue in sampling practices and many efforts have been made by various statisticians to improve the precision of the estimates by using the auxilliary information. In this paper we proposed a class of efficient estimators for estimating the population mean in SRSWOR by using the auxilliary information of coefficient of skewness and population deciles of the auxilliary variable. The properties associated with the proposed estimators are analysed through MSE and bias. We also provide an empirical study for illustration and verification.

Keywords: Coefficient of Skewness; Population Deciles; Ratio-type estimators; Mean square error; Bias; Efficiency.

1. INTRODUCTION

The use of auxiliary information in survey sampling has its own eminent role both at design and estimation stage. It is well known that the use of auxiliary information at the estimation stage improves the precision of estimates of the population mean or total. Ratio, product and regression methods of estimation are good examples in this context. If the correlation between study variate *y* and the auxiliary variate *x* is positive (high), the ratio method of estimation envisaged by Cochran is used. So in this paper we also take the advantage of correlation between study variable and auxilliary variable and thus by proposing the ratio type estimators by using the auxilliary information of coefficient of skewness and population deciles of auxilliary variable.

Consider a finite population $U=\{U_1,U_2,U_3,...,U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured of U_i, $i=1,2,3,...,N$ giving a vector $Y=\{Y_1,Y_2,Y_3,...,Y_N\}$. The objective is to estimate population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$ on the basis of a random sample. When the population parameters of the auxiliary variable, such as population mean, kurtosis, skewness, coefficient of variation, median, quartiles, correlation coefficient, deciles etc., are known, ratio estimators and their modifications are available in the literature which perform better than the usual sample mean under the simple random sampling without replacement (SRSWOR).
The notations used in this paper can be described as follows:

NOMENCLATURE

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N	Population size	n	Sample size
$f=n/N$	Sampling fraction	Y	Study variable
X	Auxiliary variable	$\bar{X},\bar{Y}$	Population means
$\bar{x},\bar{y}$	Sample means	x,y	Sample totals
s_x,s_y	Population standard deviations	s_{xy}	Population covariance between
c_x,c_y	Coefficient of variation	ρ	Correlation coefficient
$B(.)$	Bias of the Estimator	$MSE(.)$	Mean square error of the estimator
$\hat{\bar{Y}}_i$	Existing modified ratio estimator of $\bar{Y}$	$\hat{\bar{Y}}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$

D_k $k=1,2,....,10$ Deciles, β_2 Kurtosis, β_1 Skewness

Subscript

i For existing estimators, j For proposed estimators

Based on the above mentioned notations, the mean ratio estimator for estimating the Population mean, $\bar{Y}$, of the study variable Y is defined as

$$\hat{\bar{Y}}_r=\frac{\bar{y}}{\bar{x}}\bar{X} \qquad (1)$$

The bias, related constant and the mean squared error (MSE) of the ratio estimator are respectively given by

$$B(\hat{\bar{Y}}_r)=\frac{(1-f)}{n}\frac{1}{\bar{X}}(RS_x^2-\rho S_xS_Y) \quad R=\frac{\bar{Y}}{\bar{X}} \quad MSE(\hat{\bar{Y}}_r)=\frac{1-f}{n}(S_y^2+R^2S_x^2-2R\rho S_xS_Y)$$

The ratio estimator given in (1) is used for improving the precision of the estimate of the population mean as compared to usual sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable. Cochran (1940) suggested a classical ratio type estimator for the estimation of finite population mean using one auxiliary variable under simple random sampling scheme. Murthy (1967) proposed a product type estimator to estimate the population mean or total of study variable by using auxiliary information when coefficient of correlation is negative. Rao (1991) introduced difference type ratio estimator that outperforms conventional ratio and linear regression estimators. Upadhyaya & Singh (1999) modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate. Singh & Tailor (2003) proposed a family of estimators using known values of some parameters by using SRSWOR for estimation of population mean of the study variable. Sisodia & Dwivedi (1981) and Singh *et al.* (2004) utilized coefficient of variation of the auxiliary variate. Further improvements are achieved by introducing a large number of modified ratio estimators with the use of known coefficient of variation, kurtosis, skewness, median, coefficient of correlation, decile (see Subramani and Kumarpandiyan, 2012 a, b and c).
The organization of the rest of the article is as follows: Section 2 provides a description of the existing estimators. The structure of suggested modified linear regression type ratio estimator and the efficiency comparison of the suggested estimator with the existing estimators are presented in Section 3. Section 4 consists of an empirical study of proposed estimators. Finally, Section 5 summarizes the findings of the study.

2. Existing Ratio Estimators

Kadilar and Cingi (2004) suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.
Kadilar & Cingi (2004) estimators are given by

$$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}, \quad \hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x), \quad \hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2),$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x), \quad \hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2),$$

The biases, related constants and the MSE for Kadilar and Cingi (2004) estimators are respectively as follows:

$$B(\hat{\bar{Y}}_1) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2, \quad R_1 = \frac{\bar{Y}}{\bar{X}} \qquad MSE(\hat{\bar{Y}}_1) = \frac{(1-f)}{n}(R_1^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_2) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X}+C_x)} \qquad MSE(\hat{\bar{Y}}_2) = \frac{(1-f)}{n}(R_2^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_3) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X}+\beta_2)} \qquad MSE(\hat{\bar{Y}}_3) = \frac{(1-f)}{n}(R_3^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_4) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2+C_x)} \qquad MSE(\hat{\bar{Y}}_4) = \frac{(1-f)}{n}(R_4^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_5) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x+C_x)} \qquad MSE(\hat{\bar{Y}}_5) = \frac{(1-f)}{n}(R_5^2S_x^2 + S_y^2(1-\rho^2)).$$

Kadilar and Cingi (2006) developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho), \quad \hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho), \quad \hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho), \quad \hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi (2006) estimators are respectively given by

$$B(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X}+\rho} \qquad MSE(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}(R_6^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho} \qquad MSE(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}(R_7^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x} \qquad MSE(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}(R_8^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho} \qquad MSE(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}(R_9^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2} \qquad MSE(\hat{\bar{Y}}_{10}) = \frac{(1-f)}{n}(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)).$$

Yan and Tian (2010) proposed some modified ratio estimators using coefficient of skewness and kurtosis as follows:

$$\hat{\bar{Y}}_{11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)}(\bar{X} + \beta_1), \qquad \hat{\bar{Y}}_{12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + \beta_2)}(\bar{X}\beta_1 + \beta_2).$$

The biases, related constants and the MSE for Yan and Tian (2010) estimators are respectively given by

$$B(\hat{\bar{Y}}_{11}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{11}^2, \quad R_{11} = \frac{\bar{Y}}{\bar{X} + \beta_1} \qquad MSE(\hat{\bar{Y}}_{11}) = \frac{(1-f)}{n}(R_{11}^2 S_x^2 + S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{12}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{12}^2, \quad R_{12} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \beta_2} \qquad MSE(\hat{\bar{Y}}_{12}) = \frac{(1-f)}{n}(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)).$$

Yan and Tian (2010) showed that the use of coefficient of skewness and coefficient of kurtosis, respectively, provides better estimates for the population mean in comparison to the usual ratio estimator and numerous existing estimators.

3. Proposed Modified Ratio Estimator

Motivated by the mentioned estimators in Section 2, we propose new class of efficient ratio type estimators using the linear combination of coefficient of skewness and population deciles.

The proposed estimators is given below:

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_1)}(\bar{X}\beta_1 + D_1). \qquad \hat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_2)}(\bar{X}\beta_1 + D_2).$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_3)}(\bar{X}\beta_1 + D_3). \qquad \hat{\bar{Y}}_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_4)}(\bar{X}\beta_1 + D_4).$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_5)}(\bar{X}\beta_1 + D_5). \qquad \hat{\bar{Y}}_{p6} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_6)}(\bar{X}\beta_1 + D_6).$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_7)}(\bar{X}\beta_1 + D_7). \qquad \hat{\bar{Y}}_{p8} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_8)}(\bar{X}\beta_1 + D_8).$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_9)}(\bar{X}\beta_1 + D_9). \qquad \hat{\bar{Y}}_{p10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + D_{10})}(\bar{X}\beta_1 + D_{10}).$$

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows:
MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x},\bar{y}) \cong h(\bar{X},\bar{Y}) + \frac{\partial h(c,d)}{\partial c}\Big|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{\partial h(c,d)}{\partial d}\Big|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y}) \qquad (3.1)$$

Where $h(\bar{x},\bar{y}) = \hat{R}_{pj}$ and $h(\bar{X},\bar{Y}) = R.$

As shown in Wolter (1985), (3.1) can be applied to the proposed estimators in order to obtain MSE equation as follows:

$$\hat{R}_{pj} - R \cong \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x}\beta_1 + D_k)}{\partial \bar{x}}\Big|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x}\beta_1 + D_k)}{\partial \bar{y}}\Big|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y})$$

$$\cong -\left(\frac{\bar{y}}{(\bar{x}\beta_1 + D_k)^2} + \frac{b(\bar{X}\beta_1 + D_k)}{(\bar{x}\beta_1 + D_k)^2}\right)\Big|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{1}{(\bar{x}\beta_1 + D_k)}\Big|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y})$$

$$E(\hat{R}_{pj} - R)^2 \cong \frac{(\bar{Y} + B(\bar{X}\beta_1 + D_k))^2}{(\bar{X}\beta_1 + D_k)^4}V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X}\beta_1 + D_k))}{(\bar{X}\beta_1 + D_k)^3}Cov(\bar{x},\bar{y}) + \frac{1}{(\bar{X}\beta_1 + D_k)^2}V(\bar{y})$$

$$\cong \frac{1}{(\bar{X}\beta_1 + D_k)^2}\left\{\frac{(\bar{Y} + B(\bar{X}\beta_1 + D_k))^2}{(\bar{X}\beta_1 + D_k)^2}V(\bar{x}) - \frac{2(\bar{Y} + B(\beta_1 + D_k)}{(\bar{X}\beta_1 + D_k)}Cov(\bar{x},\bar{y}) + V(\bar{y})\right\}$$

Where $B=\frac{s_{xy}}{s_x^2}=\frac{\rho s_x s_y}{s_x^2}=\frac{\rho s_y}{s_x}$. Note that we omit the difference of $(E(b)-B)$.

$$MSE(\bar{y}_{pj})=(\bar{X}\beta_1+D_k)^2E(\hat{R}_{pj}-R)^2\cong\frac{(\bar{Y}+B(\bar{X}\beta_1+D_k))^2}{(\bar{X}\beta_1+D_k)^2}V(\bar{x})-\frac{2(\bar{Y}+B(\bar{X}\beta_1+D_k))}{(\bar{X}\beta_1+D_k)}Cov(\bar{x},\bar{y})+V(\bar{y})$$

$$\cong\frac{\bar{Y}^2+2B(\bar{X}\beta_1+D_k)\bar{Y}+B^2(\bar{X}\beta_1+D_k)^2}{(\bar{X}\beta_1+D_k)^2}V(\bar{x})-\frac{2\bar{Y}+2B(\bar{X}\beta_1+D_k)}{(\bar{X}\beta_1+D_k)}Cov(\bar{x},\bar{y})+V(\bar{y})$$

$$\cong\frac{(1-f)}{n}\left\{\left(\frac{\bar{Y}^2}{(\bar{X}\beta_1+D_k)^2}+\frac{2B\bar{Y}}{(\bar{X}\beta_1+D_k)}+B^2\right)S_x^2-\left(\frac{2\bar{Y}}{(\bar{X}\beta_1+D_k)}+2B\right)S_{xy}+S_y^2\right\}$$

$$\cong\frac{(1-f)}{n}(R^2S_x^2+2BRS_x^2+B^2S_x^2-2RS_{xy}-2BS_{xy}+S_y^2)$$

$$MSE(\bar{y}_{pj})\cong\frac{(1-f)}{n}(R^2S_x^2+2R\rho S_xS_y+\rho^2S_y^2-2R\rho S_xS_y-2\rho^2S_y^2+S_y^2$$

$$\cong\frac{(1-f)}{n}(R^2S_x^2-\rho^2S_y^2+S_y^2)\cong\frac{(1-f)}{n}(R^2S_x^2+S_y^2(1-\rho^2)$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{pj})\cong\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_j^2$$

Thus the bias and MSE of the proposed estimators is given below:

$$B(\hat{\bar{Y}}_{pj})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_j^2,\quad R_{j1}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+D_k}\qquad MSE(\hat{\bar{Y}}_{pj})=\frac{(1-f)}{n}(R_j^2S_x^2+S_y^2(1-\rho^2)),$$

Where $j=1,2,\ldots,10$ and $k=1,2,\ldots,12$.

4. Efficiency Comparisons

4.1. Comparisons with existing ratio estimators

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\hat{\bar{Y}}_{pj})\le MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2S_x^2+S_y^2(1-\rho^2)\le\frac{(1-f)}{n}(R_i^2S_x^2+S_y^2(1-\rho^2)),$$

$$R_{pj}^2S_x^2\le R_i^2S_x^2,$$

$$R_{pj}\le R_i,$$

Where $j=1,2,\ldots.,10$ and $i=1,2,\ldots.,12$.

5. Empirical Study

The performances of the suggested ratio estimators are evaluated and compared with the usual ratio estimator and the mentioned ratio estimators in Section 2 by using natural Population.

The percentage relative efficiency (PREs) of the proposed estimators (p), with respective to the existing estimators (e), are computed as

$$PRE=\frac{MSE\ of\ Existing\ \ Estimator}{MSE\ of\ \ propoesd\ estimator}\times 100$$

The statistics of population taken from Singh and Chaudhary (1986) is given in table 1

Table 1

Parameters	Population 1	Parameters	Population 1	Parameters	Population 1
N	34	S_x	150.2150	D_4	111.20
n	20	C_x	0.7531	D_5	142.50
$\bar{Y}$	856.4117	β_2	1.0445	D_6	210.20
$\bar{X}$	199.4412	β_1	1.1823	D_7	264.50

ρ	0.4453	D_1	60.60	D_8	304.40
S_y	733.1407	D_2	83.00	D_9	373.20
C_y	0.8561	D_3	102.70	D_{10}	634.00

Table 2: The Statistical Analysis of the Estimators for this Population

Estimators	Pop I			Estimators	Pop I		
	Constant	Bias	MSE		Constant	Bias	MSE
$\hat{\bar{Y}}_1$	4.294	10.002	17437.65	$\hat{\bar{Y}}_{12}$	4.275	9.914	17362.26
$\hat{\bar{Y}}_2$	4.278	9.927	17373.31	$\hat{\bar{Y}}_{p1}$	3.416	6.330	14293.17
$\hat{\bar{Y}}_3$	4.272	9.898	17348.62	$\hat{\bar{Y}}_{p2}$	3.176	5.472	13558.08
$\hat{\bar{Y}}_4$	4.279	9.930	17376.04	$\hat{\bar{Y}}_{p3}$	2.991	4.853	13028.48
$\hat{\bar{Y}}_5$	4.264	9.865	17319.75	$\hat{\bar{Y}}_{p4}$	2.918	4.619	12827.33
$\hat{\bar{Y}}_6$	4.285	9.957	17399.52	$\hat{\bar{Y}}_{p5}$	2.676	3.886	12199.85
$\hat{\bar{Y}}_7$	4.281	9.943	17387.08	$\hat{\bar{Y}}_{p6}$	2.270	2.796	11266.17
$\hat{\bar{Y}}_8$	4.258	9.834	17294.19	$\hat{\bar{Y}}_{p7}$	2.024	2.222	10774.62
$\hat{\bar{Y}}_9$	4.285	9.960	17401.14	$\hat{\bar{Y}}_{p8}$	1.874	1.906	10503.91
$\hat{\bar{Y}}_{10}$	4.244	9.771	17239.66	$\hat{\bar{Y}}_{p9}$	1.663	1.499	10155.96
$\hat{\bar{Y}}_{11}$	4.269	9.885	17336.98	$\hat{\bar{Y}}_{p10}$	1.164	0.735	9501.029

Table 3: PRE of the Proposed Estimators with the Estimators in Literature for this population.

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$	$\hat{\bar{Y}}_{p10}$
$\hat{\bar{Y}}_1$	121.999	128.614	133.842	135.941	142.933	154.778	161.840	166.011	171.698	183.534
$\hat{\bar{Y}}_2$	121.549	128.139	133.348	135.439	142.405	154.207	161.242	165.398	171.065	182.857
$\hat{\bar{Y}}_3$	121.377	127.957	133.159	135.247	142.203	153.988	161.013	165.163	170.822	182.597
$\hat{\bar{Y}}_4$	121.568	128.160	133.369	135.461	142.428	154.232	161.268	165.424	171.092	182.885
$\hat{\bar{Y}}_5$	121.175	127.744	132.937	135.022	141.966	153.732	160.745	164.888	170.537	182.293
$\hat{\bar{Y}}_6$	121.733	128.333	133.549	135.644	142.620	154.440	161.486	165.648	171.323	183.133
$\hat{\bar{Y}}_7$	121.646	128.241	133.454	135.547	142.518	154.330	161.370	165.529	171.200	183.002
$\hat{\bar{Y}}_8$	120.996	127.556	132.741	134.823	141.757	153.505	160.508	164.645	170.286	182.024
$\hat{\bar{Y}}_9$	121.744	128.345	133.562	135.656	142.634	154.454	161.501	165.663	171.339	183.150
$\hat{\bar{Y}}_{10}$	120.614	127.154	132.322	134.397	141.310	153.021	160.002	164.126	169.749	181.450
$\hat{\bar{Y}}_{11}$	121.295	127.871	133.069	135.156	142.108	153.885	160.905	165.052	170.707	182.474
$\hat{\bar{Y}}_{12}$	121.472	128.058	133.263	135.353	142.315	154.109	161.140	165.293	170.956	182.740

Conclusion

From the above empirical study we reveal that by proposing the class of ratio type estimators in SRSWOR by using the population deciles and coefficient of skewness as auxilliary information are found more efficient than the existing estimators as their MSE and bias is lower than the existing estimators and hence we strongly recommend that our proposed estimators preferred over existing estimators for practical applications.

References

Cochran, W. G. (1940). The Estimation of the Yields of the Cereal Experiments by Sampling for the Ratio of Grain to Total Produce. *The Journal of Agric Science*, 30. 262-275.

Kadilar, C. & Cingi, H. (2004), 'Ratio estimators in simple random sampling', Applied Mathematics and Computation 151, 893–902.

Kadilar, C. & Cingi, H. (2006), 'An improvement in estimating the population mean by using the correlation coefficient', Hacettepe Journal of Mathematics and Statistics 35(1), 103–109.

Murthy, M. (1967), Sampling Theory and Methods, 1 edn, Statistical Publishing Society, India.

Rao, T. J. (1991), 'On certain methods of improving ratio and regression estimators', Communications in Statistics-Theory and Methods 20(10), 3325–3340.

Singh, D. & Chaudhary, F. S. (1986), Theory and Analysis of Sample Survey Designs, 1 edn, New Age International Publisher, India.

Singh, H. P. & Tailor, R. (2003), 'Use of known correlation coefficient in estimating the finite population means', Statistics in Transition 6(4), 555–560.

Singh, H. P., Tailor, R., Tailor, R. & Kakran, M. (2004), 'An improved estimator of population mean using power transformation', Journal of the Indian Society of Agricultural Statistics 58(2), 223–230.

Sisodia, B. V. S. & Dwivedi, V. K. (1981), 'A modified ratio estimator using coefficient of variation of auxiliary variable', Journal of the Indian Society of Agricultural Statistics 33(1), 13–18.

Subramani, J. & Kumarapandiyan, G. (2012a), 'Estimation of population mean using co- efficient of variation and median of an auxiliary variable', International Journal of Probability and Statistics 1(4), 111–118.

Subramani, J. & Kumarapandiyan, G. (2012b), 'Estimation of population mean using known median and co-efficient of skewness', American Journal of Mathematics and Statistics 2(5), 101–107.

Subramani, J. & Kumarapandiyan, G. (2012c), 'Modified ratio estimators using known median and co-efficient of kurtosis', American Journal of Mathematics and Statistics 2(4), 95– 100.

Upadhyaya, L. N. & Singh, H. (1999), 'Use of transformed auxiliary variable in estimating the finite population mean', Biometrical Journal 41(5), 627–636.

Wolter K.M. (1985). Introduction to Variance Estimation, Springer-Verlag.

Yan, Z. & Tian, B. (2010), 'Ratio method to the mean estimation using coefficient of skewness of auxiliary variable', Information Computing and Applications 106, 103–110.